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DSGE Model for Georgia: LEGO with Emerging Market Features

by Lasha Arevadze, Shalva Mkhatrishvili, Saba Metreveli, Giorgi Tsustkiridze, Tamar Mdivnishvili and Nika Khinashvili

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National Bank of Georgia

DSGE Model for Georgia: LEGO with Emerging Market Features

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Abstract

Last couple of decades of research has significantly advanced New Keynesian DSGE modeling. While each of such models faces its own important limitations, it can still contribute to robust policy analysis as long as we consolidate relevant macroeconomic features in it and remain conscious of the limitations. With this paper we are introducing a DSGE model for Georgia with features relevant for Emerging Market Economies (EMEs), characterized with large number of real and nominal imperfections. While some model features are already standard to existing DSGE frameworks, we also emphasize aspects particularly relevant to EMEs. These include dominant currency invoicing, forward premium puzzle, breakdown of Ricardian equivalence, impaired expenditure switching mechanism, decoupled domestic and imported price levels impacting real exchange rate trend, and other non-stationarities. Additionally, we distinguish between global financial centers and other trade partner economies. This LEGO model with these building blocks is planned to be expanded further with other properties in the future to make the model suitable for analyzing FX interventions and macroprudential policies, in addition to monetary and fiscal policies. The model is intended to become the workhorse model for macro-financial analysis in Georgia, representing a key addition to the NBG's existing FPAS, though its adaptability can extend to other country contexts as well.

JEL Codes: E17, E52, F10

Keywords: DSGE model; dollar-invoicing; modified UIP; monetary policy

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1 Introduction

Since switching to inflation targeting back in 2009, the National Bank of Georgia (NBG) has been basing its monetary policy decisions on forward-looking macroeconomic modeling extensively. The main virtue of the NBG's Forecasting and Policy Analysis System (FPAS) is that it provides a structuring device for discussions on what the monetary authority should do to achieve its inflation target (which is 3% for Georgia). In more technical terms, this means that the modeling approach should take the endogeneity of monetary policy into account (see Svensson, 1997 or Freedman and Laxton, 2009) - meaning that the main model would not be expected to forecast inflation for the long term in a traditional sense; rather it should forecast the monetary policy rate such that will guarantee inflation being at the target in the long term.

This has been a feature of the core of the FPAS - Georgian Economy Model (GEMO) - providing policy prescriptions for achieving the inflation target. The documentation of the NBG's FPAS, including GEMO, is provided in Tvalodze et al. (2016). GEMO is a semi-structural, relatively small, macroeconomic model (consisting of four key building blocks). It has been used by the Macroeconomic Research Division at the NBG to produce consistent macroeconomic forecasts and provide the corresponding stories to the Monetary Policy Committee (MPC). In addition, it is frequently used for risk scenario analysis and sometimes for counterfactual simulations. The MPC has shown its appreciation of the important input that the general equilibrium modeling has provided. Inflation has averaged at around 4.5% since 2009[1,2] and its volatility has declined (even though it could still be considered high). The 10-year yields are now almost as low as the overnight interest rates - quite a rare phenomenon for small open emerging economies, like Georgia, subject to a myriad of external disturbances.

Notwithstanding the important advances, there's still a lot of room for improvement, including on the capacity development side. On the modeling front (that this paper concentrates), FPAS may enjoy an addition of a new, "fully" structural, macroeconomic model. An existing one, GEMO, is a semi-structural gap model that is useful for business cycle (real economy) analysis, however, it doesn't incorporate consistent

[1] Before the COVID-19 shock

[2] The inflation target started to be 6%, however, the NBG has made it clear from the beginning that it was intending to lower the target to 3% - something it did in 2018.

stock-flow relationships. Yet this is an important element for analyzing issues like the balance of payments and external debt, portfolio flows and exchange rates, FX interventions and central bank balance sheets, financial frictions, and commercial bank balance sheets. In addition, while semi-structural models have the virtue of being flexible enough to fit certain empirical facts, fully structural models, on the other hand, have the advantage of making sure the analysis is internally consistent which minimizes the risk of unsound policy advice.

Taking these into account we have started a DSGE project that aims at building a new dynamic stochastic general equilibrium model that is expected to fit the key empirical facts of the macro-financial environment in Georgia. Namely, the model that we develop below incorporates an elaborate external sector (balance of payments) and the FX market. We introduce relatively novel friction: dollar-invoicing and moreover, the model benefits having the feature to account for the forward premium puzzle on FX market, based on inverse relationship between the risk premium and expected depreciation (see Adolfson et al, 2005). Also, we are suggesting that by introducing foreign bond portfolio adjustment costs we are capable for replicating the features of above-mentioned modified (lagged) UIP condition. These are in addition to standard real and nominal frictions found in the literature, including habit formation, investment adjustment costs, wage and price stickiness, etc. After calibrating the model we demonstrate, using impulse response functions, how significantly the incorporation of the new frictions improves the empirical relevance and realism of the model. In the coming papers, this model is planned to be extended with the central bank balance sheet and FX interventions as well as commercial bank balance sheets and several financial frictions, like information asymmetries and financial dollarization. The resulting model is expected to become the major tool for macro-financial analysis in Georgia, including the monetary-macroprudential nexus. Given that the resulting model would feature many instruments (including monetary policy rate, FX interventions, fiscal spending, and taxes as well as different macroprudential tools) it fits quite well in the emerging literature on the Integrated Policy Framework (see Basu et al, 2020 or Adrian et al, 2020).

The paper is organized as follows: Section 2 describes the model economy, with an emphasis on intuition. Section 3 calibrates the model and does impulse response exercises. Section 4 concludes and discusses the road ahead, while the appendices provide the full description of the technical details as well as key issues in the derivations that is usually absent in the literature are provided there.

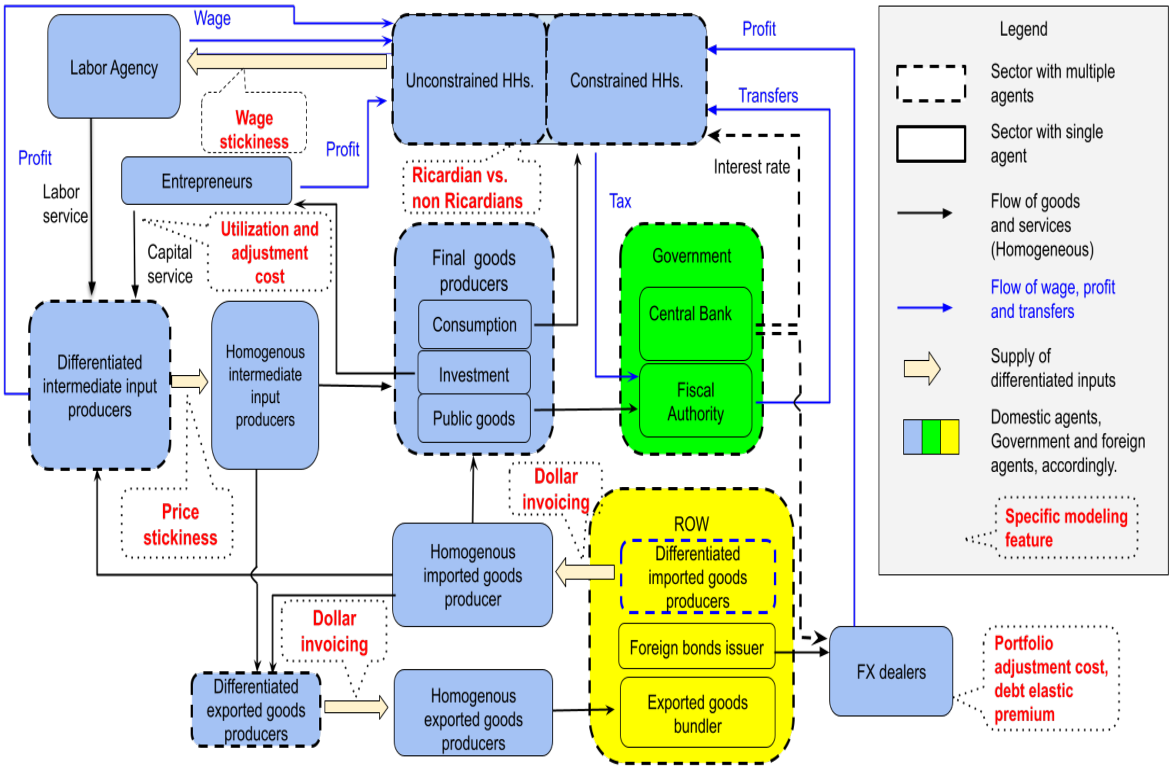
2 Model Economy

2.1 The model design

The small open economy model is developed to apply for macroeconomic analysis in Georgia. The model belongs to the class of "fully" micro-founded models, where the dynamics of economic variables are the outcome of decisions made by households and firms. The structure of the real side of the economy is quite elaborate and includes various layers of the production process, however, the financial sector is still insufficiently modeled. Adding the sector is envisaged as the next step of the model development. The model shares features of medium-scale DSGE models from the existing literature, such as price and wage stickiness (à la Calvo (1983)), investment adjustment cost in the capital production process, presence of Non-Ricardian consumers and etc., but also, we have incorporated some interesting features which could be relevant for applying the DSGE model for policy analysis in emerging markets. The rest part of the section includes a bird's eye view of the entire model, the sectoral interlinkages and some interesting features of the model are highlighted below (see, Figure

Households. Some (unconstrained) part of the households (HHs) are rational agents who make intertemporal allocations of their consumption, they own all types of firms and receive dividends from them. They are suppliers of heterogeneous labor input and have a market power to set wages. HHs have to pay consumption, wage and profit tax to the government, and some part of revenues received by the government is transferred back to HHs. Another part of households are constrained by their current period income and they consume everything available in the given period (hand to mouth behavior). Due to these properties, we could say that the model belongs to class of Two Agent New Keynesian models (TANK). 1).

Figure 1: Model design



Labor Agency picks up heterogeneous labor input provided by households and supplies labor services to the domestic intermediate input producers.

Firms. Four groups of firms are involved in the production process to produce goods consumed by households, government, entrepreneurs and foreigners (exported goods).

- Group of final goods producers (which operates in a competitive market) combines homogeneous input produced domestically and imported ones and provides the final goods to different institutional sectors, therefore, there are three different producers designated to produce final goods for households, government and entrepreneurs.

- Domestic homogeneous input is produced in two stages:

  - Differentiated intermediate input producers use labor and capital service as well as imported homogeneous input to create differentiated input (using the Cobb-Douglas production technology), those firms operate on the monopolistic competitive market and set prices in domestic currency.

- The homogeneous intermediate input producer bundles the differentiated inputs and supplies to the final goods producers as it was mentioned above.

- Homogeneous imported goods are produced by two sets of vertically integrated producers: firms operating outside of our economy produce differentiated goods and set prices in USD, on the next stage, those goods are imported and aggregated by homogeneous imported goods producers domestically, after that the homogeneous input is supplied to final goods and domestic intermediate input producers.

- Three types of firms are involved in the production of exported goods: differentiated exported goods producers use homogeneous domestic and imported inputs to produce the brand name goods on the monopolistic competitive market, which is bundled by homogeneous exported goods producers and then provided to the foreign firms (Exported goods bundler) operating outside of our economy. They bundle the goods together with exported goods from the rest of the world (their decision determines the demand on goods exported from our economy). Prices in the export and import sectors are set in USD. Which is equivalent to producer currency pricing (PCP) in import sector unless there are not shocks which imply global appreciation/depreciation of USD (the same is true about local currency pricing (LCP) vs. DCP in the export sector). In general, the implication of price stickiness in DCP which applies to LCP too in the export sector is that the expenditure switching mechanism is impaired when the economy experiences local currency swings, for example, if the domestic currency depreciates, the adjustment path of the export is muted. However, the global appreciation (depreciation) cycles of USD results in lower (higher) foreign demand even if local currency exchange rate vs trade partners' currencies does not changes at all. That said, LCP is not well suited to be applied for analysing transmission of shocks (for instance, US policy rate changes) which implies asymmetric reaction of exchange rates in emerging markets which are still dollairzed in trade relations, therefore, exposed to appreciation/depreciation cycles of USD.

Entrepreneurs accumulate capital stock subject to capital adjustment and utilization costs, the latter friction implies that capital service supplied to domestic intermediate input producers does not always equal to the capital stock.

Government sector is represented by the monetary and fiscal authorities.

- The central bank sets the policy rate in line with the Taylor-type reaction function to respond to the deviation of expected inflation from targeted inflation. The change of short-term interest rate is transmitted to the demand through different channels: on the one hand, unconstrained HHs lower consumption (if the interest rate increases) as the expected inflation drops and the real interest rate increases, as long as nominal variables are rigid, which depress demand in the current period. At the same time, entrepreneurs tend to apply the higher real rate to discount future profit stream, given the net present value of the profit declines, they reduce investment, amid demand on inputs necessary for the production of investment goods declines. An increase in nominal interest rate stimulates the substitution of foreign financial assets with domestic ones. In turn, a resulting reduction of capital outflow implies an appreciation of the domestic currency and, other things equal, it implies higher demand for imported goods and a gradual reduction of foreign demand. However, given import and export prices are sticky in USD, the expenditure switching mechanism is impaired somewhat.

- The fiscal authority collects consumption, wage income and profit taxes, issues local currency bonds to finance deficit if necessary and provides transfers to HHs. Debt-issuing is restricted by debt limits and fiscal policy rules such that fiscal authority is forced to stabilize the debt at a sustainable level in the medium run.

Forex dealers trade with foreign currency bonds. Their choice of next period portfolio is subject to risk premium which is inversely related with expected depreciation of exchange rate. The premium discourages them to reallocate the portfolio when the differential of returns of domestic relative to foreign bonds opens up; which implies deviation from simple UIP condition. Hence, the exchange rate path keeps some persistence instead of instantaneous adjustment under pure UIP.

2.2 Households

Our model economy is populated by 2 types of households: **Constrained** and **Unconstrained** ones (referred to as CHHs and UCHHs, respectively, in the rest of the paper). The fraction of CHHs relative to the whole population is λ, while the rest $(1-\lambda)$ are UCHHs. These households, as the names suggest, mainly differ in their degrees of accessibility to various financial resources. First of all, they differ in their presence in financial markets. UCHHs can afford to use financial instruments for smoothing out their consumption pattern intertemporally. While CHHs are unable to access financial markets, hence, are limited by their current disposable income generated from supplying their labor services to the production sector and government transfers. Second, constrained households do not own shares in firms, while unconstrained ones do. Therefore, the former receive no dividend payments from the firms while the latter does. Third, they have different utilities over the sequence of consumption across time. UCHHs' preferences are characterized by the presence of habit persistence, while CHHs are not.

The introduction of two types of households was motivated by the empirical fact (e.g. Campbell and Mankiw, 1989) that a significant part of households does not have enough wealth (collateral) to allocate it intertemporally, or enough income to obtain credit for consumption smoothing. Hence, this part of households consumes its current income. In addition, empirical research also indicates that the Ricardian equivalence fails to hold in practice, meaning that the effects of fiscal policy are relatively more pronounced. Introducing constrained (so-called hand-to-mouth) households do exactly that - strengthening the fiscal policy effects (Gali et al, 2007). Another, more realistic but also technically more difficult, approach is to introduce overlapping generations setting in the model.

Deviation from the representative agent economy, where all households optimize their consumption over time has its implication for determinacy analysis (Gali et al, 2003). As the existing literature shows monetary policy has to be more aggressive when the share of CHHs is low and vice versa when this share is big enough. For example, consider an inflationary shock that hits the economy. If monetary policy is aggressive enough (i.e. Taylor principle holds) consumption and real wage of unconstrained households drop initially, however, firms' profits increase (due to lower real marginal cost) which, due to higher dividends inflows, implies a leftward shift in labor supply (among UCHHs) on the latter stage. If labor supply is relatively less elastic, we end up with higher real wages that have a positive effect on CHHs' consumption levels (since CHHs consume all of their resources in every period). Therefore, the aggregate demand could increase, implying some pressure on inflation. According to Bilbiie (2005), this latter effect dominates when the share of CHHs is large enough. As a result, we get that aggressive monetary policy implies self-fulfilling inflation expectations (i.e. indeterminacy). However, the indeterminacy brought by aggressive monetary policy that is described in the example above breaks if wages are sticky, as the wage increase which has a positive effect on CHHs' consumption is muted in this case. As a result, the Taylor principle is still necessary (see Colciago, 2011) when the "CHHs meet sticky wages" as it is in our specification .

Optimization Problem of Unconstrained Households. Let's consider unconstrained households first. We assume a continuum of monopolistic competitive households of this type, each of which supplies differentiated labor ($L_t^{uc}(i)$) to the production sector. Every unconstrained HH has preferences over consumption ($C_t^{uc}(i)$) and labor supply and is subject to preference (ψ_t) as well as labor supply (θ_t) shocks. Furthermore, its utility from consumption in the current period is affected by the average consumption level in the previous period (hence, external habit formation is present in our model). In order to finance its own consumption UCHH generates income from various sources. First of all, it supplies differentiated labor input to the Labor Agency, earning $W_t(i)$ wage rate for a unit of household i's labor variety. Second, it receives transfers from the government ($T_t^{uc}(i)$). Third, it owns shares in the firms of the economy generating dividend inflows ($D_t^{uc}(i)$), and finally, UCHH has financial income, which comes from two types of financial assets: risk-free government bonds ($B_t(i)$) paying off R_{t-1} nominal gross return and Arrow-Debreau securities ($a_t(i)$) paying off 1 unit of nominal currency in the respective state of nature purchased in the previous period. Furthermore, UCHH's income sources are subject to various taxes described below.

As one might have already noticed UCHHs are heterogeneous in their labor services which gives them some pricing power when setting their wage rates. However, they are not freely able to choose their wages optimally in each period. Following the staggered price setting mechanism in Calvo (1983), in each period the probability that a household is able to optimize its wage is $1 - \theta_w$. With the probability of θ_w, it is stuck with the same wage for one period, θ_w^2 for two periods, and so on. The reason we assume so, like in most DSGE models, is to get nominal wage rigidity in the model economy, even if the literal interpretation of the assumption may not be the most realistic.

Note that, in equilibrium, the unconstrained household i cannot choose labor supply and wage rate independently. Since it is a monopolist in the labor market of its variety, it faces downward sloping labor demand curve ($L_t(i) = d(W_t(i))$) and once it chooses either labor supply or wage rate, the equilibrium value for the other one is automatically determined. For this reason, WLOG, we can assume that a household sets a wage rate and then chooses a labor supply that clears the market at that wage rate. Thus, for each variety i, $L_t(i) = L_t^{uc}(i)$ and in equilibrium UCHH faces additional constraint:

$$L_t^{uc}(i) = d(W_t(i)) \tag{1}$$

There is one more thing in 1 that we need to determine before we head to solving UCHH's problem. The functional form for $d(W_t(i))$ is unknown. To find out that, we need an agent whose optimal behavior defines the demand function for the labor of type i. Labor Agency is such an agent, the role of which will be discussed below.

Labor Agency is a competitive firm that aggregates different types of labor into a "composite" homogeneous labor good (L_t) that it then leases to intermediate goods firms at the wage rate W_t. There are some alternative ways of thinking about the role of the Labor Agency. One of them is viewing it as a firm's HR department that recruits differentiated labor varieties, then trains them and provides homogeneous labor input to domestic intermediate goods producers. We adopt this interpretation and use CES technology for aggregating differentiated labor inputs into homogeneous one. Furthermore, Labor Agency takes both wages, one for variety i of labor ($W_t(i)$) and another for homogeneous labor good (W_t) as given and chooses $L_t(i)$ and L_t to maximize its profit in each period t subject to its aggregation technology. Hence, its problem has the following form:

$$\underset{L_t(i),\, L_t}{\text{maximize}} \quad W_t L_t - \int_0^1 W_t(i) L_t(i) di \tag{2a}$$

$$\text{subject to} \quad L_t = \left(\int_0^1 L_t(i)^{\frac{\eta_t^l - 1}{\eta_t^l}} di \right)^{\frac{\eta_t^l}{\eta_t^l - 1}} \tag{2b}$$

where η_t^l is the elasticity of labor substitution, we assume that it is time-varying and in steady state $\eta^l > 1$.

The optimization problem of the Labor Agency implies the following demand function for household $i's$ labor input:

$$L_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\eta_t^l} L_t \tag{3}$$

If we substitute equation 3 into 2 and use the fact that the Labor Agency earns zero profit (because it is a competitive firm), we end up with equation 4 for aggregate wage index:

$$W_t = \left(\int_0^1 W_t(i)^{1-\eta_t^l} di \right)^{\frac{1}{1-\eta_t^l}} \tag{4}$$

Taking into account the functional form of demand for household i's labor variety (equation 3) and wage setting mechanism, the unconstrained household's problem can be formulated as:

$$\underset{\substack{\{C_t^{uc}(i), B_{t+1}^{uc}(i),\\ W_t^*(i), a_{t+1}(i)\}_{t=0}^{\infty}}}{\text{maximize}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \psi_t ln(C_t^{uc}(i) - hC_{t-1}^{uc}) - \chi \theta_t \frac{L_t^{uc}(i)^{1+\zeta}}{1+\zeta} \right\} \tag{5a}$$

$$\begin{aligned} \text{subject to} \quad & (1+\tau^c) P_t^c C_t^{uc}(i) + B_{t+1}^{uc}(i) + E_t\{Q_{t,t+1} a_{t+1}(i)\} \\ & = (1-\tau^w) W_t(i) L_t^{uc}(i) + R_{t-1} B_t^{uc}(i) + a_t(i) \\ & + T_t^{uc}(i) + (1-\tau^{\pi r}) D_t^{uc}(i) \end{aligned} \tag{5b}$$

$$L_t^{uc}(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\eta_t^l} L_t \tag{5c}$$

$$W_t(i) = \begin{cases} W_t^*(i) & \text{if } W_t(i) \text{ is chosen optimally} \\ \Pi^w_{t-1|t-k-1} W^*_{t-k}(i) & \forall k \in \{0, \dots, t-1\}, \text{ otherwise} \end{cases} \tag{5d}$$

where, in addition to what is described above, β is the household's discount factor, ζ stands for the inverse of Frisch labor supply elasticity and h determines the degree of habit persistence. As for taxes, τ^c, τ^w and τ^p are VAT, income and profit (or dividend income) tax rates, respectively. Also, $\Pi^w_{t-1|t-k-1} = \frac{W_{t-1}}{W_{t-k-1}}$. Hence, we assume that at the moment when the household is not able to set the optimal wage, it uses the wage indexation rule i.e., the household updates its wage based on average wage inflation in the previous period.

The first order conditions (FOCs) of the problem 5 with respect to $C_t^{uc}(i)$ and $B_{t+1}^{uc}(i)$ are the following:

$$[C_t^{uc}] : \qquad \psi_t \frac{1}{C_t(i)^{uc} - hC_{t-1}^{uc}} - \lambda_t(i)(1+\tau^c)P_t^c = 0 \tag{6}$$

$$\left[B_{t+1}^{uc}\right] : \qquad -\beta^t \lambda_t(i) + E_t \beta^{t+1} \lambda_{t+1}(i) R_t = 0 \tag{7}$$

where, $\lambda_t(i)$ is the Lagrange multiplier. By combining equations (6) and (7) we arrive at the Euler equation 8. Note that we dropped index i. Following Erceg et al (2000) when the market for Arrow-Debreau securities is complete and period utility is separable in labor, households fully insure idiosyncratic wage-adjustment risk and we get symmetric equilibrium for optimal consumption level, where everybody consumes the same irrespective of its own wage history.

$$R_t = E_t \frac{\psi_t (C_{t+1}^{uc} - hC_t^{uc}) \Pi_{t+1}^c}{\beta \psi_{t+1} (C_t^{uc} - hC_{t-1}^{uc})} \tag{8}$$

The Euler equation reflects the equilibrium condition on the domestic bonds market when the nominal interest rate equals the inverse of stochastic discount factor[3]. Which in turn describes the marginal rate of substitution between consumption today and

[3]We can show that the price of the portfolio of Arrow Debreu securities Q_{t+1} equals to the stochastic discount factor too. That said, we will apply Q_{t+1} to discount the firms' future profit streams in the next sections as those problems implicitly are part of the HHs' optimization problem.

tomorrow.

The Linear version of the Euler equation has the following form:

$$\begin{aligned}\widehat{C_t^{uc}} =& \frac{h}{1+h+\gamma^z}\widehat{C_{t-1}^{uc}} + \frac{1+\gamma^z}{1+h+\gamma^z}E_t\widehat{C_{t+1}^{uc}} + \frac{1+h+\gamma^z}{1+h+\gamma^z}(\widehat{\psi}_t - E_t\widehat{\psi_{t+1}})+ \\ &+ \frac{\gamma^z}{1+\gamma^z+h}\left(\frac{1}{1+\gamma^z}E_t\widehat{\gamma_{t+1}^z} - \widehat{\gamma_t^z}\right) - \frac{1+\gamma^z-h}{1+h+\gamma^z}\left(\frac{1}{R}\widehat{i}_t - \frac{1}{1+\pi^c}(E_t\pi_{t+1}^c - \pi^c)\right)\end{aligned} \tag{9}$$

which highlights that consumption depends on its own lag and lead as well as demand shocks and real interest rate. The first order condition with respect to wages ($W_t^*(i)$) is more involved and it requires solving the following optimization problem (see detailed derivations in the Appendix A.1):

Wage-setting Problem

$$\underset{W_t^*(i)}{\text{maximize}} \quad E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\Big\{-\chi\theta_{t+k}\frac{L_{t+k}(i)^{1+\zeta}}{1+\zeta} - \lambda_{t+k}(i)(-(1-\tau^w)W_{t+k}(i)L_{t+k}(i))\Big\} \tag{10a}$$

$$\text{subject to} \quad L_{t+k}(i) = \left(\frac{W_{t+k}(i)}{W_{t+k}}\right)^{-\eta_{t+k}^l} L_{t+k} \tag{10b}$$

$$W_{t+k}(i) = \Pi_{t+k-1|t-1}^w W_t^*(i) \tag{10c}$$

The resulting optimal wage equation is given by:

$$\begin{aligned}& E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\Big\{-\theta_{t+k}\chi(-\eta_{t+k}^l)W_t^{*-\eta_{t+k}^l(1+\zeta)-1}\left(\left(\frac{\Pi_{t+k-1|t-1}^w}{W_{t+k}}\right)^{-\eta_{t+k}^l}L_{t+k}\right)^{1+\zeta} - \\ &- E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\Big\{\lambda_{t+k}(i)(\eta_{t+k}^l-1)W_t^{*-\eta_{t+k}^l}\left(\frac{\Pi_{t+k-1|t-1}^w}{W_{t+k}}\right)^{1-\eta_{t+k}^l}W_{t+k}L_{t+k}\Big\} = 0\end{aligned} \tag{11}$$

If wages were flexible (i.e. if $\theta_w = 0$) then the optimal wage could be interpreted as a markup over the marginal rate of substitution between consumption and leisure.

After linearization around the steady state, or balanced growth path more precisely (see the detailed derivations in the Appendix A.1), the wage Phillips curve is given by:

$$\pi_t^w = \frac{1}{1+\beta}\pi_{t-1}^w + \frac{\beta}{1+\beta}E_t\pi_{t+1}^w + \frac{\Pi^w(1-\theta)(1-\beta\theta_w)}{\theta_w(1+\beta)(1+\eta^l\zeta)}(\widehat{MRS}_t - \widehat{W}_t^r) + \frac{1-\beta\theta_w}{1+\eta^l\zeta}\frac{\Pi^w}{1-\eta^l}\widehat{\eta}_t^l \tag{12}$$

where, $\widehat{MRS}_t$ and $\widehat{W}_t^r$ are the gaps of the marginal rate of substitution and the real wage, while the gap of the marginal rate of substitution is given by:

$$\widehat{MRS}_t = \widehat{\theta}_t + \zeta\widehat{L}_t + \frac{1+g}{1+g-h}\widehat{C_t^{uc}} - \frac{h}{1+g-h}\widehat{C_{t-1}^{uc}} \tag{13}$$

This linearized wage inflation equation is also close to standard one with wages following some inertia, reaction to expectations, and depending on demand conditions in the labor market.

Optimization Problem of Constrained Households. Constrained (sometimes also called hand-to-mouth) households, as described above, do not have access to financial markets, hence they can not make intertemporal decisions. Neither owns their shares in firms. They only earn labor income and get transfers from the government (T_t^c). We assume that their utility is strictly increasing in consumption (they don't give themselves the luxury of forming consumption habits) and they receive no disutility from labor (they always accept employment if its available). Hence their optimal decision is to consume whatever they earn net of taxes plus net transfers from the government. Constrained HHs set their wages at the same level as the average wage of unconstrained HHs (or, put differently, competition makes the wages of the two types of HHs equal). Given the wage, they supply working hours as much as to meet firms' labor demand. Hence, the consumption of credit-constrained HH depends on after-tax wage income and government transfers and is given by:

$$(1+\tau^c)C_t^c = (1-\tau^w)\frac{W_t}{P_t^c}L_t + T_t^c \tag{14}$$

Since constrained and unconstrained HHs face the same labor demand function, the working hours of CHHs equal to the average working hours of the unconstrained ones and as a result, consumption is the same across CHHs as well.

After linearization around the steady state (BGP) we get:

$$\widehat{C_t^c} = \frac{1-\tau^w}{1+\tau^c}\frac{\widetilde{L}\widetilde{W^r}}{\widetilde{C^c}}(\widehat{W_t}+\widehat{L_t}) + \frac{1}{1+\tau^c}\frac{\widetilde{T^{cr}}}{\widetilde{C^c}}\widehat{T_t^c} \tag{15}$$

Where the tilde over the variable (without time subscript) refers to the stationary component of a variable in a steady state.

2.3 Entrepreneurs

In our model, entrepreneurs are firms that are ultimately owned by households. Entrepreneurs own capital stock, can vary the utilization rate of physical capital, and decide how much to invest in the next period's physical capital. We can think of capital utilization as increasing number of capital hours. Even if the amount of physical capital is the same, just by increasing the utilization rate an entrepreneur can increase the number of capital services supplied to the capital market. However, this comes at a cost. A higher utilization rate implies higher maintenance expenditures in terms of investment goods. At the beginning of each period t entrepreneurs rent capital services (which is the product of physical capital and the utilization rate) to Domestic Intermediate Input Producers to generate revenue and buy investment goods from Final investment goods producers to add to the physical stock of capital for the next period. These decisions are made with the objective of maximizing its lifetime expected discounted profit. Changing the level of investment between two consecutive periods is however costly. Only a fraction of the unit of investment goods is translated into the amount of installed physical capital. The rest is wasted in the process of installment. The adjustment cost per unit of investment is described by $\tilde{S}(x)$ who's exact functional form and properties are given in detail in appendix B.1 and B.2. The way, investment is converted into physical capital in the presence of investment adjustment cost, is specified by the law of motion of capital:

$$\overline{K}_{t+1} = (1-\delta)\overline{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \tag{16}$$

Entrepreneur's profit (πr_t^e) in period t is equal to:

$$\pi r_t^e = R_t^k \overline{K}_t u_t - \gamma(u_t)\overline{K}_t P_t^i - I_t P_t^i \tag{17}$$

where R_t^k is the rental rate for a unit of capital service. $\overline{K}_t$ is the amount of physical capital that the entrepreneur own, while $K_t = u_t\overline{K}_t$ is the total amount of utilized capital (i.e. capital service) supplied to the Domestic Intermediate Input Producer. I_t is the amount of investment that the entrepreneur undertakes each unit of which costs P_t^i. u_t is the capital utilization rate and $\gamma(u_t)$[4] is the corresponding utilization cost, which shows the number of units of domestic investment goods required for operating one unit of physical capital at rate u_t. Hence, $\gamma(u_t)\overline{K}_t P_t^i$ represents the total cost (expressed in monetary terms) that the entrepreneur incurs from renting out K_t^d capital service in period t.

Taking prices $\left\{R_t^k \text{ and } P_t^i\right\}_{t=0}^{\infty}$ as given, a representative entrepreneur[5] solves the following maximization problem:

$$\underset{\left\{u_t, I_t, \overline{K}_{t+1}\right\}_{t=0}^{\infty}}{\text{maximize}} \quad E_0 \sum_{t=0}^{\infty} \lambda_t \left[R_t^k \overline{K}_t u_t - \gamma(u_t)\overline{K}_t P_t^i - I_t P_t^i\right] \tag{18a}$$

$$\text{subject to} \quad \overline{K}_{t+1} = (1-\delta)\overline{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \tag{18b}$$

where, λ_t is the marginal utility of 1 unit of profit received in consumption units (i.e. it comes from HHs utility maximization problem).

First order conditions for the problem are:

$$[u_t]: \quad R_t^k = \gamma'(u_t) P_t^i \tag{19}$$

$$\begin{aligned}[I_t]: \quad P_t^i =& \lambda_t^e \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) \\ &+ E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\lambda_{t+1}^e \tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\frac{I_{t+1}^2}{I_t^2}\right]\end{aligned} \tag{20}$$

$$\begin{aligned}\left[\overline{K}_{t+1}\right]: \quad \lambda_t^e =& E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(R_{t+1}^k u_{t+1} - \gamma(u_{t+1})p_{t+1}^i\right)\right] \\ &+ (1-\delta)E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\lambda_{t+1}^e\right]\end{aligned} \tag{21}$$

$$[\lambda_t^e]: \quad \overline{K}_{t+1} = (1-\delta)\overline{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \tag{22}$$

[4]Exact functional form of the capital utilization cost function is given in appendix B.1

[5]A representative entrepreneur exists, since all entrepreneurs are identical - face the same problem under the same conditions.

After linearizing around the steady state, FOCs of the model in terms of percentage deviations from the steady state values of respective variables will be:[6]

$$
\begin{aligned}
[u_t]: \quad & \widehat{r_t^k} = r^k \sigma_a \widehat{u}_t + r^k \widehat{P_t^i} && (23)\\
[I_t]: \quad & \widehat{P_t^i} = \widehat{\tilde{\lambda}_t^e} + (1+\gamma^z)\tilde{S}'' \widehat{I_{t-1}} \\
& \quad - \tilde{S}''(1+\gamma^z)(1+\beta)\widehat{I}_t + \tilde{S}''\beta(1+\gamma^z)\widehat{I_{t+1}} && (24)\\
[\overline{K}_{t+1}]: \quad & \widehat{\tilde{\lambda}_t^e} = -\widehat{i_t^r} + \left(1 - \frac{(1-\delta)\beta}{1+\gamma^z}\right)\left(E_t \widehat{r_{t+1}^k} + E_t \widehat{u_{t+1}}\right) + \frac{(1-\delta)\beta}{1+\gamma^z} E_t \widehat{\tilde{\lambda}_{t+1}^e} && (25)\\
[\lambda_t^e]: \quad & \widehat{\overline{K}_{t+1}} = \frac{1-\delta}{1+\gamma^z}\widehat{\overline{K}_t} + \frac{\gamma^z+\delta}{1+\gamma^z}\widehat{I}_t - \frac{\gamma^z}{1+\gamma^z}\widehat{\gamma_t^z} && (26)
\end{aligned}
$$

2.4 Goods Producers

The structure of goods production in the model economy, as shown in figure 1, is quite elaborate. Domestic intermediate input produced by a continuum of monopolistically competitive firms under the same generic name using labor services (supplied by households), capital services (provided by entrepreneurs) and imported inputs are aggregated into the domestic homogeneous input by a representative domestic homogeneous input producer. The latter is then combined with imported goods to produce final private and government consumption, final investment and final export goods which are supplied to households, government, entrepreneurs, and the rest of the world accordingly. Below we describe each agent producing the respective type of goods, its technology, and its optimization problem.

2.4.1 Domestic Input Producers

We describe the production of domestic goods in two steps: domestic differentiated input producers (we call them intermediate firms) produce differentiated goods using capital, labor and imported inputs, while the producer of the domestic homogeneous input uses CES production technology to aggregate domestic differentiated inputs produced by the intermediate firms.

[6] For details about linearization procedure consult appendix B.2

Production of the Domestic Homogenous Inputs. Aggregate domestic homogenous input Y_t^d is produced using the following production technology:

$$Y_t^d = \left(\int_0^1 Y_t(i)^{\frac{\eta_t^d - 1}{\eta_t^d}} di \right)^{\frac{\eta_t^d}{\eta_t^d - 1}} \tag{2.4.1.27}$$

where, $Y_t(i)$ is the amount of domestic differentiated inputs produced by the i^{th} intermediate input producer and η_t^d is the time-varying elasticity of substitution of differentiated inputs. Which is assumed to follow the process:

$$\eta_t^d = (1 - \rho^{\eta^d})\eta^d + \rho^{\eta^d}\eta_{t-1}^d + \varepsilon_t^{\eta^d} \tag{2.4.1.28}$$

Y_t^d in our case is a sum of the domestic consumption (C_t^d), investment (I_t^d), public spending goods (Y_t^g) and export (X_t^d) goods.

Production of the Domestic Differentiated Inputs. The i^{th} intermediate (differentiated) input producer has the following production function:

$$Y_t(i) = \gamma_t (z_t L_t(i))^{\alpha_1} K_t(i)^{\alpha_2} \left(\frac{Y_t^m(i)}{a_t^x} \right)^{1-\alpha_1-\alpha_2} - F_t^d \tag{2.4.1.29}$$

where, $K_t(i)$ is capital rented by the i^{th} intermediate input producer, γ_t is a stationary technology process (TFP), z_t represents a (labor-augmented) productivity process and $L_t(i)$ denotes labor hired by the i^{th} intermediate firm. There is a fixed cost to enter the business in the sector, and the fixed cost F_t^d changes with labor-augmented technology process z_t over time, hence, $F_t^d = z_t F^d$. Alternatively, we could interpret the cost as an amortization payment (opportunity costs included) netting off which profit is zero in SS. $Y_t(i)^m$ is a share of the imported goods ($M_t(i)$) used as a domestic intermediate input by the i^{th} firm. a_t^x is a non-stationary technology process, which makes an imported input relatively less efficient in the production over time, we assume that its net growth rate ($\gamma_t^{a^x}$) follows AR(1) process:

$$\gamma_t^{a^x} = (1 - \rho_{\gamma^{a^x}})\gamma^{a^x} + \rho_{\gamma^{a^x}}\gamma_{t-1}^{a^x} + \varepsilon_t^{\gamma^{a^x}} \tag{2.4.1.30}$$

After solving the firm i's cost minimization problem (see C.1 and C.1b in the

Appendix) we will obtain the following marginal cost function:

$$MC_t = \frac{1}{\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}} \frac{a_t^{x1-\alpha_1-\alpha_2}}{\gamma_t z_t^{\alpha_1}} W_t^{\alpha_1} R_t^{k\alpha_2} P_t^{m1-\alpha_1-\alpha_2} \tag{2.4.1.31}$$

R_t^k, W_t and P_t^m are the input prices of capital, labor and import, respectively.

Firm i operates on a monopolistic competitive market meaning has some power to set prices. Price-setting decision is subject to Calvo frictions. Note, that $P_t(i)$ is a price of intermediate input $Y_t(i)$ produced by firm i. And the aggregate amount and price of the domestic homogenous goods are Y_t^d and P_t^d, respectively.

Also, note that MC_t does not depend on i because all firms have a symmetric problem in equilibrium. We assume that in each period, only some fraction of firms are able to update its prices optimally. While, the remaining part of the firms can only index its price to lagged inflation.
We can define indexation here as:

$$\Pi_{t+k-1,t-1}^d = \frac{P_{t+k-1}^d}{P_{t-1}^d} \tag{2.4.1.32}$$

The price that firm i can charge in period t is:

$$P_t^d(i) = \begin{cases} P_t^{*d}(i) & \text{if } \; P_t^d(i) \text{ is chosen optimally} \\ \Pi_{t+k-1,t-1}^d P_{t-1}^d(i) & \text{if otherwise} \end{cases} \tag{2.4.1.33}$$

Now, let's consider the profit maximization of the firm that adjusts its price in period t. The problem will be dynamic because the price chosen in period t will have an effect in future periods. Hence, we apply the nominal discount factor ($Q_{t+k,t}$) to derive the PV of the future profit stream. The profit maximization problem of the firm i is given by:

$$\underset{P_t^{*d}(i)}{\text{maximize}} \quad E_t \sum_{k=0}^{\infty} \theta_d^k Q_{t+k,t} \left(P_t^{*d}(i)\Pi_{t+k-1,t-1}^d Y_{t+k}(i) - MC_{t+k}\left(Y_{t+k}(i) + F_t^d\right)\right) \tag{2.4.1.34a}$$

$$\text{subject to} \quad Y_{t+k}(i) = \left(\frac{P_t^{*d}(i)\Pi_{t+k-1,t-1}^d}{P_{t+k}^d} \right)^{-\eta_{t+k}^d} Y_{t+k}^d \tag{2.4.1.34b}$$

where, $Q_{t,t+k} = \beta^k \frac{U'(C_{t+k})P_t^c}{U'(C_t)P_{t+k}^c}$ is the marginal value of one unit of profits in utility terms in $t+k$ relative to t. Detailed derivations and equilibrium conditions of the firms' price-setting problem are reported in the Appendix C. The resulting linearized price-setting equation (augmented Phillips curve), is:

$$\begin{aligned}\pi_t^d = \frac{1}{1+\beta}\pi_{t-1}^d + \frac{\beta}{1+\beta}E_t\pi_{t+1}^d + \frac{(1-\theta_d\beta)(1-\theta_d)(1+\pi^d)}{\theta_d(1+\beta)}\widehat{MC_t^{rd}} - \\ - \frac{(1-\theta_d\beta)(1-\theta_d)(1+\pi^{xf})}{\theta_d(1+\beta)(\eta^d-1)}\widehat{\eta_t^d}\end{aligned} \tag{2.4.1.35}$$

where $\widehat{MC_t^{rd}}$ is a real marginal cost gap, of the domestic intermediate firm, which is defined in (C.2.12). Also, note that positive markup shock (actually, it is mark down shock by definition) pushes inflation up, the shock would be useful to analyze supply-side drivers of inflation.

2.4.2 Final Consumption Goods Sector

Consumption Goods Retailer. Final consumption goods are purchased by households, and are produced by the competitive firm using the following production function:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} C_t^{d\,\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(\frac{C_t^m}{a_t^x} \right)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}} \tag{2.4.2.1}$$

Note, that final consumption goods production is related to nonstationary technology process a_t^x too, which makes imported inputs relatively less efficient in consumption over time. It follows that the technology process a_t^x creates a wedge between the aggregate consumer price index (CPI) and import price index that implies trend appreciation of CPI-based real exchange rate, which is a relevant property in case of emerging and developing countries with relatively faster-growing prices in the nontradeable sector than in advanced economies.

The rest of the variables are defined as: C_t is the final consumption good which is the composite of C_t^d - homogenous domestic consumption input and C_t^m - homogenous imported consumption input.

Taking the price of final consumption goods P_t^c, input prices P_t^d and P_t^{mG} (the price of imported goods in domestic currency) as given, the Georgian consumption retailer solves the profit maximization problem:

$$\underset{C_t^d,\, C_t^m}{\text{maximize}} \quad P_t^c C_t - \left(P_t^d C_t^d + P_t^{mG} C_t^m\right) \tag{2.4.2.2a}$$

$$\text{subject to} \quad C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} {C_t^d}^{\frac{\eta_c-1}{\eta_c}} + {\omega_c}^{\frac{1}{\eta_c}} \left(\frac{C_t^m}{a_t^x}\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}} \tag{2.4.2.2b}$$

Detailed derivations of the profit maximization problem are given in Appendix B.1. The resulting demand equations for domestic and imported consumption goods are the following:

$$C_t^d = (1-\omega_c)\left(\frac{P_t^d}{P_t^c}\right)^{-\eta_c} C_t \tag{2.4.2.3}$$

$$\frac{C_t^m}{a_t^x} = \omega_c \left(\frac{P_t^{mG} a_t^x}{P_t^c}\right)^{-\eta_c} C_t \tag{2.4.2.4}$$

The price index of the final consumption good is given by

$$P_t^c = \left[(1-\omega_c)\left(P_t^d\right)^{1-\eta_c} + \omega_c \left(P_t^{mG} a_t^x\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}} \tag{2.4.2.5}$$

2.4.3 Final Investment Goods Sector

Production of Final Investment Goods. Final investment goods are produced by final investment goods producers using the CES technology and sold to entrepreneurs.

$$I_t = \left[(1-\omega_i)^{\frac{1}{\eta_i}} {I_t^d}^{\frac{\eta_i-1}{\eta_i}} + {\omega_i}^{\frac{1}{\eta_i}} \left(\frac{I_t^m}{a_t^x}\right)^{\frac{\eta_i-1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i-1}} \tag{2.4.3.1}$$

Where, I_t is the final investment good which is the composite of the homogenous domestic I_t^d and imported inputs I_t^m.

Taking the price of final investment good P_t^i, input prices P_t^d and P_t^{mG} as given, the investment goods producer solves the following profit maximization problem:

$$\underset{I_t^d,\, I_t^m}{\text{maximize}} \quad P_t^i I_t - \left(P_t^d I_t^d + P_t^{mG} I_t^m\right) \tag{2.4.3.2a}$$

$$\text{subject to} \quad I_t = \left[(1-\omega_i)^{\frac{1}{\eta_i}} {I_t^d}^{\frac{\eta_i - 1}{\eta_i}} + {\omega_i}^{\frac{1}{\eta_i}} \left(\frac{I_t^m}{a_t^x} \right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}} \tag{2.4.3.2b}$$

Detailed derivations of the profit maximization problem are given in Appendix D.2. The resulting demand equations are:

$$I_t^d = (1-\omega_i) \left(\frac{P_t^d}{P_t^i} \right)^{-\eta_i} I_t \tag{2.4.3.3}$$

$$\frac{I_t^m}{a_t^x} = \omega_i \left(\frac{P_t^{mG} a_t^x}{P_t^i} \right)^{-\eta_i} I_t \tag{2.4.3.4}$$

The aggregate price index of the investment good is:

$$P_t^i = \left[(1-\omega_i) \left(P_t^d \right)^{1-\eta_i} + \omega_i \left(P_t^{mG} a_t^x \right)^{1-\eta_i} \right]^{\frac{1}{1-\eta_i}} \tag{2.4.3.5}$$

2.4.4 Final Public Goods Sector

Production of the Public Spending Goods. Final public good is produced using CES technology, in a similar fashion to the final consumption and investment goods.

$$Y_t^g = \left[(1-\omega_g)^{\frac{1}{\eta_g}} {G_t^d}^{\frac{\eta_g - 1}{\eta_g}} + {\omega_g}^{\frac{1}{\eta_g}} \left(\frac{G_t^m}{a_t^x} \right)^{\frac{\eta_g - 1}{\eta_g}} \right]^{\frac{\eta_g}{\eta_g - 1}} \tag{2.4.4.1}$$

Where G_t^d and G_t^m are homogenous domestic and imported inputs used in public goods production, respectively. The maximization problem can be written as:

$$\underset{G_t^d, G_t^m}{\text{maximize}} \quad P_t^g Y_t^g - \left(P_t^d G_t^d + P_t^{mG} G_t^m \right) \tag{2.4.4.2a}$$

$$\text{subject to} \quad Y_t^g = \left[(1-\omega_g)^{\frac{1}{\eta_g}} {G_t^d}^{\frac{\eta_g - 1}{\eta_g}} + {\omega_g}^{\frac{1}{\eta_g}} \left(\frac{G_t^m}{a_t^x} \right)^{\frac{\eta_g - 1}{\eta_g}} \right]^{\frac{\eta_g}{\eta_g - 1}} \tag{2.4.4.2b}$$

After doing similar steps as in Appendix D.1, the resulting demand functions for the domestic and imported public goods are given by:

$$G_t^d = (1-\omega_g) \left(\frac{P_t^d}{P_t^g} \right)^{-\eta_g} Y_t^g \tag{2.4.4.3}$$

$$\frac{G_t^m}{a_t^x} = \omega_g \left(\frac{P_t^{mG} a_t^x}{P_t^g} \right)^{-\eta_g} Y_t^g \tag{2.4.4.4}$$

While the price index of public spending goods is given by:

$$P_t^g = \left[(1-\omega_g) P_t^{d\,1-\eta_g} + \omega_g (P_t^{mG} a_t^x)^{1-\eta_g} \right]^{\frac{1}{1-\eta_g}} \tag{2.4.4.5}$$

2.4.5 Import Sector

Production of homogeneous imported input. The production process of imported goods, used in final goods and domestic intermediate input production, can be broken down into two stages: *foreign traders* which operate outside of our economy produce differentiated imported inputs ($M_t(i)$) using homogeneous input available on the world market. The homogeneous input is purchased with the aggregate price index of our trade partners in the trade partners' aggregate currency unit. The foreign traders could be interpreted as the representative exporter firms of our trade partners whose cost of production is the cost of inputs produced within our trade partners' economies and sold in their own currency units. Foreign traders use their market power to set their prices in USD (the dominant currency). In the second stage of production of homogeneous imported input, the *homogeneous imported input producer* (import bundler) operating in our domestic economy, purchases differentiated imported inputs produced by foreign traders, then aggregates them and supplies the homogeneous imported input within our economy (M_t) in the domestic currency units; hence, it buys imported inputs in USD and sells in domestic currency (GEL). The homogeneous imported input (M_t) is used in domestic intermediate input production (Y_t^m), as well as in the production of final consumption (C_t^m), investment (I_t^m), public (G_t^m) and exported goods (X_t^m). The homogeneous imported input producer takes the price set by foreign traders in USD P_t^{mf} as given, and it (import bundler) maximizes its profit s.t. CES aggregate of differentiated imported inputs:

$$\underset{M_t(i),\, M_t}{\text{maximize}} \quad \frac{P_t^{mG}}{e_t^{Gel/D}} M_t - \int_0^1 M_t(i) P_t^{mf}(i) di \tag{2.4.5.1a}$$

$$\text{subject to} \quad M_t = \left(\int_0^1 M_t(i)^{\frac{\varepsilon_t^m - 1}{\varepsilon_t^m}} di \right)^{\frac{\varepsilon_t^m}{\varepsilon_t^m - 1}} \tag{2.4.5.1b}$$

Where P_t^{mG} is the domestic currency price of homogenous imported goods, and $e_t^{Gel/D}$ is the GEL/USD exchange rate. Also, ε_t^m is the time-varying elasticity of substitution of differentiated imported inputs, which is assumed to follow the AR(1) process.

$$\varepsilon_t^m = (1 - \rho^{\varepsilon^m})\varepsilon^m + \rho^{\varepsilon^m} \varepsilon_t^m + \varepsilon_t^{\varepsilon^m} \tag{2.4.5.2}$$

From the maximization problem we can derive a demand function for differentiated imported input i:

$$M_t(i) = \left(\frac{P_t^{mf}(i)}{P_t^{mf}} \right)^{-\varepsilon_t^m} M_t \tag{2.4.5.3}$$

Where the price index of homogeneous imported input is given by:

$$P_t^{mf} = \left[\int_0^1 P_t^{mf}(i)^{1-\varepsilon_t^m} di \right]^{\frac{1}{1-\varepsilon_t^m}} \tag{2.4.5.4}$$

Profit maximization problem of differentiated imported input producers. As mentioned, differentiated imported input producers, i.e. foreign traders, purchase aggregated bundles of homogeneous input produced in our trading partner economies at the aggregate price in trade partners' aggregate currency unit (effective exchange rate). Therefore, foreign traders' cost of production is the purchase of homogeneous input on the world market price in trade partners' aggregate currency unit. The differentiated imported input produced by foreign traders (outside of our economy) is used in the production process of homogeneous imported input (within our economy).

The differentiated imported input producers (i.e. foreign traders) are monopolistic competitive firms and use their market power to set prices optimally in USD, subject to Calvo friction. Hence, here our modeling approach is based on dollar invoicing in trade relations, as the price is sticky in USD. Meaning only a part of the firms $(1 - \theta_m)$ get a chance to set a price of differentiated imported inputs optimally in each period in USD. There is no additional layers of price setting in local currency, and imported goods purchased in USD is resoled in local currency on perfectly competitive market, that implies a perfect pass-trough of GEL/USD while there is incomplete path trough from GEL against our trade partners currencies. The price of imported goods in GEL could be defined as:

$$P_t^{mG} = e_t^{Gel/D} P_t^{mf} \tag{2.4.5.5}$$

The differentiated imported input producer i (foreign trader) maximizes its profit subject to demand by homogeneous imported input producer operating in our domestic economy, and sets a price in t period optimally. The firm takes into account that it may not be able to change its price optimally in the next periods, but updates its price following an indexation scheme. Therefore, the profit maximization problem of differentiated imported input producers is the following:

$$\underset{P_t^{*mf}(i)}{\text{maximize}} \quad E_t \sum_{k=0}^{\infty} \left\{ \theta^k Q_{t,t+k}^f \left[P_{t+k}^{mf}(i) M_{t+k}(i) - e_{t+k}^{D/R} P_{t+k}^R M_{t+k}(i) \right] \right\} \tag{2.4.5.6a}$$

$$\text{subject to} \quad P_{t+k}^{mf}(i) = P_t^{*mf}(i) \Pi_{t+k-1|t-1}^{mf} \tag{2.4.5.6b}$$

$$M_{t+k}(i) = \left(\frac{P_{t+k}^{mf}(i)}{P_{t+k}^{mf}} \right)^{-\epsilon_{t+k}^m} M_{t+k} \tag{2.4.5.6c}$$

where, $P_t^{*mf}(i)$ is the price set by the differentiated imported input producer who had a chance of updating its price back in the t period. $e_{t+k}^{D/R}$ is the aggregated nominal exchange rate of trade partners' currencies w.r.t. USD (i.e. nominal effective exchange rate of the dollar). Also, P_{t+k}^R is the price of homogeneous inputs (produced in our trading partner economies). While $Q_{t,t+k}^f$ is the foreign discount rate applied by foreign traders to discount future profits; and :

$$\Pi_{t+k-1|t-1}^{mf} = \frac{P_{t+k-1}^{mf}}{P_{t-1}^{mf}} \tag{2.4.5.7}$$

is the price index used by the differentiated imported input producers to update its price (in k period) whenever optimization isn't possible.

Given the foreign trader assesses its profit in USD, while the purchase of the homogeneous input on the world market is made in our trade partners' aggregate currency units, we can express the marginal cost of production of differentiated imported input in USD as:

$$MC_t^m = e_t^{D/R} P_t^R \tag{2.4.5.8}$$

While the real marginal cost faced by foreign traders is:

$$MC_t^{m^r} = \frac{e_t^{D/R} P_t^R}{P_t^{mf}} \tag{2.4.5.9}$$

Alternatively, the real marginal cost could be rewritten as:

$$MC_t^{m^r} = REER_t \frac{P_t^c}{P_t^{mG}} \tag{2.4.5.10}$$

where, $REER_t$ is the real effective exchange rate of GEL, and $P_t^{mG} = e_t^{Gel/D} P_t^{mf}$ is the price of imported goods expressed in domestic currency.
From the maximization problem we can derive the following optimality condition for differentiated imported input producers (see, Appendix E.2):

$$\begin{aligned} E_t \sum_{k=0}^{\infty} \Bigg\{ \theta_m^k Q_{t,t+k}^f P_{t+k}^{mf} \Bigg[(1 - \varepsilon_{t+k}^m) P_t(i)^{*mf^{-\varepsilon_{t+k}^m}} \left(\frac{\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}} \right)^{1-\varepsilon_{t+k}^m} M_{t+k} + \\ + \varepsilon_{t+k}^m MC_{t+k}^{m^r} P_t(i)^{*mf^{-\varepsilon_{t+k}^m - 1}} \left(\frac{\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}} \right)^{-\varepsilon_{t+k}^m} M_{t+k} \Bigg] \Bigg\} = 0 \end{aligned}$$

If we apply a log-linear transformation to the optimality condition (note, that the optimization problem is symmetric and firms that are able to reset price in the t period set the same prices in USD), the following linear version of the Phillips curve is derived from the profit maximization problem:

$$\pi_t^{mf} = \frac{1}{1+\beta^*} \pi_{t-1}^{mf} + \frac{\beta^*}{1+\beta^*} E_t \pi_{t+1}^{mf} + \frac{(1-\beta^*\theta_m)(1-\theta_m)(1+\pi^{mf})}{\theta_m (1+\beta^*)} (\widehat{MC_t^{r^m}} - \frac{1}{\varepsilon^m - 1} \widehat{\varepsilon_t^m}) \tag{2.4.5.11}$$

where, π_t^{mf} measures inflation in the import sector in USD, and π^{mf} is its value in SS, while $\widehat{MC_t^{m^r}}$ is real marginal cost gap in the sector (in foreign currency). The marginal cost, the driver of the imported inflation, in turn, is determined by the REER (See, Appendix E.4). As the import price is sticky in USD in the short run, GEL/USD bilateral exchange rate plays role too in the inflation dynamics through the imported input channel.

2.4.6 Export Sector

The export sector within our economy is structured into two layers of firms. The *homogeneous exported goods producer* uses inputs ($X_t(i)$) produced by *differentiated exported goods producers* to transform it into homogeneous exported goods (X_t). Latter, exported to the rest of the world. Besides the firms operating in our domestic economy (differentiated exported goods and homogeneous exported goods producers), we, also, analyze the decisions made by foreign firms (operating outside of our economy). Which try to optimally combine homogeneous goods exported from all countries around the world, together with homogeneous exported goods from our economy. In turn, the demand on homogeneous exported goods from our economy is the outcome of the cost minimization problem of the foreign firm. Hence, first, we formally derive the demand function for our homogeneous exported goods (which cross the border) by looking at the cost minimization problem of foreign firms). Second, we analyze decisions made by firms involved in the export sector in the domestic economy.

Demand for homogeneous exported goods. Homogeneous exported goods on world market X_t^w is the aggregate of homogeneous goods exported from each country using the following aggregation technology (i.e, the exported goods from our economy is one of the inputs among exported goods from other countries):

$$X_t^w = \left(\sum_{j=0}^{J} \alpha_t(j)^{\frac{1}{\varepsilon_w}} X_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \tag{2.4.6.1}$$

where $X_t(j)$ is the export of homogeneous exported goods from country j and $\alpha_t(j)$ approximates foreigners' preference for exported goods from country j, in steady state it approximates the share of country $j's$ export in world aggregate export. For a small open economy like Georgia, $\alpha_t(j) \to 0$, however, the term serves us to analyze export demand shocks not related to the economic conditions in our trade partners, but it could reflect the changes in preferences toward goods produced in our economy, for example, trade-related measures. On the aggregate level, we assume that all countries set the price of their homogeneous goods in USD. The profit maximization problem of the aggregator of exported goods on the world market, determines demand on the homogeneous exported goods from country j:

$$X_t(j) = \alpha_t(j)\left(\frac{P_t^{xf}(j)}{P_t^{xf,w}}\right)^{-\varepsilon_w} X_t^w \tag{2.4.6.2}$$

where $P_t^{xf,w}$ is the aggregate export price index in the world (in USD):

$$P_t^{xf,w} = \left(\sum_{j=0}^{J} \alpha_t(j) P_t^{xf}(j)^{1-\varepsilon_w}\right)^{\frac{1}{1-\varepsilon_w}} \tag{2.4.6.3}$$

With the demand function for exported goods from country j in hand, we can apply this function as the demand on our homogeneous exported goods too. However, note, that at the moment the demand on homogeneous exported goods from our economy is expressed as the function of world aggregate export. Then few more steps are needed to derive demand as the function of world aggregate output. We assume that homogeneous exported goods from different countries which are transformed into X_t^w, are used as homogeneous imported input together with domestic inputs in final goods production in the rest of the world. Hence, on an aggregate level, world output is produced with imported and homogeneous domestic (their own) inputs using CES technology. After solving the profit maximization problem in the world output production, we come up with the following demand function on aggregate import:

$$M_t^w = \omega_w \left(\frac{P_t^{mf,w}}{P_t^*}\right)^{-\eta_w} Y_t^* \tag{2.4.6.4}$$

where, ω_w is the share of imported input in the world output production, $P_t^{mf,w}$ and P_t^* are the world aggregate import price (in USD) and the aggregate price index of world output, respectively, while Y_t^* is world aggregate output. It is straightforward to note that on an aggregate level world export equals world import. Also, assuming elasticity of substitution of exported goods in different countries and the elasticity of substitution among imported and domestic inputs in the world output production (on an aggregate level) are same, the resulting demand function for our exported goods looks:

$$X_t = \omega_w \alpha_t \left(\frac{P_t^{xf}}{P_t^*}\right)^{-\varepsilon_w} Y_t^* \tag{2.4.6.5}$$

Production of homogeneous exported goods. As said, in our domestic export sector there are two layers of firms: the homogeneous exported goods producer purchases differentiated exported goods and pays the price in USD (i.e. differentiated exported goods producers are price setters), aggregate them and resell on the world market in the same currency; in turn, differentiated exported good producers use domestic and imported inputs to produce differentiated goods for export production. Therefore, the profit maximization problem of the homogeneous exported goods producer is given by:

$$\underset{X_t(i), X_t}{\text{maximize}} \quad P_t^{xf} X_t - \int_0^1 P_t^{xf}(i) X_t(i)^{xf} di \tag{2.4.6.6a}$$

$$\text{subject to} \quad X_t = \left(\int_0^1 X_t(i)^{\frac{\varepsilon_t^x - 1}{\varepsilon_t^x}} di \right)^{\frac{\varepsilon_t^x}{\varepsilon_t^x - 1}} \tag{2.4.6.6b}$$

where, P_t^{xf} and $P_t(i)^{xf}$ are the aggregate price index of homogeneous exported goods and the price of differentiated exported goods i, respectively. Also, ε_t^x is time-varying elasticity of substitution of differentiated exported goods, which follows the AR(1) process:

$$\varepsilon_t^x = (1 - \rho^{\varepsilon^x})\varepsilon^x + \rho^{\varepsilon^x} \varepsilon_{t-1}^x + \varepsilon_t^{\varepsilon^x} \tag{2.4.6.7}$$

From the maximization problem, we can derive the following demand function for differentiated exported goods produced by firm i:

$$X_t(i) = \left(\frac{P_t^{xf}(i)}{P_t^{xf}} \right)^{-\varepsilon_t^x} X_t \tag{2.4.6.8}$$

While the aggregate price of homogeneous exported goods in USD is given by the equation:

$$P_t^{xf} = \left[\int_0^1 (P_t^{xf}(i))^{1-\varepsilon_t^x} di \right]^{\frac{1}{1-\varepsilon_t^x}} \tag{2.4.6.9}$$

Cost minimization problem of differentiated exported goods producers. The i^{th} differentiated exported goods producer is a monopolistic competitive firm and produces $X_t(i)$ by aggregating domestically produced intermediate (X_t^d) and imported (X_t^m) inputs with CES technology. Entering the sector is related to fixed cost F_t^x, the growth of which follows a stochastic trend of real export Γ_t^x over time, hence,

$F_t^x = \Gamma_t^x F^x$. The i^{th} differentiated exported goods producer minimizes its cost s.t. the CES production technology to determine the optimal combination of inputs, given the prices of domestic intermediate P_t^d and imported inputs P_t^{mG}:

$$\underset{X_t^d;\, X_t^m}{\text{minimize}} \quad P_t^d X_t^d + P_t^{mG} X_t^m \tag{2.4.6.10a}$$

$$\text{subject to} \quad X_t(i) = a_t^{r\,\frac{2}{\eta_x - 1}} \left[\omega_x^{\frac{1}{\eta_x}} \left(X_t^d a_t^x \right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} X_t^{m\,\frac{\eta_x - 1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x - 1}} - F_t^x \tag{2.4.6.10b}$$

The production process of differentiated exported goods is characterized with the two distinct nonstationary technology processes: the aggregate export-specific (a_t^r) and import inefficiency (a_t^x) technologies. We assume a potentially faster quality improvement in the export sector relative to other sectors in our domestic economy, which, in turn, makes quality-adjusted exported goods relatively cheaper (than implied with domestic input prices), while the real effective exchange rate is appreciating. It follows that exporters can expand output faster than implied by the demand from trade partners' economies. We define the growth rate of the general technology process in export production as $\gamma_t^r = \frac{a_t^r}{a_{t-1}^r} - 1$, where $\gamma_t^{a^r}$ follows the AR(1) process:

$$\gamma_t^{a^r} = (1 - \rho_{\gamma^{a^r}})\gamma^{a^r} + \rho_{\gamma^{a^r}} \gamma_{t-1}^{a^r} + \varepsilon_t^{\gamma^{a^r}} \tag{2.4.6.11}$$

From the cost minimization problem, we can derive the marginal cost function of the i^{th} firm. Since the minimization problem is symmetric across all differentiated exported goods producers, we write the marginal cost function without the i subscript (Detailed derivations are reported in Appendix F.2):

$$MC_t^x = a_t^{r - \frac{2}{\eta_x - 1}} \left[\omega_x \left(\frac{P_t^d}{a_t^x} \right)^{1-\eta_x} + (1-\omega_x)(P_t^{mG})^{1-\eta_x} \right]^{\frac{1}{1-\eta_x}} \tag{2.4.6.12}$$

Profit maximization problem of differentiated exported goods producers. As said, the differentiated exported goods producer firm i operates on the monopolistic competitive market, in each period of time the firm has $(1-\theta_x)$ probability of setting a new price in USD while with probability of θ_x the exporter firm (that last reset its price in the period t) follows price indexation rule given by:

$$P_{t+k}^{xf}(i) = P_t^{*xf}(i)\Pi_{t+k-1|t-1}^{xf} \tag{2.4.6.13}$$

where $\Pi_{t+k-1|t-1}^{xf}$ is price index in USD from $t-1$ to $t+k-1$ period:

$$\Pi_{t+k-1|t-1}^{xf} = \frac{P_{t+k-1}^{xf}}{P_{t-1}^{xf}} \tag{2.4.6.14}$$

With marginal cost function, that is the outcome of the cost minimization problem of differentiated exported goods producer and demand function (from homogeneous exported goods producer's profit maximization problem) in hand, the profit maximization problem of differentiated exported goods producer is the following:

$$\underset{P_t^{*xf}(i)}{\text{maximize}} \quad \sum_{k=0}^{\infty} E_t \Big\{ \theta^k Q_{t+k|t} \Big[e_{t+k}^{Gel/D} P_{t+k}^{xf}(i) X_{t+k}(i) - MC_{t+k}^{x}\left(X_{t+k}(i) + F_t^x\right) \Big] \Big\} \tag{2.4.6.15a}$$

$$\text{subject to} \quad P_{t+k}^{xf}(i) = P_t^{*xf}(i)\Pi_{t+k-1|t-1}^{xf} \tag{2.4.6.15b}$$

$$X_{t+k}(i) = \left(\frac{P_{t+k}^{xf}(i)}{P_{t+k}^{xf}}\right)^{-\varepsilon_t^x} X_{t+k} \tag{2.4.6.15c}$$

where $Q_{t,t+k} = \beta^k \frac{U'(C_{t+k}^{uc})P_t^c}{U'(C_t^{uc})P_{t+k}^c}$ is households' nominal kernel used to discount future profits. From the maximization problem, we can derive the optimal price of differentiated exported good producer firms (detailed derivations are reported in Appendix F.3 and F.5):

$$\begin{aligned} E_t \sum_{k=0}^{\infty} \Bigg\{ \theta_x^k \beta^k \frac{\psi_{t+k}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t(C_{t+k}^{uc} - hC_{t+k-1}^{uc})\Pi_{t+k|t}^c} P_{t+k}^{xG} \Bigg[(1-\varepsilon_{t+k}^x) \left(\frac{P_t^{*xf}}{P_t^{xf}}\right)^{-\varepsilon_{t+k}^x} \left(\frac{\Pi_t^{xf}}{\Pi_{t+k}^{xf}}\right)^{1-\varepsilon_{t+k}^x} X_{t+k} + \\ + \varepsilon_{t+k}^x MC_{t+k}^{x^r} \left(\frac{\Pi_t^{xf}}{\Pi_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x} X_{t+k} \left(\frac{P_t^{*xf}}{P_t^{xf}}\right)^{-\varepsilon_{t+k}^x - 1} \Bigg] \Bigg\} = 0 \end{aligned} \tag{2.4.6.16}$$

The linear version of inflation (in USD) in the export sector looks:

$$\pi_t^{xf} = \frac{1}{1+\beta}\pi_{t-1}^{xf} + \frac{\beta}{1+\beta}E_t\pi_{t+1}^{xf} + \frac{(1-\theta_x\beta)(1-\theta_x)(1+\pi^{xf})}{\theta_x(1+\beta)}\widehat{MC_t^{x^r}} -$$

$$-\frac{(1-\theta_x\beta)(1-\theta_x)(1+\pi^{xf})}{\theta_x(1+\beta)(\varepsilon^x-1)}\widehat{\varepsilon_t^x} \tag{2.4.6.17}$$

Given the price of exported goods is sticky in USD, movements in exchange rate markets do not have an instantaneous effect on competitiveness.

2.5 Foreign exchange market

The investment decision in foreign currency bonds is determined by the infinitely-lived perfectly competitive forex dealers, operating on behalf of households. The forex dealers participate in the foreign currency bond market and maximize their lifetime profit subject to risk premium which is inversely related to expected depreciation. The feature is introduced to account for the empirical evidence on "forward premium puzzle". (see Adolfon et al, 2007). The intuition is that if the exchange rate movement is predictable (consecutive depreciations) then lower return is required by investors for holding foreign currency bonds.

The problem of the forex dealers is[7]

$$\max_{B_t^f} E_0 \sum_{t=0}^{\infty} B_t^f \Bigg\{ \lambda_{t+1} e_{t+1}^{Gel/D} R_t^f \quad R_t^{\rho} exp\left(-\xi^{dl}\left(b_t^f - b^f\right) - \xi^{fp}\left(\frac{e_{t+1}^{Gel/D}}{e_t^{Gel/D}}\frac{e_t^{Gel/D}}{e_{t-1}^{Gel/D}} - 1\right)\right) - \\ -\lambda_t e_t^{Gel/D} \Bigg\}$$

[7]We also tied an alternative version of forex dealers' profit maximization problem when the decision is subject to portfolio adjustment cost, for example, $B_t^f \Bigg\{ \lambda_{t+1} e_{t+1}^{Gel/D} R_t^f R_t^{\rho} exp\Big(-\xi^{dl}\left(b_t^f - b^f\right)\Big) - \lambda_t e_t^{Gel/D} - \lambda_t e_t^{Gel/D} \frac{\xi^{adj}}{2}\left(\frac{B_t^f e_t^{Gel/D}}{B_{t-1}^f e_{t-1}^{Gel/D}} - \Gamma^{B^f}\right)^2 \Bigg\}$, The Γ^{B^f} is the gross rate of growth of foreign bonds in a steady state, while the parameter ξ^{adj} reflects the impact of portfolio adjustment cost. The latter implies deviation from standard UIP condition. Although the term isn't fully structural, we can show that the extension works quite well to replicate empirical facts. It is related to the recent theoretical advances on modeling deviations from standard UIP condition, for instance, Gourinchas, et al. (2022), where the deviation from pure UIP is driven by the behavior of risk-averse arbitrageurs, trying to avoid risk of excess accumulation of certain assets. While the aforementioned model feature (the preferred agent models belong to the class of partial equilibrium models) isn't easy to integrate into the general equilibrium setup, the extension of our model in this regard is quite simple, but still useful to reconcile empirical facts on exchange rate dynamics.

where B_t^f is the amount of the foreign currency bonds chosen by the forex dealer, while b_t^f is the foreign asset to output ratio, hence, this part of the premium is endogenous and elastic to the country's foreign asset position.[8]. The dealer takes it as exogenously given when it makes a portfolio choice. Also, risk premium is inversely related to expected depreciation and ξ^{fp} measures the sensitivity of risk premium to it, we can note that the parameter approximately reflects the share of backward looking agents on the FX market. R_t^ρ is the exogenous currency risk premium and follows the AR(1) process. The sovereign risk premium is given by

$$R_t^\rho = (1 - \rho_{prem})R^\rho + \rho_{prem}R_{t-1}^\rho + \eta_t \tag{2.5.1}$$

where R^ρ is the steady-state gross FX risk premium, and η_t is iid shock.

FOC of (2.5.1) yields:

$$R_t = E_0 R_t^f R_t^\rho (1 + \gamma_{t+1}^{e^{Gel/D}}) exp \left(-\xi^{dl} \left(b_t^f - b^{f^{ss}} \right) - \xi^{fp} \left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1 \right) \right) \tag{2.5.2}$$

In a linear form it can be written as:

$$i_t = i_t^f + (R_t^\rho - R^\rho) + (1 - \xi^{fp}) E_0 \gamma_{t+1}^{e^{Gel/D}} - \xi^{fp} \gamma_t^{e^{Gel/D}} - \xi^{dl} \left(b_t^f - b^{f^{ss}} \right) \tag{2.5.3}$$

The equation (2.5.3) is a modified uncovered interest rate parity condition (UIP), where $E_0 \gamma_{t+1}^{e^{Gel/D}}$-represents expected rate of depreciation of the local currency against USD.

2.6 Fiscal Sector

Fiscal authority changes its primary balance (defined as $GB_t = T_t - G_t - TR_t$, where $G_t = P_t^g Y_t^g$) to maintain debt at a sustainable level in the medium term by the following

[8] See Schmitt-Grohe and Uribe (2003).

rule:[9]:

$$gb_t = (1-\rho^{gb})gb + \rho^{gb} gb_{t-1} + \phi\,(d_t - d) + u_t^g \tag{2.6.1}$$

where, $gb_t = \frac{GB_t}{P_t^d Y_t}$ is the government's primary balance to output ratio, T_t denotes taxes (total), G_t is the government spending, $\phi > 0$ is the fiscal reaction parameter, $d_t = \frac{D_t}{P_t^d Y_t}$ is government debt to output ratio at time t, while d is its value in SS. The latter could be 40%, like it was in a pre-pandemic period when the debt-to-GDP ratio fluctuated around this level, and it has stabilized to the level in the recent period after the sharp rise during the pandemic while the government keeps comfortable buffer until 60% ceiling. Note that, $D_t - (1+i_{t-1})D_{t-1} = GB_t$, could be given as $d_t = (1+i_{t-1})\frac{1}{\Pi_t^d(1+\gamma_t^y)}d_{t-1} - gb_t$ in stationary form, where i_{t-1} is the interest rate on government bonds at $t-1$ while π_t^d and γ_t^y are domestic goods inflation and growth rate of intermediate goods (Y_t) (output), respectively. The government receives the following tax revenues:

$$T_t = T_t^c + T_t^w + T_t^\pi = \tau_t^c P_t^c C_t + \tau_t^w W_t L_t + \tau_t^\pi \pi r_t^T \tag{2.6.2}$$

where, T_t is a sum of consumption (VAT) tax (T_t^c), labor income (wage) tax (T_t^w) and profit tax (T_t^π). While, $\tau^c, \tau^w, \tau^{\pi r}$ are the tax rates, respectively. πr_t^T is the total profit earned by the firms operating in different sectors of the economy.

Government transfers part of its revenue ($TR_t = TR_t^{cr} + TR_t^{ucr}$) to constrained and unconstrained HHs. We assume that transfers to output ratios follow AR(1) processes:

$$t_t^{cr} = (1-\rho_{tr}^{cr})t^{cr} + \rho_{tr} t_{t-1}^{cr} + \epsilon_t^{t^{cr}} \tag{2.6.3}$$

$$t_t^{ucr} = (1-\rho_{tr}^{ucr})t^{ucr} + \rho_{tr} t_{t-1}^{ucr} + \epsilon_t^{t^{ucr}} \tag{2.6.4}$$

Note that, $TR_t^{cr} = t_t^{cr} P_t^d Y_t$ and $TR_t^{ucr} = t_t^{ucr} P_t^d Y_t$.

[9]We note, that the rule augmented with the reaction to cycles of output works better to fit the model to the data, therefore, we augment the equation to make it counter-cycle on the data filtration stage

2.7 Monetary Policy

The central bank sets the nominal interest rate, i_t, according to the Taylor-type reaction function, and reacts to deviation of expected inflation from the target. Specifically, the monetary policy rule has the following functional form:

$$i_t = \delta_1 i_{t-1} + (1-\delta_1)\left[i_t^N + \delta_2 E_t\left(\pi_{4.t+4} - \pi_{t+4}^{tar}\right)\right] + \epsilon_t^i \tag{2.7.1}$$

where, i_t^N is the neutral nominal interest rate, π_{t+4}^{tar} and $E(\pi_{4,t+4})$ are the inflation target and headline inflation expectations over the next year, respectively. While the ϵ_t^i is monetary policy shock which follows the AR(1) process. The targeted inflation has the following dynamics:

$$\pi_t^{tar} = \pi_{t-1}^{tar} + \epsilon_t^{tar} \tag{2.7.2}$$

While the nominal neutral rate is given by:

$$i_t^N = r_t^{nut} + \pi_t^{exp} \tag{2.7.3}$$

Where the dynamics of the real neutral interest rate (r_t^{nut}) is defined using the real UIP condition when relevant variables are on their neutral level and by adding its lag to allow some persistence to the variable[10]:

$$1 + r_t^{nut} = \rho^{rnut}(1 + r_{t-1}^{nut}) + (1-\rho^{rnut})\left(\frac{1}{1+\gamma_t^{ax}} R_t^{\rho^{nut}}(1 + r_t^{fnut})\right) + \varepsilon_t^{rnut} \tag{2.7.4}$$

where, r_t^{fnut} is the neutral real foreign interest rate, b_t^f is the foreign assets to output ratio, $R_t^{\rho^{nut}}$ is the neutral level of sovereign risk premium (which is defined as a AR(1) process), the γ_t^{ax} is the growth rate of import inefficiency technology (hence, representing trend real appreciation), π_t^{exp} is the expected inflation in central bank's mind, i.e. what do they think how the expectations are determined in the economy, though, the definition of the expected inflation is non structural.

$$\pi_t^{exp} = \rho^{exp1}\pi_{t-1}^{exp} + (1-\rho^{exp1})(\omega^\pi \pi_{t-1}^c + (1-\omega^\pi)(\rho^{exp2}\pi_{t+1}^c +$$

[10]The weighted sum of real neutral interest rate defined based on real UIP and the Euler equation fits the data better, therefore, we accept the form of real neutral interest rate to confront the model with the data.

$$+(1-\rho^{exp2})\pi_t^{tar})) + \varepsilon_t^{exp} \tag{2.7.5}$$

From the perspective expected inflation is formed based on lag, lead and targeted inflation.

2.8 Balance of Payments

By combining HH's and fiscal authorities' budget constraints, we can write the balance of payment equation:

$$B_t^f = CA_t + R_t^f R_t^\rho exp\left(-\xi^{dl}\left(b_t^f - b^f\right) - \xi^{fp}\left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1\right)\right) B_{t-1}^f \tag{2.8.1}$$

where B_t^f is the foreign bond portfolio at the end of the period t and CA_t is the current account at time t. All variables are in USD.

We do not model cross-border flows other than the export and import of goods in our model, therefore, the current account balance is defined as:

$$CA_t = P_t^{xf} X_t - P_t^{mf} M_t \tag{2.8.2}$$

2.9 Foreign Sector

Given the economy under our considerations is small relative to the rest of the world, foreign variables are exogenously given for agents within the domestic economy. Also, to keep it as simple as possible, we do not model possible interactions between foreign variables, therefore, instead, they are modeled as separate AR(1) processes. For example, the growth rate of foreign output (which determines the demand for exported goods) is defined as:

$$(1+\gamma_t^{Y^*}) = \frac{Y_t^*}{Y_{t-1}^*} \tag{2.9.1}$$

We assume that it follows the AR(1) process:

$$\gamma_t^{Y^*} = (1-\rho_{\gamma^{Y^*}})\gamma^{Y^*} + \rho_{\gamma^{Y^*}}\gamma_{t-1}^{Y^*} + \varepsilon_t^{\gamma^{Y^*}} \tag{2.9.2}$$

Foreign price (Π_t^R) shocks could be transmitted to our local economy through the import sector by changing the marginal cost of imported goods, or through the export sector, as the price shocks in trade partners' economies alter the demand on goods produced in our economy. Inflation of the price index (Π_t^R) of homogenous goods produced in our trade partners' economies in aggregate currency units is assumed to follow AR(1) process as well:

$$\Pi_t^R = (1 - \rho_{\Pi^R})\,\Pi^R + \rho_{\Pi^R}\Pi_{t-1}^R + \varepsilon_t^{\Pi^R} \tag{2.9.3}$$

In the model, three pairs of exchange rates play a role. Firstly, lari against USD ($e_t^{GEL/D}$), as long as exported and imported goods are priced in USD, also, forex dealers keep foreign assets position in USD. Secondly, despite the price stickiness in USD, the exchange rate w.r.t. our trade partners economies ($e_t^{Gel/R}$) is important as well in the medium run. Thirdly, even if the lari does not change against our trade partners' economies but USD appreciates/depreciates globally ($e_t^{R/D}$), it still could have a material effect on our economy by changing the cost of imported goods and the relative price of exported goods due to dollar invoicing.

The effective exchange rate of USD against the basket of our trade partners' currencies is given by the following UIP condition:

$$\frac{(1 + i_t^{rw})(1 + \gamma_t^{e^{R/D}})}{1 + i_t^f} = \exp(\rho^{rwuip}((1 + \gamma_t^{R/D})(1 + E_t\gamma_{t+1}^{R/D}) - 1)) \tag{2.9.4}$$

Given the $e_t^{D/R}$ and $e_t^{Gel/D}$ in hand, the $e_t^{Gel/R}$ is defined as cross exchange rate.
The lari's exchange rate against the USD is influenced by the interest rate in the USA, which is assumed to follow the process:

$$i_t^f = (1 - \rho_{i^f})i^f + \rho_{i^f} i_{t-1}^f + \varepsilon_t^{i^f} \tag{2.9.5}$$

The foreign real rate could be decomposed as:

$$r_t^f = r_t^{fnut} + r_t^{fgap} \tag{2.9.6}$$

where, $r_t^{fnut}$and r_t^{fgap} are foreign real neutral interest rate and the gap of foreign real rate, respectively. Also, we could write that:

$$i_t^f = r_t^f - E_t \pi_{t+1}^f \tag{2.9.7}$$

where, π_t^f is USA inflation rate, and it matters for real foreign rate determination. To clarify, we can note that the model setup here is the three economy version to some extent. Our economy has real interlinkages with trade partner economies (where we export in and import goods from), in this regard, the foreign inflation Π_t^R matters for analysing trade competitiveness, for instance. However, we have the financial ineterlinkages with "financial center", we are trading with the center with foreign bonds denominated in USD.

While the interest rate in the rest of the world is given by *Foreign interest rate (ROW)*

$$i_t^{rw} = (1 - \rho_{irw}) i^{rw} + \rho_{irw} i_{t-1}^{rw} + \varepsilon_t^{irw} \tag{2.9.8}$$

2.10 Market Clearing and Aggregation

Domestic input market clearing. Firstly, we start by aggregating the input produced by differentiated input producers :

$$\int_0^1 Y_t(i)di = \int_0^1 \left(\gamma_t (z_t L_t(i))^{\alpha_1} K_t(i)^{\alpha_2} \left(\frac{Y_t^m(i)}{a_t^x} \right)^{1-\alpha_1-\alpha_2} - F_t^d \right) di \tag{2.10.1}$$

The equation can be written in terms of capital to labor and imported intermediate input to labor ratios:

$$\int_0^1 Y_t(i)di = \gamma_t \int_0^1 (z_t L_t(i)) \left(\frac{K_t(i)}{z_t L_t(i)} \right)^{\alpha_2} \left(\frac{Y_t(i)^m}{a_t^x z_t L_t(i)} \right)^{1-\alpha_1-\alpha_2} di - F_t^d \tag{2.10.2}$$

Constant return to scale production function implies that factor shares should be same across firms. Then, the the aggregated production function can be rewritten as:

$$\int_0^1 Y_t(i)di = \gamma_t \left(\frac{K_t}{z_t L_t} \right)^{\alpha_2} \left(\frac{Y_t^m}{a_t^x z_t L_t} \right)^{1-\alpha_1-\alpha_2} z_t \int_0^1 L_t(i)di - F_t^d \tag{2.10.3}$$

Let's define firms' total labor demand as:

$$L_t = \int_0^1 L_t(i)di \tag{2.10.4}$$

Finally,

$$Y_t = \int_0^1 Y_t(i)di = (z_t L_t)^{\alpha_1} \gamma_t K_t^{\alpha_2} Y_t^{m1-\alpha_1-\alpha_2} - F_t^d \tag{2.10.5}$$

The aggregated demand for differentiated intermediate inputs is given by:

$$\int_0^1 Y_t(i)di = \int_0^1 \left(\frac{P_t^d(i)}{P_t^d}\right)^{-\eta_t^d} Y_t^d di = Y_t^d \int_0^1 \left(\frac{P_t^d(i)}{P_t^d}\right)^{-\eta_t^d} di \tag{2.10.6}$$

Let's define price dispersion in domestic input production sector as $d_t^d \equiv \int_0^1 \left(\frac{P_t^d(i)}{P_t^d}\right)^{-\eta_d} di$. Then the domestic intermediate inputs market clears when:

$$Y_t = d_t^d Y_t^d \tag{2.10.7}$$

We can show that (see, Appendix G) the price dispersion can be written recursively as:

$$d_t^d = (1-\theta_d)\left(\frac{P_t^{*d}}{P_t^d}\right)^{-\eta_t^d} + \theta_d {\Pi_{t-1}^d}^{-\eta_t^d} {\Pi_t^d}^{\eta_t^d} d_{t-1}^d \tag{2.10.8}$$

Labour market clearing On the labor market HHs set wages and supply labor input as much as to satisfy labor demand determined by labor bundlers optimization problem. In turn the labor agency aggregates labor input to meet firms' demand on labor, i.e.:

$$L_t^s = \int_0^1 L(i)di = \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\eta_t^l} L_t di = d_t^w L_t \tag{2.10.9}$$

where the term $d_t^w \equiv \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\eta_t^l} di$ is the measure of wage dispersion, which can be write recursively as:

$$d_t^w = (1-\theta_w)\left(\frac{W_t^*}{W_t}\right)^{-\eta_t^l} + \theta_w {\Pi_{t-1}^w}^{-\eta_t^l} {\Pi_t^w}^{\eta_t^l} d_{t-1}^w \tag{2.10.10}$$

Capital market clearing. On physical capital market, aggregate demand on capital input K_t is met by total utilized capital rented out by entrepreneurs:

$$K_t = u_t \overline{K}_t \tag{2.10.11}$$

Aggregate consumption. Before deriving aggregate resource constraint of our economy, it is more convenient to aggregate its parts separately in advance. Let's start from aggregate consumption, in our framework $(1-\lambda)$ part of HHs are unconstrained HHs, while the rest part is constrained non ricardian type consumers, Then the aggregate consumption could be derived as:

$$C_t = \int_0^1 C_t(i)di = \int_0^\lambda C_t^c(i)di + \int_\lambda^1 C_t^{uc}(i)di = \lambda C_t^c + (1-\lambda)C_t^{uc} \tag{2.10.12}$$

Aggregate profit functions. Profit earned by monopolistic competitive firms in any sector is transferred to HHs.

Aggregate profit function in domestic input production:

$$\begin{aligned}
\pi r_t^d =& \int_0^1 \left(P_t^d(i)Y_t(i) - MC_t^d\left(Y_t(i)+F_t^d\right)\right)di = \int_0^1 \left(\frac{P_t^d(i)}{P_t^d}\right)^{-\eta_t^d} Y_t^d P_t^d(i)di - \\
&-MC_t^d \int_0^1 \left(\frac{P_t^d(i)}{P_t^d}\right)^{-\eta_t^d} Y_t^d - MC_t^d F_t^d = Y_t^d P_t^{d\eta_d} \int_0^1 P_t^d(i)^{1-\eta_d} di - MC_t^d d_t^d Y_t^d - MC_t^d F_t^d \\
=& Y_t^d P_t^{d\eta_d} P_t^{d1-\eta_d} - MC_t^{r^d} P_t^d d_t^d Y_t^d - MC_t^{r^d} P_t^d F_t^d = P_t^d Y_t^d \left(1 - MC_t^{r^d} d_t^d\right) - MC_t^{r^d} P_t^d F_t^d = \\
=& P_t^d Y_t^d - R_t^k K_t - W_t L_t - P_t^{mG} Y_t^m
\end{aligned} \tag{2.10.13}$$

Aggregate profit of differentiated exported goods producers:

$$\begin{aligned}
\pi r_t^x =& \int_0^1 \left(e_t^{Gel/D} P_t^{xf}(i)X_t(i) - MC_t^{xf}\left(X_t(i)+F_t^x\right)\right) = \int_0^1 e_t^{Gel/D} P_t^{xf}(i)\left(\frac{P_t^{xf}(i)}{P_t^{xf}}\right)^{-\varepsilon_t^x} X_t di - \\
&-MC_t^x \int_0^1 \left(\frac{P_t^{xf}(i)}{P_t^{xf}}\right)^{-\varepsilon_t^x} X_t di - MC_t^x F_t^x = e_t^{Gel/D} P_t^{xf^{\varepsilon_t^x}} X_t \int_0^1 P_t^{xf}(i)^{1-\varepsilon_t^x} di - \\
&-MC_t^x X_t \int_0^1 \left(\frac{P_t^{xf}(i)}{P_t^{xf}}\right)^{-\varepsilon_t^x} di - MC_t^x F_t^x = e_t^{Gel/D} P_t^{xf} X_t - MC_t^x d_t^x X_t - MC_t^x F_t^x = \\
&= P_t^{xG} X_t \left(1 - MC_t^{r^x} d_t^x\right) - MC_t^{r^x} P_t^{xG} F_t^x
\end{aligned} \tag{2.10.14}$$

where, d_t^x is the measure of price dispersion in exported goods sector, and it could be written recursively as:

$$d_t^x = (1-\theta_x)\left(\frac{P_t^{*xf}}{P_t^{xf}}\right)^{-\varepsilon_t^x} + \theta_x {\Pi_{t-1}^{xf}}^{-\varepsilon_t^x} {\Pi_t^{xf}}^{\varepsilon_t^x} d_{t-1}^x \tag{2.10.15}$$

The last term in profit function is aggregated total cost in production of differenciated exported goods which is produced using domestic and imported inputs, therefore, the function could be written as:

$$\pi r_t^x = e_t^{Gel/D} P_t^{xf} X_t - MC_t^x d_t^x X_t - MC_t^x F_t^x = e_t^{Gel/D} P_t^{xf} X_t - P_t^d X_t^d - P_t^{xG} X_t^m \tag{2.10.16}$$

Aggregate profit functions of entrepreneurs, forex dealers, final consumption, investment and government goods producers are given by, accordingly:

$$\pi r_t^e = R_t^k \overline{K}_t u_t - \gamma(u_t)\overline{K}_t P_t^i - I_t P_t^i \tag{2.10.17}$$

$$\pi r_t^{fx} = e_t^{Gel/D} B_{t-1}^f R_t^f R_t^\rho exp\left(-\xi_{dl}\left(b_t^f - b^{f^{ss}}\right) - \xi^{fp}\left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1\right)\right) - e_t^{Gel/D} B_t^f \tag{2.10.18}$$

$$\pi r_t^c = P_t^c C_t - P_t^d C_t^d - P_t^{mG} C_t^m \tag{2.10.19}$$

$$\pi r_t^i = P_t^i I_t - P_t^d I_t^d - P_t^{mG} I_t^m \tag{2.10.20}$$

$$\pi r_t^g = G_t - P_t^d G_t^d - P_t^{mG} G_t^m \tag{2.10.21}$$

Finally, total profit generated in the economy is given by:

$$\pi {r_t}^T = \pi r_t^d + \pi r_t^x + \pi r_t^e + \pi r_t^{fx} + \pi r_t^c + \pi r_t^i + \pi r_t^g \tag{2.10.22}$$

Aggregate resource constraint. The aggregate resource constraint could be derived by summing up budget constraints of HHs and the government:

$$
\begin{aligned}
&\int_0^\lambda \left((1+\tau^c)P_t^c C_t^c - (1-\tau^w)W_t L_t - T_t^c\right) di + \\
&\quad + \int_\lambda^1 \left((1+\tau^c)P_t^c C_t^{uc}(i) + B_t^{uc}(i) + E_t Q_{t,t+1} a_{t+1}(i)\right) di - \\
&\quad - \int_\lambda^1 \Big((1-\tau^w)W_t(i)L_t(i) + R_{t-1}B_{t-1}^{uc}(i) + a_t(i) + T_t^{uc} + (1-\tau^{\pi r})D_t^{uc}(i)\Big) di + \\
&\quad + G_t + T_t^T - \tau^c P_t^c C_t - \tau^w W_t L_t - \tau^\pi \pi r_t^T + R_{t-1}D_{t-1} - D_t = 0
\end{aligned}
\tag{2.10.23}
$$

Trade with Arrow Debreu security implies that individual consumption equals to average consumption. Also, we use the equivalences that aggregate profit $\pi r_t^T = (1-\lambda)D_t^{uc}$, where D_t^{uc} is aggregate dividend received by unconstrained HHs (actually, that is profit received by firms owned by individual unconstrained HHs and aggregated over the subset $(1-\lambda)$). Also, $D_t = (1-\lambda)B_t^{uc}$, and T_t^T is the total transfer from the government to constrained and unconstrained HHs. Then the resource constraint can be written as:

$$
\begin{aligned}
&\lambda\Big((1+\tau^c)P_t^c C_t^c - (1-\tau^w)W_t L_t - T_t^c\Big) + (1-\lambda)((1+\tau^c)P_t^c C_t^{uc} + B_t^{uc} - \\
&\qquad -(1-\tau^w)W_t L_t - R_{t-1}B_{t-1}^{uc} - T_t^{uc} - (1-\tau^\pi)\pi r_t^T) + \\
&\qquad + G_t + T_t^T - \tau^c P_t^c C_t - \tau^w W_t L_t - \tau^\pi \pi r_t^T + R_{t-1}D_{t-1} - D_t = 0
\end{aligned}
\tag{2.10.24}
$$

After collecting same terms and putting profit functions in the aggregate constraint, we get:

$$
\begin{aligned}
&P_t^c C_t + G_t - W_t L_t - \\
&\quad -(P_t^d Y_t^d - R_t^k K_t - W_t L_t - P_t^{mG} Y_t^m) - (e_t^{Gel/D} P_t^{xf} X_t - P_t^d X_t^d - P_t^{mG} X_t^m) - \\
&\quad -(R_t^k \overline{K}_t u_t - \gamma(u_t)\overline{K}_t P_t^i - I_t P_t^i) - \\
&\quad -e_t^{Gel/D} B_{t-1}^f R_t^f R_t^\rho exp\left(-\xi_{dl}\left(b_t^f - b^{f^{ss}}\right) - \xi^{fp}\left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1\right)\right) \\
&\quad -(P_t^c C_t - P_t^d C_t^d - P_t^{mG} C_t^m) - (P_t^i I_t - P_t^d I_t^d - P_t^{mG} I_t^m) - \\
&\quad -(G_t - P_t^d G_t^d - P_t^{mG} G_t^m) = 0
\end{aligned}
\tag{2.10.25}
$$

Taking into account that wage per unit of effective worker $w_t = \frac{W_t}{z_t}$, also, aggregate demand on imported inputs $M_t = C_t^m + I_t^m + G_t^m + X_t^m + Y_t^m$; and taking into account balance of payment identity : $e_t^{Gel/D} P_t^{xf} X_t - e_t^{Gel/D} P_t^{mf} M_t = e_t^{Gel/D} B_t^f - e_t^{Gel/D} B_{t-1}^f R_t^f R_t^\rho exp\left(-\xi_{dl}\left(b_t^f - b^{f^{ss}}\right) - \xi^{fp}\left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1\right)\right)$ after collecting same terms in the above equations, we end up with:

$$P_t^d Y_t^d = P_t^d C_t^d + P_t^d I_t^d + P_t^d G_t^d + P_t^d X_t^d + P_t^i \gamma(u_t)\overline{K}_t \tag{2.10.26}$$

This equation defines aggregate demand function on domestic produced goods. Now, let's introduce definition of nominal GDP:

$$GDP_t = (1+\tau^c) P_t^c C_t + P_t^g G_t + P_t^i I_t + (e_t^{Gel/D} P_t^{xf} X_t - e_t^{Gel/D} P_t^{mf} M_t) \tag{2.10.27}$$

GDP deflater. The GDP deflator is not determined within our model. In order to derive the real GDP, we define the GDP deflator as the weighted average of price indexes of the corresponding components of nominal GDP.

$$P_t^y = P_t^{c s_c} P_t^{g s_g} P_t^{i s_I} \left(e_t^{Gel/D} P_t^x\right)^{s_x} \left(e_t^{Gel/D} P_t^{mf}\right)^{-s_m} \tag{2.10.28}$$

where, s_c , s_i , s_g , s_x , s_m are the shares of the aggregate consumption expenditure, investment, government spending, export and import in the nominal GDP at steady state accordingly; note, that in our model nominal shares are stationary. Then the *real GDP* is given by:

$$GDP_t^r = \frac{GDP_t}{P_t^y} \tag{2.10.29}$$

Also, the aggregate absorption is definition as:

$$ABS_t = P_t^c C_t + P_t^g G_t + P_t^i I_t \tag{2.10.30}$$

Alternatively, the aggregate resource constraint implies that:

$$\begin{aligned} e_t^{Gel/D} B_t^f - e_t^{GEL/D} B_{t-1}^f R_{t-1}^f R_t^\rho exp\left(-\xi_{dl}\left(b_t^f - b^{f^{ss}}\right) - \xi^{fp}\left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1\right)\right) \\ = GDP_t - ABS_t \end{aligned} \tag{2.10.31}$$

Taking into account the definition of the current account balance $CA_t = (e_t^{Gel/D} P_t^{xf} X_t - e_t^{Gel/D} P_t^{mf} M_t)$ it could be written as:

$$CA_t = GDP_t - ABS_t \tag{2.10.32}$$

3 Properties of the Model

3.1 Initial Calibration

Before we estimate the model more formally, various strategies are used to calibrate model parameters. This could be used as our initial guess on the model parameters (or priors) and would be applied to analyse impulse response functions (IRFs). In turn, IRFs could be useful to re-calibrate those parameters to match the model implied properties. The parameters which determines the growth rates in steady state are calibrated based on averages of historical time series. Also, parameters which directly impacts the shares of model variables in steady state is calibrated based on data (the "great ratios", for instance). Additionally, some parameters are calibrated based on steady state restrictions. The rest of the parameters are calibrated using literature, also, data is used to calibrate a few more parameters, for instance, the Elasticity of Substitution (EoS) parameter in domestic intermediate input production is calibrated based on firms' micro-data. More details on the methodology used in the calibration process is given below, as well as calibrated parameters are summarized in the table 1

Steady State parameters

- The labor force is stationary process by assumption, then it follows that the growth rate of potential GDP and labor augmented technology coincides to each other. Therefore, we set value of γ^z to match potential GDP growth in Georgia, as few available studies on potential growth about Georgia show (for example, Liqokeli(2017)) it was around 4.5% annually befor the pandemic.

- We assume that the growth rate of foreign GDP, γ^{Y^*} coincides its potential growth γ^{z^*} in SS. The parameter is calibrated as sample average (over the last 10 years) of trade partners economies' weighted GDP growth rates.

- NBG's current inflation target (3% annual) is chosen as SS value of consumer price inflation in Georgia, while we assume that steady-state foreign inflation is 2% annually.
- We assume that foreign nominal interest rates (in USD as well as in RoW currency) in SS are 5% annually.
- The country risk premium is calibrated as 300 bp. annually.
- Taking into account above facts, the real UIP condition in SS implies that the rate of trend appreciation of the real exchange rate is $\gamma^{a^x} = 0.01$ annually (which is quite close to the historical average of the growth rate of REER. Then the trend relationship between export-specific and import inefficiency technologies implies that $\gamma^{a^r} = 0.0012$.

Steady State shares

- The initial guess of steady state shares of GDP components is derived from the data, however, calibration using only historical data could be misleading in our case, given the shares are not stationary in the data. Therefore, those parameters are calibrated using judgment, as well as, taking into account the adequate size of the trade deficit to maintain external debt sustainability (to calibrate the share of export and import in GDP).
- We use BEC classification (Classification by Broad Economic Categories) of imported goods to estimate the import components of private and government final consumption, investment, export and imported intermediate input in domestic production. We treat food and beverages, and other consumer goods not elsewhere classified as pure consumption goods. In addition, we add part of imported fuels and lubricates to consumption goods (proportional to the share of fuel consumption in the CPI basket). The rest part of the fuels and lubricates are treated as an intermediate input. Also, part of the motor cars not re-exported is added to consumption goods. While the rest part of the imported (but not re-exported) transport equipment is treated as investment goods and added to imported capital

goods classified with BEC. Finally, we treat re-export as an imported component of exported goods.

As for the allocation of service import, we made the following assumptions: debit of travel in BOP is treated as an import component of consumption, while the import of transportation service is attributed to the relevant imported categories based on categories' shares in imported goods. The rest of the services in BOP are treated as import of intermediate input.

Neither BEC classification of imported goods, nor BOP categories, are useful to disaggregate consumption into private and government parts. Therefore, we use the following assumptions to estimate the share of imported input in government consumption (instead of simply assuming that the share is zero). In the first stage, we subtract the government wage bill from the government's total consumption. Afterward, we make an assumption that the share of imported input into the rest part of the government consumption (goods and services) equals the same share in private consumption.

By applying above mentioned assumptions and modifications to the data, we calibrate the share of imported goods in private consumption at 0.3, and the share of imported input in investment goods production is higher (0.44) as expected, and the share is about 0.14 in government's consumption. Finally, the share of imported input in exported goods production is calibrated at 0.21.

- To calibrate the input shares in domestic intermediate input production, we use output disaggregation by income components, for example, Gomme and Lkhagvasuren (2012); however, in our model, the output generated in the intermediate production stage is not directly linked to the data, given that imported input is used in domestic intermediate input production. Therefore, we use the steady state relationship between domestic intermediate input and GDP to recover the latter one in the data:

$$\widetilde{GDP} = p^d\widetilde{Y} - p^{mG}\widetilde{Y^m} \tag{3.1.1}$$

Hence, we add imported intermediate input back to GDP to get output. As long as the operating surplus in the dis-aggregation of GDP by income categories does not include profit related to public capital consumption, we have two options to calibrate input shares. Either to add the profit earned with public capital (which is not available) to operating surplus, or to exclude government wage bill from the compensation of employees; we use the latter strategy. Also, the treatment of mixed incomes is challenging for calibrating input shares. Instead of adding the mixed incomes to employees' compensation and operating surplus proportionally, we have discarded this component completely to generate appropriate series of compensation for employees and return on capital input (see Gomme and Lkhagvasuren (2012)). Those shares calibrated with the data could be insightful, however, given those shares are not stationary (within the entire dataset), we have revised long-run averages based on judgment. Finally, the labor share is calibrated as $\alpha_1 = 0.42$ while the share of the capital $\alpha_2 = 0.35$, the rest part of the production of domestic intermediate input is accounted with imported input.

- We assume that the government aims to keep the public debt to GDP ratio below 40% and the external debt to GDP ratio is assumed to be 100% in SS (annually).
- We assume that the transfers to unconstrained HHs is zero, then the $\widetilde{tr^{cr}}$ is calibrated based on the historical average of government transfers to GDP ratio.

Preference and technology parameters

- Our initial guess about capital depreciation rate quarterly is $\delta = 0.025$ which implies 10% depreciation of the physical capital annually.
- The habit persistence parameter plays an important role to confront model-implied moments to the data. For example, Havranek, et. al (2017) shows that different values assigned to the habit formation parameter imply different response of output to the monetary policy shock, higher persistence implies more hump-shape reaction of output to the shock, which is related to lower variability of consumption using habit formation. The parameter is not estimated for Georgia, therefore, we assume its value to be 0.7 as starting point to match impulse responses, which is close to the mean value of the persistence parameter estimated within DSGE models (for example, Havranek, et. al (2017)).

- The presence of constrained HHs together with habit persistence, makes the deviation from the permanent income hypothesis (PIH) in the model. However, the distinction is clear, the former one implies that consumption immediately reacts to current income, while the reaction is delayed in the case of habit persistence. As Fuhrer, J.(2000), shows both of them improve the empirical fit of the model. The share of the rule of thumb consumers was first estimated by Campbell and Mankiw (1989), the estimated share of this type of consumer represents half of all Households in the case of the US. The method used by Campbell and Mankiw (1989) holds on the assumption of PIH, implying that the consumption of unconstrained HHs is a random walk and therefore, the remaining sensitivity of consumption growth to current income growth estimates the share of constrained HHs. Unfortunately, this type of estimate is not available in our case. Alternatively, the share of constrained HHs is calibrated using the share of bank account holders to the total population in the literature. Given that a person can hold multiple accounts in different banks nowadays, the number of accounts is larger than the population size, therefore, it is hard to estimate the share of non-optimizers based on this kind of data. Hence, we are left to pick $\lambda = 0.3$ which is a widely used value in the literature to calibrate the share of rule-of-thumb consumers.

- The EoS of differentiated input in domestic homogeneous intermediate input production, is calibrated based on the estimation of mark-ups collected from financial reports of companies. As the data shows, the average EBITDA margin is approximately 0.18 in Georgia, which suggests a close estimate of EoS used in literature. Therefore, we assume that $\eta_d = 6$. Unfortunately, disaggregation of profit margins is not possible for companies operating in domestic intermediate input production and export-oriented sectors. Therefore, we assume that the profit margin is same for all sectors, hence, we assign the same values to $\varepsilon^x = 6$ and $\varepsilon^m = 6$.

- Estimates of the elasticity of substitution of domestic and imported inputs in final goods production varies in the literature. For example, Bajzik, et al. 2019, based on a meta-analysis of estimated Armington parameters summarize that it varies from 0 to 8; we assume this value to be at 1.5 in our case, which sets within 95% confidence interval estimates of Armington parameter of elasticity (weighted) of substitution for developing countries. Developing countries are characterized by high elasticity of substitution given limited production capabilities domestically, i.e. it is not as difficult to find substitute products abroad. It is worth mentioning that the parameter plays an important role to make the model implied properties of trade balance consistent with data (for example, Backus et. al, 1994).

- Among the parameters σ_a and σ_b determining the shape of capital utilization cost: the latter is calibrated using steady-state restrictions, while our initial guess is $\sigma_a = 0.5$ based on the literature (for example, Copaciu et al.2015); in most cases this parameter is estimated with the data, given its effect on dynamic properties of the model. Furthermore, the steady state parameter S'' is estimated with data too. We use relatively low value of the parameter $S'' = 2.5$ (For example, Christiano et al. 2005 set its value as 5) to allow higher volatility of investment, this could be intuitive for emerging markets like Georgia.

- In the model, there are three parameters related to sticky prices. We use the estimation results of the Phillips curve in the case of Georgia (Arevadze, et al. 2020) and set the value of $\theta_d = 0.6$. The price stickiness of exported and imported goods in USD isn't estimated there, therefore, we assume that firms operating in those sectors keep prices unchanged with same duration as in case of domestic differentiated input producers; i.e. $\theta_m = \theta_x = 0.6$. Also, we assume that wage contracts are stickier than the price set by producers, therefore we assume that $\theta_w = 0.75$, which implies that wages are updated once a year.

- The sensitivity of risk premium to expected depreciation is calibrated as 0.5 as a baseline to analyze model properties, it could be related with the share of backward looking agents on the FX market. In the alternative version of forex dealer's profit optimization problem where we have portfolio adjustment cost to account possible liquidity shortages if market is shallow, we calibrate the foreign portfolio adjustment coefficient as $\xi_{adj} = 1$. As it is given in the next section, under the calibration, we achieve almost the same reaction of the exchange rate to various shocks as suggested by more standard (lagged) modified UIP. Moreover, the parameter could be re-calibrated to match IRFs of the nominal exchange rate to the interest rate shock. The larger the parameter, the dynamics of the exchange rate would be smoother.

- We calibrate elasticity of risk premium w.r.t. foreign debt to GDP ratio at 0.0025, implying the rise of risk premium by 1 pp annually if the debt to GDP ratio annually increases by 100 bp. The value is consistent with DSGE literature, but some empirical estimates suggests different values of the parameter (see, for example, Brzoza-Brzezina and Kotłowski, 2016).

Policy parameters and tax rates

- The primary surplus reaction coefficient is assumed to be $\psi = 0.0625$ and fiscal policy persistence parameter is calibrated as $\rho_b = 0.7$. These values are calibrated based on a hypothetical case to close 5% deviation from the sustainable level of public debt (50% of GDP) in 3 years of budget planning horizon without excessive fiscal measures.

- The effective tax rates are calibrated to match sample ratios of relevant tax revenue to output.

Table 1: Calibrated values of the model parameters

Parameter	Parameter Value	Name
	Preference and technology	
β	0.996	subjective discount factor
h	0.7	habit persistence
λ	0.3	share of constrained HHs
η_l	6	EoS of differentiated labor
θ_w	0.75	wage stickiness
ζ	1.5	inverse of labor supply elasticity
χ	9.79	scaling factor of labor disutility[11]

[11] The parameter is calibrated using steady-state restrictions of the model, same is true about σ_b

σ_a	0.5	capital utilization cost parameter
σ_b	0.09061	capital utilization cost parameter
S''	2.5	investment adjustment cost parameter
δ	0.025	rate of depreciation of capital
η_d	6	EoS of differentiated domestic inputs
θ_d	0.6	price stickiness , domestic
α_1	0.43	labor share in production
α_2	0.35	capital share in production
η_c	1.5	EoS, final consumption goods
η_i	1.5	EoS, final investment goods
η_g	1.5	EoS, final public goods
ε^m	6	EoS, differentiated imported input
θ_m	0.6	price stickiness, import
ε^x	6	EoS, differentiated exported goods
θ_x	0.6	price stickiness, export
η_x	1.5	EoS, final exported goods
ξ_{fp}	0.5	Sensitivity to expected depreciation
ξ_{dl}	0.0025	elasticity to external debt
	Shares	
ω_c	0.30	share of imported input in consumption
ω_i	0.44	share of imported input in investment
ω_x	0.21	share of imported input in Export
ω_g	0.14	share of imported input in public goods production
S^c	0.650	share of private consumption in GDP
S^i	0.245	share of investment in GDP
S^g	0.130	share of government consumption in GDP
S^x	0.600	share of export in GDP
S^m	-0.624	share of import in GDP
	policy parameters and tax rates	
δ_1	0.5	monetary policy persistence
δ_2	1.5	reaction to inflation deviation

ρ^{exp}	0.5	persistence of expected inflation
ρ^{exp2}	0.5	contribution of inflation lead
ρ^{exp2}	0.5	share of agents who forms expectations based on lagged inflation
ρ_b	0.7	persistence of primary balance
ψ	0.0625	primary surplus reaction coefficient
τ^w	0.18	labor income tax rate
τ^c	0.14	value-added tax rate
$\tau^{\pi r}$	0.07	profit tax rate
	Steady State parameters	
b^f	-4.0	external assets to GDP (quarterly) ratio
d	1.6	public debt to GDP (quarterly) ratio
tr^{c^r}	0.089	government transfers to output ratio
R^{ρ}	0.0074	risk premium
γ^z	0.011	growth rate, labor augmented technology
γ^{a^x}	0.0024	growth rate, inefficiency of imported input
γ^{a^r}	0.0003	growth rate, export-specific technology
$\gamma^{D/R}$	0.0	rate of appreciation of USD w.r.t. rest of the world currencies
γ^{Y^*}	0.0062	growth rate of foreign GDP
γ^{z^*}	0.0062	growth rate, potential foreign GDP
π^{tar}	0.0074	domestic inflation target
π^R	0.005	foreign inflation target
i^f	0.015	foreign nominal interest rate
	Persistence of autoregressive process[12]	
ρ_θ	0.8	persistence of preference shock
ρ_ψ	0.8	persistence of labor supply shock
ρ^{η^l}	0.7	persistence of EOS of labor inputs

[12] We set persistence of exogenous process as 0.7 in most cases, except the parameters related to Households' preferences and labor supply, also, we set a low value (0.5) of the persistence of monetary policy shock, because high values of the parameter imply negative response of nominal interest rate to the positive monetary policy shock, (for example, Gali, et al, 2007)

ρ^{η^d}	0.7	persistence of EOS of domestic intermediate inputs
ρ^{ε^x}	0.7	persistence of EOS of differentiated exported goods
ρ^{ε^m}	0.7	persistence of EOS of differentiated imported goods
ρ_{γ}	0.7	persistence of TFP shock
ρ_{γ^z}	0.7	persistence of labor augmented technology rate
$\rho_{\gamma^{a^x}}$	0.7	persistence of import inefficiency technology
$\rho_{\gamma^{a^r}}$	0.7	persistence of export specific technology
ρ_i	0.5	persistence of monetary policy shock
ρ_r	0.5	persistence of domestic real int. rate
ρ_u	0.7	persistence of government spending shock
ρ_{π^R}	0.7	persistence of foreign inflation
ρ_{i^f}	0.7	persistence of foreign interest rate
$\rho_{\gamma^{z*}}$	0.7	persistence of growth rate of foreign potential GDP
$\rho_{\gamma^{Y*}}$	0.7	persistence of foreign growth rate
$\rho_{e^{D/R}}$	0.7	persistence of appreciation rate of USD w.r.t. rest of the world
ρ_{prem}	0.7	persistence of risk premium shock
ρ_{rwuip}	0.6	share of backward-looking agents on (USD/RW) fx market
ρ_{irw}	0.7	persistence of foreign interest rate (in RW)

3.2 Impulse Response Functions

In order to validate the model properties and analyse shock propagation mechanism, we have conducted an impulse response analysis. The main objective is to demonstrate that the model is capable to replicate predictions in line with the New Keynesian theory, both in qualitative as well as in quantitative terms. Moreover, new features introduced here show their usefulness in making the model-implied results more consistent with empirical facts. This encourages us to claim that the setup of the model is appropriately designed to be employed for policy analysis and as a storytelling device in our context.

3.2.1 Policy Shocks

Monetary policy shock. The nominal short term rate increases by 20 bps in response to the +25 bps. of monetary policy shock[13] and the real interest rate increases gradually too. The endogenous reaction of monetary policy dampens the reaction of nominal rate to the shock. Also, the relatively muted response of the nominal interest rate to the shock could be explained by decrease of nominal neutral rate to rsponse to the shock. We could analyze the transmission of the shock through different channels: **Demand channel -** consumption drops as long as the real interest rate increases. The negative effect is more evident in the case of credit-constrained HHs whose consumption is tightly related to their current income. Which could be explained with the deterioration of employment prospects (indirect effect of monetary policy). On the other hand, the government has to have a pro-cyclical reaction (due to the debt rule, when the interest rate payment increases and the ratio raises also as a result of output dropping) and cut expenditures and transfers to HHs after the shock. Facing lower demand, the domestic input producers reduce the employment of labor as well as capital service in the production, but the lower demand is also driven by higher input prices initially. As long as consumption drops, the cost of working decreases, therefore, HHs are getting to set lower wages. However, due to the wage stickiness the reduction in nominal wage is insufficient and real wage increases initially. Also, the nominal rental rate increases due to the arbitrage condition, i.e. the marginal product of capital needs to increase, which could happen if capital usage decreases. However, due to capital utilization cost, which drops after the shock, the fewer capital in service could be met without a significant rise in the rental rate. Therefore, the rental rate increases, but with lesser pace than implied by models without the frictions. The real wage increases too, therefore, we could think that marginal costs would increase. However, due to the REER appreciation, imported inputs become cheaper in the production having downward pressure on marginal costs. On top of that, domestic intermediate input producers are setting prices on their products by taking into account future marginal costs, which decreases definitely regardless of the initial hike in real wages, because of

[13] The size of all other shocks are 25 bps., too; also, all variables (policy rate included) are quarterly, therefore, the size of monetary policy transmission in annualized terms would be same as expressed on IRFs figures.

wage stickiness, a drop in nominal wages takes time. As a result, domestic inflation decreases gradually, and reaches its peak at the end of the third quarter (-13 bps.). **Exchange rate channel** - after the shock, the REER appreciates, which makes imported goods cheaper. Because of high share of imported final goods in the CPI basket, CPI inflation decreases sharply (by 15 bps). Hence, no hump shape pattern of the CPI inflation could be explained with the exchange rate dynamics. Related to the monetary policy shock, we also check model properties under different specifications of monetary policy reaction function. The policy reactions to deviations of 1 quarter vs. 4 quarters ahead of expected inflation from the target are compared. The results are intuitive, real costs are lower if monetary authority reacts to one quarter ahead expected inflation since inflationary expectations are better anchored in this case relative to reaction to the 4 quarters ahead expected inflation. In the latter case this policy reaction is weaker and the real side of the economy is less affected (see the Figure 2[14]).

Inflation target shock. If the monetary authority decides to rise the inflation target it needs to ease the monetary policy initially, to anchor inflationary expectations at a higher level. All nominal variables rise after the shock in the steady state too. In contrast, the effect on real variables is only transitory. The key variable, which largely determines the transmission of the shock to the real variables, is the nominal exchange rate. If the UIP is partly backward-looking, then the nominal exchange rate needs substantial time to depreciate until the new high level, while inflation rises much faster. As a result, the real exchange rate appreciates in the meantime. Moreover, delayed adjustment of the nominal exchange rate also makes the adjustment of the nominal interest rate more persistent, meaning monetary authority needs to keep the policy rate below new neutral for a longer time to anchor inflation expectations at a higher level. Therefore, the real variables: consumption, investment and GDP remain above the steady state for a longer period of time. The transmission of the shock is much different in the case of a fully forward-looking UIP. Since, the nominal exchange rate depreciates immediately after the shock, helping the central bank to raise the policy rate much quicker, as long as inflationary expectations increase faster. Therefore, the

[14]Note, that subplots on GEL/ROW and GEL/USD shows the rate of change of respective exchange rates

real effects of the shock are significantly muted relative to the previous case (see, figure 3).

Government spending shock (consolidation). Fiscal consolidation has a contractionary effect on output, while inflation decreases as demand shrinks. The nominal exchange rate depreciates after monetary policy easing, consequently, export expands and import shrinks. The weaker demand also contributes to a decrease in import and improves the current account balance. That said, exchange rate is likely to appreciate, as long as foreign debt declines, however, it is outweighed by the depreciation pressure resulted from the endogenous resection of interest rate to the shock. As for the other components of GDP, investment expands given lower expected real interest rates. However, aggregate consumption declines, mainly because of the sharp reduction of consumption expenditures by hand-to-mouth consumers, while the Ricardian consumers benefit with lower interest rate and expand their consumption. Therefore, the introduction of constrained consumers in the model implies a negative response of consumption to fiscal consolidation (i.e. positive relationship) (see Figure 4) that is in line with empirical evidence on the response of consumption to the fiscal shock, contrary to the models with only Ricardian consumers (see Gali, Lopez-Salido, Vales, 2003). Reaction to the fiscal shock largely depends on the parametrization of the debt rule. Fiscal consolidation initially reduces the debt-to-GDP ratio below the target, which implies the expansionary fiscal policy in later periods to ensure that debt-to-GDP stabilizes on a sustainable level. If the reaction is faster (or stronger), it implies higher volatility of model variables.

As mentioned, real and nominal exchange rates depreciate after the contractionary fiscal shock, which is the ambiguous property of standard DSGE models. While the opposite dynamic is evident in empirical research (for example, Iawata, 2012; Ravn, Schmitt-Grohe, Uribe, 2012). It means that the model features are not appropriate to account for the stylized facts in this regard. Therefore, further extensions are required, for example, productive government spending, adequate frictions on financial markets and etc.

Transfers to HHs. Contrary to the exogenous expansion of fiscal deficit (which has an expansionary effect on output), the positive shock on transfers to HHs has a contractionary effect on output. This dynamic is implied by the debt rule, after expanding transfers to HHs debt rule is activated and the endogenous part of government spending on public goods decreases automatically. This implies lower demand for domestically produced goods and consequently real GDP falls. Therefore, the positive impact on consumption is outweighted by the negative endogenous reaction of expenditure on public goods. As aggregate demand falls, the real marginal costs decrease too, which has downward pressure on inflation and interest rates. Nominal and real exchange rates depreciate initially which helps improvement in CA balance. The last findings come with a caveat. In general, we expect a deterioration of CA balance after the shock. As mentioned the result could be explained by the automatic opposite reaction of government spending in order to keep debt-to-GDP back to target. As an alternative specification we set an exogenous path of government spending on public goods deterministically (we assumed that government keeps the component unchanged). The response of the model variable to the shock is significantly different now. In line to our expectations, the positive demand pressure increases marginal costs and inflation rises slightly. In response to monetary policy tightening, the nominal exchange rate appreciates too while the CA balance deteriorates as import increases. Therefore, if transfers are exogenously determined (i.e. the deviation from the debt rule is tolerated in the meantime and there is no corresponding adjustment in the budget to keep debt unchanged), it would have a sizeable positive impact on consumption, import increases and CA balance deteriorates as expected (see Figure 5).

3.2.2 Demand Side Shocks

Preference shock. Positive preference shock increases marginal utility from current consumption irrespective of its cost, consequently, consumption increases. Domestic firms experience higher demand and tend to increase production as prices are sticky, i.e. prices and wages remain below optimal in the meantime. Higher demand creates upward pressure on prices, therefore, the monetary policy rate needs to be tightened (see Figure 6). Higher nominal rate drives exchange rate appreciation on impact.

Together with higher domestic demand, import increases substantially and CA balance deteriorates. Also, it is interesting to discuss how the shock is propagated in labor and capital markets. There are two opposite drivers on the supply side of the labor market: on the one hand, higher preference for consumption reduces MRS of labor with consumption. Meaning that marginal disutility of working in terms of marginal utility of consumption decreases, subsequently, HHs get to set lower wages and opt to work longer to finance the consumption which is more preferable now, therefore, labor supply increases. However, on the other hand, the preference shock increases utility for a given amount of consumption level, which pushes MRS up and makes upward pressure on wages. On top of that, higher demand drives wages and employment up too. In our case, the latter effects outweighs the former one, and nominal, as well as, real wages increases. The rental rate increases after the shock (arbitrage condition), which makes supply of capital service temporarily more profitable. Additionally, higher demand also pushes capital service up. In contrast, investment decreases, given the higher consumption, fewer resource could be allocated to produce investment goods. The last point is evident in the literature too, for example, Adolfson, et al. (2005).

Real neutral interest rate shock. Real neutral interest rate shock reduces both domestic absorptions, as well as, foreign demand for domestic goods. An important point to mention related to the dynamics of CA balance is that it deteriorates on impact, but starts improving quickly. It could be explained by the muted response of export to REER appreciation, which is driven by the small price elasticity of exported goods' demand (under baseline parametrization); something that needs to be confronted with data (see Figure 7).

3.2.3 Supply Side Shocks

Labor supply shock is another type of preference shock, which makes working less desirable (it is a time when agents decide to work less and enjoy leisure more intensively). One may think that the same type of shock could be one of the explanations for tight labor market conditions after the pandemic, i.e. workers tend to work less as they enjoy leisure more nowadays. If this is the case, then the shock is the right candidate to explain supply side drivers of the tight labor market and subsequent pressure on wages and inflation, which is evident after the shock. However, it needs to be mentioned that during the pandemic tight labor market from the supply side had not been driven by changing preferences toward less working, more leisure was "forced" by restrictions, and it was not the outcome of the optimization problem of HHs, difficult to say that the effects would have been same. Moreover, after the pandemic, low participation could be explained by the sectoral mismatches (see Shibata and Pizzineli, 2022).

Going back to the IRF, as workers decide to reduce working hours to save more time for leisure, they get to set higher wages (i.e. it is an upward shift of labor supply curve), hence, domestic inflation increases through anticipating an increase in marginal costs. Nominal interest rate rise to response to the inflationary pressure, which implies appreciation of nominal exchange rate on impact. Due to appreciation, the CPI inflation drops initially slightly and then starts rising. The shock plays a central role to calibrate the elasticity of labor supply. In the case of low elasticity, a more muted reaction of labor supply is enough to compensate for the increased disutility of working due to changes in preferences (see Figure 8). Here, we see that in the case of high elasticity (1) vs. low elasticity (1/4), implied volatility in the economy is much greater.

Temporary productivity shock. We find that output increases (rather weakly) and employment falls after the positive temporary productivity shock (see Figure 9). Though the empirical facts support the negative reaction of employment tothe shock, for instance, Gali, 1999, those findings are more likely about permanent technology shocks, and on the other hand, there are findings opposite to this too – productivity shock implies an increase in employment. The stylized fact (positive co-movement) holds in case of permanent technology shock in our model, when the wealth effect dominates (it would be discussed latter). Two things worth to mention about temporary productivity shock. The relatively muted reaction of output and fall of consumption after the shock is related with presence of non-ricardian consumers in the model economy. They are reducing their consumption after experiencing fall in employment and labor income. After putting aside the friction, consumption reacts positively and output increases strongly too. As for the negative reaction of employment to the shock, it turns to positive when the nominal frictions are absent. The explanation could be the following: if prices are rigid demand can not keep up to productivity improvement in the meantime; while, productivity improvement gives firms possibility for going with less labor, therefore, employment falls. The last argument is provided by Gali, (1999), for instance, to reconcile the negative correlation of employment and productivity shock in data with frictions embodied in New-Keynesian models. Moreover, wage flexibility contributes to positive employment effect to the temporary productivity shock. Therefore, we are coming closer to RBC model findings about positive co-movement after drawing the number of nominal rigidities down, together with returning back to full Ricardian set-up of the model.

Labor augmented technology shock. The effects of the permanent technology shock on macroeconomic variables are hard to make a firm consensus about it in the literature. The positive co-movement between employment and the productivity shock suggested by the RBC literature was challenged with empirical findings that the direction of relationship is opposite, for example, Gali, (1999), concludes that there must be other shocks to explain business cycles. However, the findings were tested intensively and some explanations are suggested to elucidate the puzzle. Mainly arguments are twofold, one the one hand, the negative co-movement in the data is the result of treating the growth rate of employment as stationary, when it is level stationary, for example, Christiano et al. (2003). They showed that the assumption on stationarity of hours worked is important to identify the effect of the permanent technology shock on employment falling if hours are growth stationary after the positive shock, and it rising if hours are stationary in levels. Beyond the issues related to empirical identification, some authors also find that the sign also depends on model parameters, for instance, Linde (2004), finds that high persistence of technology shock implies fall in employment and investment too, while the sign is opposite in case of low persistence. The explanation is the following: if the technology improvement is prolonged then agents prefer to postpone employment and investment to exploit better prospects in the future. The employment rises in our model after the shock and the sign is sensitive to calibration of some model parameters, while the high persistence of technology process implies positive response of output gap, which turns to negative if the value of the parameter is low enough. Same reaction is especially noticeable in the case of consumption gap, which turns to positive if the persistence of technology shock is high enough, i.e. consumers are trying to frontload part of their consumption from the future if the shock appears to deliver its benefits over the longer period of time. The wage stickiness is the most important to determine the sign of change in employment to the shock. If wages were flexible the model predicts negative reaction of employment. The mechanism could be following, if wages are sticky the wage is lagged to keep up with productivity gain and the real wage falls, which increases the demand on labor. The fall in real wages and marginal costs as a consequence moves the production above capacity, and output gap[15] becomes positive too. As said the results are opposite if wages were flexible (see Figure 10). Also, some other parameters, for example, the share of imported goods in production, have implications to analyse the transmission of the shock within the model.

Inefficiency technology shock in imported goods. Imported goods lose quality after the shock, therefore, the relative price of imported goods without quality adjustment drops which implies trend appreciation of REER. As long as domestic and foreign inflation do not change in equilibrium, the nominal exchange rate should appreciate. Its later implication could be a decrease of inflation through imported inflation channel, however, the price of imported goods after quality adjustment drops, therefore, those two effects act against each other and if the exchange rate persistence is low enough the nominal exchange rate appreciates quicker and inflation decreases. The real GDP expands as imported goods are substituted with domestic inputs in the production, as for the positive gap of consumption it is driven by drop of real interest rate (see Figure 11).

Export-specific technology shock. The shock creates a wedge between the price of exported goods and domestic one. It would be useful to match the relatively higher growth rate of export than growth rate of trade partners' economies observed in the data. Therefore, the shock would help us to filter the data to explain differences between trends of real export and trade partners' real GDP. After the shock, the trend of export shifts up while the trend of its price moves in the opposite direction. In

[15]By saying –gap – we mean percentage deviation of the respective variable from the trend

the meantime, due to price stickiness in the export sector, export price does not drop sufficiently, in consequence demand on exported goods remains below the trend on impact. Hence, the gaps of exported goods and output become negative.

Markup shocks. The negative markup shocks, i.e. price reductions in the case of domestic intermediate, export and import sectors have an expansionary effect. For example, domestic inflation drops down after the markup shock of domestic intermediate input production. The monetary policy reacts to the shock by reducing the policy rate, which implies exchange rate depreciation. In the meantime, consumption and investment are boosted due to lower rates. Import markup shocks imply responses of model variables in the same direction in most cases (as in the previous situation), but the magnitudes are different. After the shock, terms-of-trade improves which has a positive demand effect on the economy (as noted, by Jaaskela and Smith, 2011). The improvement in CA balance is relatively persistent, as well as, exchange rate swings. As for the export markup shock, terms-of-trade move in the opposite direction, CA deteriorates, but real export expands and real GDP increases. One should also note that reaction of most of the domestic variables (especially nominal ones) is relatively muted. Hence, the shock does not propagate changes in the domestic economy, except for its direct effect on real GDP. As long as prices do not change noticeably, the interest rate remains relatively unresponsive, therefore, there is no pressure on the FX market, and the exchange rate remains practically unchanged too (see Figure 12).

The real wage set by households decreases after the negative wage markup (i.e. mark-down) shock, in the meantime, due to price stickiness, prices remain relatively higher than implied by marginal costs, which creates a favorable condition for firms to expand production and rise employment. Therefore, the shock has an expansionary effect. REER depreciates along with nominal exchange rates depreciation due to lower interest rates (see Figure 13).

3.2.4 Foreign Sector Shocks.

Foreign inflation shock. Foreign inflation shock is transmitted to the local economy through the following channels: it could have upward pressure on domestic prices through imported inflation channel. Although local currency needs to appreciate as foreign price level shifts up. In contrast to the first channel, the second one implies lower inflation. Which of them dominates depends on the values of parameters. For example, if a change in nominal exchange rate is more gradual then price effect dominates and CPI inflation increases after the positive foreign price shock, while the effect is opposite in case of less persistent UIP. Although, the dollar pricing plays the role for transmitting the shock. Sticky import prices in USD dampens the imported inflation channel in the meantime and the inflation does not increase, and it could be outweighed with appreciation pressure on prices if stickiness is high enough (see Figure 14).

Foreign interest rate shock (Fed) (i_t^f, interest rate on USD-denominated assets) implies a depreciation of GEL vs USD. On the other hand, if exchange rate depreciation in our trade partners economies is lower to the global cycle of USD, the GEL depreciates vs RW too. Note that we keep three economy model set-up at some extent here, as long as we are trading in USD with our trade partners, determination of the bilateral exchange rate of GEL vs USD is important too, beyond the standard three equation foreign sector. Therefore, shocks to trade partners' economies are transmitted through imported inflation and export demand channels, while the shocks to foreign interest rate is analysed through the effect of US policy rate change. It is the case in our model that the depreciation of GEL w.r.t. USD is larger in magnitude then the depreciation of RW's currencies after the shock to Fed Funds rate, consequently, the drop in the local economy is less severe than it would have been if the responses of GEL and RW currencies had been similar to foreign interest rate shock. Though, one should note that the asymmetric reaction of local and RW currencies to foreign interest rate shock is conditional to calibration, and there is no structural behavior in the model which is responsible for the stronger depreciation of local currency relative to RW's currencies to the shock. It worth to further discuss how the shock is transmitted through the **demand channel**- an increase in foreign interest rate impose a tighter financial condition domestically (domestic interest rate increases too), which causes a drop in consumption and investment (as the rental rate on capital increases indeed). The reaction of government spending to the shock depends on calibration. If the share of backward-looking agents is higher enough on the FX market (i.e. more persistent UIP), then the shock has a more moderate impact on inflation. Consequently, the government debt burden is not relaxed sufficiently to stimulate government's expenditure motivated with the fiscal rule (the debt deflation channel). However, government reaction is opposite if inflation hikes after the shock when the forward-looking agents are dominant at FX market (see Figure 15).

Exchange rate channel - the exchange rate reaction to the shock largely depends on the persistence of UIP, therefore, the transmission to other variables differs too. Our baseline calibration is that half of agents are backward-looking on the FX market. Under the parametrization, exchange rate depreciates substantially (see Figure 15), the real exchange rate depreciates too, consequently, the CA balance and real GDP improve. However, if UIP were less persistent, the improvement in CA balance would have been quicker enough to balance the drop in domestic demand caused by the high interest rates, consequently, improvement in real GDP is more significant in this case. Propagation of the shock depends on other parameters as well. For instance, as price of exported goods are set in USD, export drops or at least is not sensitive to exchange rate depreciation in the short run. If the rigidity is relaxed in the model and price flexibility is assumed in the export sector, then the export performance would be positive and stronger. The output improves after the shock, hence, exchange rate behaves as a shock absorber in this situation (see Figure 16).

Finally, we would like to emphasise the dynamics of imported goods. Although the prices are sticky in USD, it is sold in local currency, hence, there is a perfect pass-through of the exchange rate in place. Therefore, the depreciation of local currency implies a substantial drop in import as foreign goods become expensive automatically. Hence, exchange rate depreciation after the shock still implies expenditure switching through imported goods, but the export sector does not take the benefit immediately.

Risk premium shock tightens external financial condition to the local economy. The nominal and real exchange rates depreciate while the neutral interest rate increases. Monetary policy tightens after the shock, consequently, domestic absorption sharply contracts. The drop in real GDP is much severe than in the case of the foreign interest rate shock (see Figure 17). In general, both of the shocks tighten financial condition and imply exchange rate depreciation and higher interest rates which induce sharp contraction domestically. However, the key difference is the asymmetric behavior of local currency against trade partners' currencies and USD in case of foreign interest rate shock, while there are the same responses of domestic currency in case of risk premium shock. It is quite apparent, since the risk premium shock is an idiosyncratic for our economy, while the Fed funds rate shock is global and could imply an asymmetric reaction of exchange rates. If it is a case then the negative pressure from the tightening of global financial condition is softened with expenditure switching due to depreciation against our trade partners' currencies. Thus the drop in domestic absorption is not as sharp as in the case of risk premium shock.

Foreign GDP growth shock. Foreign GDP and preference shocks have a positive effect on the domestic economy. Both are foreign demand shocks and have qualitatively the same effects on the domestic economy. However, maintaining those two shocks separate within the model is important in the estimation stage. The recent rise in foreign demand while foreign output is falling in our region (trade partners) could be matched with positive foreign preference shock. Higher foreign demand improves CA balance and implies an appreciation of the local exchange rate, as a consequence imported inflation decrease. Opposite to the decline in imported inflation, increase in demand on output creates upward pressure on marginal costs, hence, domestic inflation increases. In the short-run, the former effect (appreciation) dominates and CPI inflation drops and starts increasing as domestic cost pressure gains the power in later quarters. Once more, the response of CPI inflation largely depends on the persistence of UIP equation. If the share of backward-looking agents is large enough the exchange rate path is more muted and domestic cost drivers dominate in the determination of CPI inflation and it increases after the shock (see Figure 18).

Modified UIP condition. We have incorporated new (or at least uncommon) features into the model by adding Forex dealers who are taking portfolio adjustment costs into account when making portfolio choices as a alternative way to account for deviations from the standard UIP condition evident in the data. Although the feature as part of a DSGE model is quite new, we can show that by adding portfolio adjustment cost into Forex dealer's optimization problem (instead of standard partially backward-looking UIP), it is sufficiently useful to replicate the properties of standard lagged

UIP built in our baseline specification of the model. We assume that change in the portfolio growth rate beyond its growth on BGP is costly (see the discussion in the section 2.5). The key shocks to calibrate the parameter of portfolio adjustment cost are monetary policy, foreign interest rate, and risk premium shocks. As the figure 19 shows the fit between IRFs of standard lagged UIP and UIP with the adjustment cost becomes better as the adjustment cost parameter increases in case of monetary policy and foreign interest rate shocks. If the parameter is larger than 1.5, the responses to the shocks are similar to standard specification. However, it seems that the IRFs from the modified UIP never become as smooth as in the case of standard UIP. Moreover, the bigger value of adjustment cost is not always better. For instance, in the case of risk premium shock, the parameter value larger than 1.5 implies an appreciation of the nominal exchange rate after the shock, which is at odds with both the empirical and theoretical findings. Therefore, we suggest that if we keep the parameter close to 1, it is possible to apply UIP with adjustment cost to replicate the model properties of standard lagged UIP with sufficient accuracy. This encourages us to employ the modification in our DSGE setup in the future (though it still requires further testing and analysis), as the feature seems intuitive about the behavior of the agents.

3.3 Filtering the Data

3.3.1 Excess Trend Treatment and Modifications of Some Non-structural Model Equations

Although the model includes three unit root technology processes, we still need additional treatment of the trend process to match model variables to the respective observables. Here, instead of pre-filtering the data we prefer to add excess trend variables into observable equations. The model consistent filtering is useful to avoid the need for pre-treating the data by applying the univariate filters. Adding the excess trends on filtration stage is well adopted in the literature (see Andrle et al, 2009 and Argove et al, 2012). The trend of the labor productivity is not enough to account for the trend dynamic of real GDP. The productivity growth calibrated by us is lower in

SS than the average growth rate observed in the data over the history [16]. Also, we add the wedge between the nominal policy rate and discount rate applied by households. The excess interest rate, could be interpreted as a demand side risk premium which express risk perception of HHs. We can look at the modification through discussing the definition of real neutral interest rate (see, equation 3.3.1), which was augmented at the filtration stage by combining the real long term interest rate derived from real UIP condition with the real rate derived from the Euler equation [17].

$$1 + r_t^{nut} = \rho^{rnut}(1 + r_{t-1}^{nut}) + (1 - \rho^{rnut})\left(w_1 \frac{1}{1 + \gamma_t^{ax}} R_t^{\rho^{nut}}(1 + r_t^{fnut}) + \right.$$
$$\left. + (1 - w_1)\frac{(1 + \gamma_t^z)}{\beta(1 + i_t^{ex})}\right) + \varepsilon_t^{rnut} \tag{3.3.1}$$

where, the i_t^{ex} creates wedge between the rate applied by households for discounting and risk free rate. We, also, have modified the fiscal policy rule based on the results from filtering the data, we find the counter-cyclical fiscal policy rule seems more useful to improve the fit of the model to the data.

$$gb_t = (1 - \rho^{gb})gb + \rho^{gb} gb_{t-1} + \psi^1 \widehat{gdp_{t+4}} + \psi^2(d_{t+4} - d) + \varepsilon_t^g \tag{3.3.2}$$

Where $\psi^1 > 0$. and $\widehat{gdp_{t+4}}$ is the GDP gap (deviation from the trend four quarters ahead).

To filter the data, we also add oil and food prices to the model, the respective shocks seem important to account for the dynamics of headline inflation which is used by the NBG as a referance to target inflation to it. The averages of food and oil price inflations are larger than the targeted inflation, therefore, we also add excess inflation trends of those two variables to match the model to the data. Although trends are already incorporated in the model, we also pre-filter some observables, such as, the government primary deficit and CA deficit[18]. The need for pre-filtering the data comes

[16]The Euler equation links productivity growth and long run real rate to each other. The productivity growth consistent with the respective first moment of data implies unrealistically high value of real rate in SS

[17]The excess interest rate is also, applied to other equations as well where the HHs' stochastic discount rate participates in

[18]Note that the CA deficit has recently declined to more sustainable levels, but the issue of modeling large deficits in the past still needs to be dealt with.

from systematic deviation of SS values of primary and CA deficits in the model and the first moments of the respective variables in the data. This is especially obvious in the case of CA deficit, the large deficit over the history has been driver of rising foreign debt level in the data while the relatively smaller value of CA deficit is required to keep the debt at chosen sustainable level given the parametrization of the model. In the case of primary fiscal balance, the problem could be related to interest payments, as long as the main part of the government debt is with concessional terms[19] and government is able to run larger deficit in practice than implied by the steady state restrictions in the model. Therefore, we use the de-meaned value of primary deficit, as for the current account, it shows a trend over history, thus we pre-filterd respective observable [20] before providing to the model (see Figure 20).

3.3.2 Historical Decomposition

Beyond the IRFs analysis, to validate the model-implied results we confronted it with the data. Here, we briefly discuss the main cycles and episodes from the perspective of historical decomposition of key macroeconomic variables. As long as, the BGP is built into the model, there is no need to pre-filter the data, such as headline inflation, key monetary policy rate as a proxy of short term nominal rate, GDP growth, annual changes in REER, and NEER, as well as depreciation rate of GEL vs USD. Also, the fiscal balance to GDP ratio and current account deficit seem to keep important information for identifying cycles of the economy in question. The data covers the period from 2003q1 to 2023q1.

The inflation targeting framework is in place in Georgia since 2009. Therefore, to check the model's usefulness for policy analysis, it is essential to understand how well the model identifies structural shocks and economic cycles. We could split the recent history into three episodes. Before 2014 the real and neutral interest rates were largely declining (see Figure 22), mainly driven by trend appreciation of REER (see Figure 21). Inflation was largely below the target, explained with positive shocks to UIP (we could interpret it as exchange rate pass-through to inflation) (see Figure 23) while the global food price shock in 2011 has been a disturbing factor to inflation in the episode

[19]This, naturally, won't go on forever as Georgian economy develops.

[20]Using HP filter

of history, but it appeared short-lived.

The period from the end of 2014 to the Covid-19 shock could be characterized as a sequence of negative external shocks, reflected in the loss of relative competitiveness of our economy (see Figure 27). The negative shock to import inefficiency technology is suggested to be one of the main (and persistent) contributor to the rising real neutral rate and REER had plateaued (see Figure 26). Relatedly, the UIP shocks and subsequent depreciation of NEER was the main driver of rising headline inflation at the end of 2015 and 2017. Moreover, the shock seems to be the main contributor to the inflation deviation from the target since mid-2019, when the ban on direct flights from Russia and political uncertainty had a negative effect on expectations and implied exchange rate depreciation. On the supply side, the declining productivity could explain the relative slowdown of economic activity during the period (see Figure 24). Nevertheless, improved export-specific technology and fiscal stimuli explain the short term improvement in economic outlook at the end of 2019.

It remains a hot topic of discussion in the literature whether the COVID shock and the inflation in the subsequent period was driven by supply or demand factors. Our case provides important insights to the discussion too. At the beginning, the slowdown of labor productivity contributed to inflation from the supply side, on the other hand, declining markup shocks had an opposite contribution to inflation until 2021. The mark-up shocks could be related to the firms' negative expectations on demand condition. According to the model, the UIP shock, i.e. the exchange rate depreciation, has been the main contributor to high inflation during the Covid-19 period. The supply and demand factors were broadly equally important to explain the dramatic fall of the GDP growth rate. The contraction of local as well as external demand have added to the negative productivity shock during Covid-19. The economy has started recovering since mid-2021. On the demand side, the positive contribution of external demand has been noticeable since then. It needs to be mentioned that the fast (V shape) recovery has been characterized not only for the cyclical economic activity, but also, trend GDP has raised rapidly. However, the improvement is not solely accounted with labor productivity, but some other exogenous factors possibly explain high trend growth. Therefore, we need further research to uncover the factors driving the higher growth after the Covid-19 shock[21].

As for the inflationary performance, the sudden rise of food and oil prices has contributed to the largest-ever deviation of headline inflation from the target, together with negative UIP shocks since mid-2021. The recent decline in inflation could be explained by moderation of oil and food prices. Meanwhile, the favorable external conditions contributed to quick exchange rate appreciation, while due to mark-up shocks, inflation keeps resistance. Last but not least, the inefficiency technology shock to imported goods (or relative improvement in productivity) also explains remaining resistance of inflation. This could be a reflection of relatively high persistence of domestic and non-tradable inflation recently. The quick rebound in REER has been remarkable as well. According to the model, the important part of it could be assigned to recovery of relative productivity and subsequent trend appreciation of REER.

To conclude the section, the narrative of the model seems quite consistent from the historical perspective. The model well identifies that external factors (for example, UIP shocks) were dominant in explaining inflation before the Covid-19, along with declining relative productivity and REER depreciation. The Covid-19 recession was supply and demand driven largely to the same extent. While the recovery is mainly driven by external demand (together with local demand) and trend improvement, part of which still requires further research. It should be mentioned that CA and primary deficit help trend-cycle decomposition. For example, as long as the fiscal rule is counter-cyclical, the recent fiscal consolidation plays a role to explain the high economic activity as a cyclical process, while the large CA deficit during Covid-19 times implies moderation of negative output gap and larger fall in trend GDP.

4 Conclusions and Future Work

This work is the first in-house project on developing a "fully" structural DSGE model at the NBG. Although the model to a large extent shares the properties of a standard medium-scale DSGE, there are still additional features introduced relevant for analyzing monetary policy transmission in emerging markets, such as, dollar invoicing.

[21]Migrant inflows from Russia after outbreak of war between Russia and Ukraine could possible explains it together with fast expansion of IT sector recently

In addition, we incorporate the forward premium puzzle into our model. We have also tried an alternative way of modeling deviations from standard UIP with similar results: namely, by using portfolio adjustment costs in Forex dealers optimization problem. Additionally, some degree of heterogeneity is introduced based on two agent setup. Moreover, we have built various trend processes into the model, which would be practical to filter data and conduct historical analysis using the model in the future.

On the one hand, the narrative of the model, as described, is consistent with the predictions made in the literature. Indeed, it is capable to explain some empirical facts that are overlooked by standard DSGE models. For example, the model predicts low sensitivity of export to exchange rate movements, i.e. lower benefit from exchange rate flexibility than predicted with more standard models. The model generates a delayed reaction of exchange rate to shocks, i.e. the deviation from UIP condition, which is an empirically justified fact, and needs to be captured within small open economy models intended for policy analysis. Also, the three pairs of exchange rate embodied in our model would be an useful extension for emerging markets, as long as, ceteris paribus, all the changes in exchange rate w.r.t. to USD do not have implication on the trade balance if domestic currency does not change w.r.t. to trade partners' economies. The standard models with the single pair of currencies are not capable to conduct same analysis. Those extensions of the model and subsequent results seem promising for us to put further efforts in making the model applicable for macroeconomic and policy analysis, as well as, for forecasting. Although we are employing a semi-structural model for forecasting and policy analysis at the moment, alternative models (like this one) always serve more informed and better decisions.

Finally, developing the model on our own was a valuable experience for the team, and those skills would also be helpful for us in the next stages of model development, as this remains a work in progress. Moreover, we find it sensible to publish the entire documentation of the model to get feedback and facilitate discussion around the model setup and its properties. Last but not least, we hope that detailed derivations provided in the paper would help new entrants/students in the field of DSGE modeling[22],

[22]footnote: We hope this transparency will help us as well, since interested readers may spot some typos in any place of the whole derivations, making sure the analysis is based on a correctly built model.

since some mathematical derivations, as well as, conceptual details sometimes are not appropriately provided or freely accessible in the literature.

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Appendix A Household Sector

A.1 Linearization of Euler Equation

To derive the linear version of the Euler equation given by 8, firstly, we take the first order derivatives of the equation w.r.t. ψ_t, ψ_{t+1}, π^c_{t+1} , $\widetilde{C^{uc}_{t-1}}$, $\widetilde{C^{uc}_t}$, $\widetilde{C^{uc}_{t+1}}$, γ^z_t, and γ^z_{t+1} and then we find their values in SS.

$$[\partial\psi_t] : \qquad \frac{E_t((1+\gamma^z_{t+1})\widetilde{C^{uc}_{t+1}} - h\widetilde{C^{uc}_t})\Pi^c_{t+1}}{\beta E_t\psi_{t+1}(\widetilde{C^{uc}_t} - \frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}})} \tag{A.1.1}$$

In SS.

$$\frac{(1+\pi^c)((1+\gamma^z)\widetilde{C^{uc}} - h\widetilde{C^{uc}})}{\beta\psi(\widetilde{C^{uc}} - \frac{h}{1+\gamma^z}\widetilde{C^{uc}})} = \frac{(1+\pi^c)(1+\gamma^z)}{\beta\psi} \tag{A.1.2}$$

$$[\partial\psi_{t+1}] : \qquad -\frac{E_t\psi_t((1+\gamma^z_{t+1})\widetilde{C^{uc}_{t+1}} - h\widetilde{C^{uc}_t})\Pi^c_{t+1}}{E_t(\psi_{t+1})^2(\widetilde{C^{uc}_t} - \frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}})} \tag{A.1.3}$$

In SS.

$$-\frac{(1+\pi^c)(1+\gamma^z)}{\beta\psi} \tag{A.1.4}$$

$$\left[\partial\Pi^c_{t+1}\right] : \qquad \frac{E_t\psi_t((1+\gamma^z_{t+1})\widetilde{C^{uc}_{t+1}} - h\widetilde{C^{uc}_t})}{\beta E_t\psi_{t+1}(\widetilde{C^{uc}_t} - \frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}})} \tag{A.1.5}$$

In SS.

$$\frac{(1+\gamma^z)}{\beta} \tag{A.1.6}$$

$$\left[\partial\widetilde{C^{uc}_t}\right] :$$

$$\frac{-h\psi_t E_t(1+\pi^c_{t+1})(\beta E_t\psi_{t+1}(\widetilde{C^{uc}_t} - \frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}})) - \psi_t E_t(1+\pi^c_{t+1})((1+\gamma^z_{t+1}\widetilde{C^{uc}_{t+1}} - h\widetilde{C^{uc}_t})\beta E_t\psi_{t+1}}{(\beta E_t\psi_{t+1}(\widetilde{C^{uc}_t} - h\widetilde{C^{uc}_{t-1}}))^2} \tag{A.1.7}$$

In SS.

$$-\frac{(1+\pi^c)(1+\gamma^z)(1+h+\gamma^z)}{\beta(1-h+\gamma^z)\widetilde{C^{uc}}} \tag{A.1.8}$$

$$\left[\partial\widetilde{C^{uc}_{t+1}}\right]: \qquad \frac{\psi_t E_t(1+\pi^c_{t+1})(1+\gamma^z_{t+1})}{\beta E_t\psi_{t+1}(\widetilde{C^{uc}_t}-\frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}})} \tag{A.1.9}$$

In SS.

$$\frac{(1+\pi^c)}{\beta(1-h+\gamma^z_t)\widetilde{C^{uc}}} \tag{A.1.10}$$

$$\left[\partial\widetilde{C^{uc}_{t-1}}\right]: \qquad \frac{\beta E_t\psi_{t+1}\frac{h}{1+\gamma^z_t}\psi_t((1+\gamma_{t+1})\widetilde{C^z_{t+1}}-h\widetilde{C}_t)(1+\pi^c_{t+1})}{(\beta E_t\psi_{t+1}(\widetilde{C^{uc}_t}-\frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}}))^2} \tag{A.1.11}$$

In SS.

$$\frac{h(1+\pi^c)(1+\gamma^z)^2}{\beta(1-h+\gamma^z)\widetilde{C^{uc}}} \tag{A.1.12}$$

$$[\partial\gamma^z_t]: \qquad -\frac{\psi_t E_t\left((1+\gamma^z_{t+1})\widetilde{C^{uc}_{t+1}}-h\widetilde{C^{uc}_t}\right)\Pi^c_{t+1}\beta\psi_{t+1}\frac{h}{(1+\gamma^z_t)^2}\widetilde{C^{uc}_t}}{\left(\beta E_t\psi_t\left(\widetilde{C^{uc}_t}-\frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}}\right)\right)^2} \tag{A.1.13}$$

In SS:

$$-\frac{(1+\pi^c)h}{\beta(1+\gamma^z-h)} \tag{A.1.14}$$

Also,

$$\left[\partial\gamma^z_{t+1}\right]: \qquad \frac{E_t\psi_t\widetilde{C^{uc}_{t+1}}\Pi^c_{t+1}}{\beta E_t\psi_{t+1}\left(\widetilde{C^{uc}_t}-\frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}}\right)} \tag{A.1.15}$$

In SS:

$$-\frac{(1+\pi^c)(1+\gamma^z)}{\beta(1+\gamma^z-h)} \tag{A.1.16}$$

Taking into account SS values of FOCs of Euler equation, we can write its linear approximation in the following way:

$$\begin{aligned}
R+(R_t-R) = &\frac{(1+\pi^c)(1+\gamma^z)}{\beta}+\frac{(1+\pi^c)(1+\gamma^z)}{\beta\widetilde{\psi}}(\widetilde{\psi}_t-\widetilde{\psi})\\
&-\frac{(1+\pi^c)(1+\gamma^z)}{\beta\widetilde{\psi}}(E_t\widetilde{\psi_{t+1}}-\widetilde{\psi})+\frac{1+\gamma^z}{\beta}(E_t\pi^c_{t+1}-\pi^c)-\\
&-\frac{(1+\pi^c)(1+\gamma^z)(1+h+\gamma^z)}{\beta(1-h+\gamma^z)\widetilde{C^{uc}}}(\widetilde{C^{uc}_t}-\widetilde{C^{uc}})+\frac{(1+\pi^c)(1+\gamma^z)^2}{\beta(1-h+\gamma^z)\widetilde{C^{uc}}}(E_t\widetilde{C^{uc}_{t+1}}-\widetilde{C^{uc}})+\\
&+\frac{h(1+\pi^c)(1+\gamma^z)}{\beta(1-h+\gamma^z)\widetilde{C^{uc}}}(\widetilde{C^{uc}_{t-1}}-\widetilde{C^{uc}})-\frac{h(1+\pi^c)}{\beta(1+\gamma^z-h)}(\gamma^z_t-\gamma^z)+\frac{(1+\pi^c)(1+\gamma^z)}{\beta(1+\gamma^z-h)}(E_t\gamma^z_{t+1}-\gamma^z)
\end{aligned} \tag{A.1.17}$$

Finally, the Euler equation for the subset of unconstrained HHs is:

$$\begin{aligned}
\widehat{C^{uc}_t} =&\frac{h}{1+h+\gamma^z}\widehat{C^{uc}_{t-1}}+\frac{1+\gamma^z}{1+h+\gamma^z}E_t\widehat{C^{uc}_{t+1}}+(\widehat{\psi}_t-E_t\widehat{\psi_{t+1}})+\\
&+\frac{\gamma^z}{1+\gamma^z+h}\left(\frac{1}{1+\gamma^z}E_t\widehat{\gamma^z_{t+1}}-\widehat{\gamma^z_t}\right)-\frac{1+\gamma^z-h}{1+h+\gamma^z}[\frac{1}{R}\widehat{i}_t-\frac{1}{1+\pi^c}(E_t\pi^c_{t+1}-\pi^c)]
\end{aligned} \tag{A.1.18}$$

The linear version of budget constraint of constrained HHs could be written as:

$$\widehat{C^c_t}=\frac{1}{(1+\gamma^c)\widetilde{C^c}}\left((1-\tau^w)\widetilde{W^r}L\widehat{L}_t+(1-\tau^w)\widetilde{W^r}L\widehat{W^r_t}+\widetilde{T^{cr}}\widehat{T^{cr}_t}\right) \tag{A.1.19}$$

While the aggregate consumption function in gaps is given as:

$$\widehat{C}_t=(1-\lambda)\frac{\widetilde{C^{uc}}}{\widetilde{C}}\widehat{C^{uc}_t}+\lambda\frac{\widetilde{C^c}}{\widetilde{C}}\widehat{C^c_t} \tag{A.1.20}$$

Finally, Euler equation for aggregate consumption reads:

$$\widehat{C}_t =\frac{h}{1+h+\gamma^z}\widehat{C_{t-1}}+\frac{1+\gamma^z}{1+h+\gamma^z}E_t\widehat{C_{t+1}}+(\widehat{\psi}_t-E_t\widehat{\psi_{t+1}})+$$

$$+\frac{\gamma^z}{1+\gamma^z+h}\left(\frac{h}{1+\gamma^z}E_t\widehat{\gamma^z_{t+1}}-\widehat{\gamma^z_t}\right)-\frac{1+\gamma^z-h}{1+h+\gamma^z}[\frac{1}{R}\widehat{i_t}-\frac{1}{1+\pi^c}(E_t\pi^c_{t+1}-\pi^c)]$$

$$+\frac{\lambda(1-\tau^w)}{1+\tau^c}\frac{\widetilde{L}\widetilde{W^r}}{\widetilde{C^c}}\Big[\Big(\widehat{W^r_t}-\frac{h(1+\gamma^z)}{1+h+\gamma^z}\widehat{W^r_{t-1}}-\frac{1}{1+h+\gamma^z}E_t\widehat{W^r_{t+1}}\Big)+$$

$$+\Big(\widehat{L_t}-\frac{h(1+\gamma^z)}{1+h+\gamma^z}\widehat{L_{t-1}}-\frac{1}{1+h+\gamma^z}E_t\widehat{L_{t+1}}\Big)\Big]+$$

$$+\frac{\lambda}{1+\tau^c}\frac{\widetilde{T^{cr}}}{\widetilde{C^r}}\Big(\widehat{T^r_t}-\frac{h(1+\gamma^z)}{1+h+\gamma^z}\widehat{T^r_{t-1}}-\frac{1}{1+h+\gamma^z}E_t\widehat{T^r_{t+1}}\Big) \tag{A.1.21}$$

A.2 Aggregate Wage

As mentioned above, in our framework, unconstrained HHs set wages to maximize their utility s.t. demand function on labor input, in each period $(1-\theta_w)$ part of HHs get to set their wages optimally, while the rest part (θ_w) follows the wage indexation rule given by:

$$W_t(i)=\Pi^w_{t-1}W_{t-1}(i) \tag{A.2.1}$$

where, $\Pi^w_{t-1}=\frac{W_{t-1}}{W_{t-2}}$. While credit-constrained HHs equalize their wages to the average wage of unconstrained HHs. Hence, firstly we discuss the aggregate wage supply function for the continuum of unconstrained HHs it is still possible to represent the subset of unconstrained HHs as a continuum from 0 to 1 within the subset of all HHs from λ to 1. We assume that the wage-setting problem is symmetric across HHs, hence, the optimal wages are the same. Then we can rewrite the aggregate wage equation (for the subset of unconstrained HHs) as:

$$W_t=\left[\int_0^{\theta_w}\Big(\Pi^w_{t-1}W_{t-1}(i)\Big)^{1-\eta^l_t}+(1-\theta_w)W_t^{*1-\eta_w}\right]^{\frac{1}{1-\eta^l_t}} \tag{A.2.2}$$

Using the assumption by Calvo, that the HHs who get to set optimal wage in each period is a random selection, then the aggregate wage for any subset of HHs is the same as the average wage, then we can write:

$$W_t=\left[\theta_w(\Pi^w_{t-1}W_{t-1})^{1-\eta^l_t}+(1-\theta_w)W_t^{*1-\eta^l_t}\right]^{\frac{1}{1-\eta^l_t}} \tag{A.2.3}$$

Let's divide both sides by W_t:

$$1 = \left[\theta_w(\Pi^w_{t-1}\Big(\frac{W_{t-1}}{W_t}\Big)^{1-\eta^l_t} + (1-\theta_w)\Big(\frac{W^*_t}{W_t}\Big)^{1-\eta^l_t}\right]^{\frac{1}{1-\eta^l_t}} \tag{A.2.4}$$

Then the equation for gross wage inflation can be written as:

$$\Pi^{w\,1-\eta^l_t}_t = \theta_w \Pi^w_{t-1}{}^{1-\eta^l_t} + (1-\theta_w)\Pi^{w\,1-\eta^l_t}_t \left(\frac{W^*_t}{W_t}\right)^{1-\eta^l_t} \tag{A.2.5}$$

The first order linear approximation implies:

$$\begin{aligned}\Pi^{w\,1-\eta^l_t} + (1-\eta^l_t)\Pi^{w\,-\eta^l_t}(\Pi^w_t - \Pi^w) = \theta_w \Pi^w + (1-\eta^l_t)\theta_w \Pi^{w\,-\eta^l_t}(\Pi^w_{t-1} - \Pi^w) +\\ +(1-\theta_w)\Pi^{w\,1-\eta^l_t}\left(\frac{W^*}{W}\right) + (1-\theta_w)(1-\eta^l_t)\Pi^{w\,-\eta^l_t}\left(\frac{W^*}{W}\right)^{1-\eta^l_t}(\Pi^w_t - \Pi^w)+\\ +(1-\theta_w)(1-\eta^l_t)\Pi^{w\,1-\eta^l_t}\left(\frac{W^*}{W}\right)^{1-\eta^l_t}\Big(\frac{W^*_t}{W_t} - 1\Big)\end{aligned} \tag{A.2.6}$$

That can be written as:

$$\begin{aligned}\Pi^{w\,-\eta^l_t}(\Pi^w_t - \Pi^w) = &\theta_w \Pi^{w\,-\eta^l_t}(\Pi^w_{t-1} - \Pi^w) + (1-\theta_w)\Pi^{w\,-\eta^l_t}(\Pi^w_t - \Pi^w)+\\ &+(1-\theta_w)\Pi^{w\,1-\eta^l_t}\Big(\frac{W^*_t}{W_t} - 1\Big)\end{aligned} \tag{A.2.7}$$

If we devide the both parts by $\Pi^{w\,-\eta^l_t}$, we get:

$$\Pi^w_t - \Pi^w = \theta_w(\Pi^w_{t-1} - \Pi^w) + (1-\theta_w)(\Pi^w_t - \Pi^w) + (1-\theta_w)\Pi^w\Big(\frac{W^*_t}{W_t} - 1\Big) \tag{A.2.8}$$

After collecting the same terms:

$$\theta_w(\Pi^w_t - \Pi^w) = \theta_w(\Pi^w_{t-1} - \Pi^w) + (1-\theta_w)\Pi^w\Big(\frac{W^*_t}{W_t} - 1\Big) \tag{A.2.9}$$

Finally:

$$\Pi^w_t = \Pi^w_{t-1} + \frac{1-\theta_w}{\theta_w}\Pi^w\Big(\frac{W^*_t}{W_t} - 1\Big) \tag{A.2.10}$$

A.3 Wage Setting Problem

If we substitute all constraints into the utility maximization problem in 5, the wage setting problem can be rewritten as:

$$\underset{W_t^*(i)}{\text{maximize}} \quad \sum_{k=0}^{\infty}(\beta\theta_w)^k\Bigg\{-\theta_{t+k}\chi\frac{\left(\left(\frac{\Pi^w_{t+k-1|t-1}W_t^*(i)}{W_{t+k}}\right)^{-\eta^l_{t+k}}L_{t+k}\right)^{1+\zeta}}{1+\zeta}- \\ -\lambda_{t+k}(i)(-(1-\tau^w)\Pi^w_{t+k-1|t-1}W_t^*(i)\left(\frac{\Pi^w_{t+k-1|t-1}W_t^*(i)}{W_{t+k}}\right)^{-\eta^l_{t+k}}L_{t+k})\Bigg\} \tag{A.3.1}$$

The FOC of the household wage-setting problem is the following:

$$[\partial W_t^*(i)]: \quad E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\Bigg\{-\theta_{t+k}\chi(-\eta^l_{t+k})W_t^{*-\eta^l_{t+k}(1+\zeta)-1}\left(\left(\frac{\Pi^w_{t+k-1|t-1}}{W_{t+k}}\right)^{-\eta^l_{t+k}}L_{t+k}\right)^{1+\zeta}- \\ -E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\Bigg\{\lambda_{t+k}(i)(\eta^l_{t+k}-1)W_t^{*-\eta^l_{t+k}}\left(\frac{\Pi^w_{t+k-1|t-1}}{W_{t+k}}\right)^{1-\eta^l_{t+k}}W_{t+k}L_{t+k}\Bigg\}=0 \tag{A.3.2}$$

In order to derive the equation for an optimal wage (before writing the equation recursively in the next section), we treat the elasticity of substitution as a parameter initially and after deriving the equation, we reconsider it as a variable again. Then we rewrite the equation w.r.t W_t^* and divide both sides of equation by W_t , and substitute $\lambda_{t+k}(i)$. from the equation 6 we end up with:

$$\left(\frac{W_t^*}{W_t}\right)^{-(1+\eta^l\zeta)}= \\ =\frac{(1-\eta^l)(1-\tau^w)}{\eta^l(1+\tau^c)}\frac{E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\Bigg\{\frac{\psi_{t+k}}{P^c_{t+k}(C^{uc}_{t+k}-hC^{uc}_{t+k-1})}\left(\frac{\Pi^w_{t+k-1|t-1}}{W_{t+k}/W_t}\right)^{1-\eta^l}W_{t+k}L_{t+k}\Bigg\}}{E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\Bigg\{\theta_{t+k}\chi\left(\left(\frac{\Pi^w_{t+k-1|t-1}}{W_{t+k}/W_t}\right)^{-\eta^l}L_{t+k}\right)^{1+\zeta}\Bigg\}} \tag{A.3.3}$$

Let's define real wage $W^r_{t+k}=\frac{W_{t+k}}{P^c_{t+k}}$; also, note that $\Pi^w_{t+k}=\frac{W_{t+k}}{W_{t+k-1}}$, and $\Pi^w_t=\frac{W_t}{W_{t-1}}$ are gross wage inflations in the period $t+k$ and t accordingly. Then the above equation can be rewritten as:

$$\left(\frac{W_t^*}{W_t}\right)^{-(1+\eta_t^l\zeta)} = \frac{(1-\eta^l)(1-\tau^w)}{\eta^l(1+\tau^c)} \frac{E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left\{\frac{\psi_{t+k}}{(C_{t+k}^{uc}-hC_{t+k-1}^{uc})}\left(\frac{\Pi_t^w}{\Pi_{t+k}^w}\right)^{1-\eta^l}W_{t+k}^r L_{t+k}\right\}}{E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left\{\theta_{t+k}\chi\left(\left(\frac{\Pi_t^w}{\Pi_{t+k}^w}\right)^{-\eta^l}L_{t+k}\right)^{1+\zeta}\right\}} \tag{A.3.4}$$

A.3.1 Recursive form of optimal wage

Let's introduce the following definitions:

$$C_{1t} = E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left\{\frac{\psi_{t+k}}{(C_{t+k}^{uc}-hC_{t+k-1}^{uc})}\left(\frac{\Pi_t^w}{\Pi_{t+k}^w}\right)^{1-\eta^l}W_{t+k}^r L_{t+k}\right\} \tag{A.3.1}$$

and,

$$C_{2t} = E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left\{\theta_{t+k}\chi\left(\left(\frac{\Pi_t^w}{\Pi_{t+k}^w}\right)^{-\eta^l}L_{t+k}\right)^{1+\zeta}\right\} \tag{A.3.2}$$

Time-varying elasticity of substitution introduces difficulties to derive wage PC recursively. Hence, at the stage of derivation, we treat it as a parameter, then after deriving the equation in the recursive form we would re-introduce it as a variable. Regretfully, it does not sound mathematically correct. However, it is a popular approach in the literature for making it easier to rewrite the equation in recursive form when the elasticity of substitution is time-varying (i.e. variable). That said, we can rewrite C_{1t} in the following way:

$$\begin{aligned}
C_{1t} =& \frac{\psi_t}{(C_t^{uc}-hC_{t-1}^{uc})}W_t^r L_t + E_t\sum_{k=1}^{\infty}(\beta\theta_w)^k\left\{\frac{\psi_{t+k}}{(C_{t+k}^{uc}-hC_{t+k-1}^{uc})}\left(\frac{\Pi_t^w}{\Pi_{t+k}^w}\right)^{1-\eta^l}W_{t+k}^r L_{t+k}\right\}\\
&= \frac{\psi_t}{(C_t^{uc}-hC_{t-1}^{uc})}W_t^r L_t + \beta\theta_w E_t\left(\frac{\Pi_t^w}{\Pi_{t+1}^w}\right)^{(1-\eta^l)} \times\\
&\times\sum_{k=0}^{\infty}(\beta\theta_w)^k\left\{\frac{\psi_{t+k+1}}{(C_{t+k+1}^{uc}-hC_{t+k}^{uc})}\left(\frac{\Pi_{t+1}^w}{\Pi_{t+k+1}^w}\right)^{1-\eta^l}W_{t+k+1}^r L_{t+k+1}\right\} =\\
&= \frac{\psi_t}{(C_t^{uc}-hC_{t-1}^{uc})}W_t^r L_t + \beta\theta_w E_t\left(\frac{\Pi_t^w}{\Pi_{t+1}^w}\right)^{(1-\eta^l)} E_t C_{1t+1}
\end{aligned} \tag{A.3.3}$$

Also, the C_{2t} is rewritten as:

$$
\begin{aligned}
C_{2t} =& \chi\theta_t L_t^{1+\zeta} + E_t \sum_{k=1}^{\infty} (\beta\theta_w)^k \left\{ \theta_{t+k}\chi \left(\left(\frac{\Pi_t^w}{\Pi_{t+k}^w} \right)^{-\eta^l} L_{t+k} \right)^{1+\zeta} \right\} \\
&= \chi\theta_t L_t^{1+\zeta^l} + \beta\theta_w E_t \left(\frac{\Pi_t^w}{\Pi_{t+1}^w} \right)^{-\eta^l} \sum_{k=0}^{\infty} (\beta\theta_w)^k \left\{ \theta_{t+k+1}\chi \left(\left(\frac{\Pi_{t+1}^w}{\Pi_{t+k+1}^w} \right)^{-\eta^l} L_{t+k+1} \right)^{1+\zeta} \right\} \\
&= \chi\theta_t L_t^{1+\zeta^l} + \beta\theta_w \left(\frac{\Pi_t^w}{\Pi_{t+1}^w} \right)^{-\eta^l} E_t C_{2t+1} \qquad \text{(A.3.4)}
\end{aligned}
$$

Finally, the recursive form of wage PC is given by (note that we are reintroducing η_t^l as a variable here):

$$
\left(\frac{W_t^*}{W_t} \right)^{-(1+\eta_t^l \zeta)} = \frac{(1-\tau^w)(\eta_t^l - 1)}{(1+\tau^c)\eta_t^l} \frac{C_{1t}}{C_{2t}} \qquad \text{(A.3.5)}
$$

A.3.2 Linearization of Wage PC

Let's derive the linear version of wage PC. Using the stationary variables. It could be rewritten as:

$$
\begin{aligned}
&E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left\{ \theta_{t+k}\chi\eta_{t+k}^l \left(\frac{W_t^*}{W_t} \right)^{-\eta_{t+k}^l(1+\zeta)-1} \left(\left(\frac{\Pi_t^w}{\Pi_{t+k}^w} \right)^{-\eta_{t+k}^l} L_{t+k} \right)^{1+\zeta} - \right. \\
&\left. - E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left\{ \frac{\psi_{t+k}}{(z_{t+k}\widetilde{C_{t+k}^{uc}} - h z_{t+k-1}\widetilde{C_{t+k-1}^{uc}})} (\eta_{t+k}^l - 1) \left(\frac{W_t^*}{W_t} \right)^{-\eta_{t+k}^l} \left(\frac{\Pi_t^w}{\Pi_{t+k}^w} \right)^{1-\eta_{t+k}^l} z_{t+k}\widetilde{W_{t+k}} L_{t+k} \right\} = 0 \right. \\
&\qquad \text{(A.3.1)}
\end{aligned}
$$

$\Longrightarrow$

$$
\begin{aligned}
&E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left\{ \theta_{t+k}\chi\eta_{t+k}^l \left(\frac{W_t^*}{W_t} \right)^{-\eta_{t+k}^l(1+\zeta)-1} \left(\left(\frac{\Pi_t^w}{\Pi_{t+k}^w} \right)^{-\eta_{t+k}^l} L_{t+k} \right)^{1+\zeta} - \right. \\
&\left. - E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left\{ \frac{\psi_{t+k}}{(\widetilde{C_{t+k}^{uc}} - \frac{h}{1+\gamma_{t+k}^z}\widetilde{C_{t+k-1}^{uc}})} (\eta_{t+k}^l - 1) \left(\frac{W_t^*}{W_t} \right)^{-\eta_{t+k}^l} \left(\frac{\Pi_t^w}{\Pi_{t+k}^w} \right)^{1-\eta_{t+k}^l} \widetilde{W_{t+k}} L_{t+k} \right\} = 0 \right. \\
&\qquad \text{(A.3.2)}
\end{aligned}
$$

Let's make the following definitions, first:

$$
LHS_{t+k}^l \equiv \chi\eta_{t+k}^l \theta_{t+k} \left(\left(\frac{\Pi_t^w}{\Pi_{t+k}} \right)^{-\eta_{t+k}^l} L_{t+k} \right)^{1+\zeta} \left(\frac{W_t^*}{W_t} \right)^{-\eta_{t+k}^l(1+\zeta)-1} \qquad \text{(A.3.3)}
$$

Its value in SS would be:

$$LHS^l = \chi\eta^l\theta L^{1+\zeta} \tag{A.3.4}$$

Also, let's define:

$$RHS^l_{t+k} \equiv \frac{(1-\tau^w)}{(1+\tau^c)} \frac{\psi_{t+k}(\eta^l_{t+k}-1)}{(\widetilde{C^{uc}_{t+k}} - \frac{h}{1+\gamma^z_{t+k}}\widetilde{C^{uc}_{t+k-1}})} \left(\frac{\Pi^w_t}{\Pi^w_{t+k}}\right)^{1-\eta^l_{t+k}} \left(\frac{W^*_t}{W_t}\right)^{-\eta^l_{t+k}} \widetilde{W^r_{t+k}} L_{t+k} \tag{A.3.5}$$

note, that the equation in the steady state can be written as:

$$RHS_l = \frac{(\eta^l-1)(1-\tau^w)}{(1+\tau^c)} \frac{\psi(1+\gamma^z)}{(1+\gamma^z-h)} \frac{\widetilde{W^r}L}{\widetilde{C^{uc}}} \tag{A.3.6}$$

Firstly, make linear transformation of the $\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l_{t+k}$:

$$\begin{aligned}
&E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l_{t+k} \approx \\
\approx&(1-\theta_w\beta)RHS^l + E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{1}{\psi}(\psi_{t+k}-\psi)- \\
-&E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{1+\gamma^z}{1+\gamma^z-h}\frac{1}{\widetilde{C^{uc}}}(\widetilde{C^{uc}_{t+k}}-\widetilde{C^{uc}})+ \\
+&E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k \widetilde{RHS^l}\frac{h}{1+\gamma^z-h}\frac{1}{\widetilde{C^{uc}}}(\widetilde{C^{uc}_{t+k-1}}-\widetilde{C^{uc}})+ \\
+&(1-\eta^l)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\left(\frac{\Pi^w_t}{\Pi^w}-1\right) - (1-\eta^l)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\left(\frac{\Pi^w_{t+k}}{\Pi^w}-1\right)+ \\
+&\eta^l E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{1}{\widetilde{W^r}}(\widetilde{W^r_{t+k}}-\widetilde{W^r}) + E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{1}{L}(L_{t+k}-L) \\
+&E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{\eta^l}{\eta^l(\eta^l-1)}(\eta^l_{t+k}-\eta^l) - E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{h}{1+\gamma^z-h}\frac{\gamma^z}{\gamma^z(1+\gamma^z)}(\gamma^z_{t+k}-\gamma^z)- \\
-&E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\eta^l RHS^l\left(\frac{W^*_t}{W_t}-1\right)
\end{aligned} \tag{A.3.7}$$

This expression can be written in gaps as:

$$E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l_{t+k} \approx$$

$$
\begin{aligned}
\approx &(1-\theta_w\beta)RHS^l + E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\widehat{\psi_{t+k}} - E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{1+\gamma^z}{1+\gamma^z-h}\widehat{C^{uc}_{t+k}}+\\
&+E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{h}{1+\gamma^z-h}\widehat{C^{uc}_{t+k-1}} + (1-\eta^l_t)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\left(\frac{\Pi^w_t}{\Pi^w}-1\right)-\\
&-(1-\eta^l)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\left(\frac{\Pi^w_{t+k}}{\Pi^w}-1\right) + E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\widehat{W^r_{t+k}}+\\
&+E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\widehat{L_{t+k}} + E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{\eta^l}{\eta^l-1}\widehat{\eta^l_{t+k}}-\\
&-E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k RHS^l\frac{h}{1+\gamma^z-h}\frac{\gamma^z}{1+\gamma^z}\widehat{\gamma^z_{t+k}}-\\
&-E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\eta^l RHS^l\left(\frac{W^*_t}{W_t}-1\right)
\end{aligned}
\tag{A.3.8}
$$

Now, we can make the linear transformation of the left-hand side:

$$
\begin{aligned}
&E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l_{t+k}\approx\\
\approx &(1-\theta_w\beta)\chi\theta(L)^{1+\zeta} + E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\frac{1}{\theta}(\theta_{t+k}-\theta)-\\
&-\eta^l(1+\zeta)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\left(\frac{\Pi^w_t}{\Pi^w}-1\right) + \eta^l(1+\zeta)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\left(\frac{\Pi^w_{t+k}}{\Pi^w}-1\right)+\\
&+(1+\zeta)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\frac{1}{L}(L_{t+k}-L) - (1+\eta^l\zeta)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\left(\frac{W^*_t}{W_t}-1\right)+\\
&+E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\frac{1}{\eta^l}\left(\eta^l_{t+k}-\eta^l\right)
\end{aligned}
\tag{A.3.9}
$$

we can rewrite the equation in gaps as:

$$
\begin{aligned}
&E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l_{t+k}\approx\\
\approx &(1-\beta\theta_w)\chi\theta(L)^{1+\zeta} + E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\widehat{\theta_{t+k}}-\\
&-\eta^l(1+\zeta)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\left(\frac{\Pi^w_t}{\Pi^w}-1\right) + \eta^l(1+\zeta)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\left(\frac{\Pi^w_{t+k}}{\Pi^w}-1\right)+\\
&+(1+\zeta)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\widehat{L_{t+k}} - (\eta^l(1+\zeta)+1)E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k LHS^l\left(\frac{W^*_t}{W_t}-1\right)+
\end{aligned}
$$

$$+ E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k LHS^l \widehat{\eta^l_{t+k}} \tag{A.3.10}$$

After collecting the same terms in the linear version of the left and right-hand sides of the wage setting problem, it can be written as:

$$\frac{1+\eta^l\zeta}{1-\beta\theta_w}\left(\frac{W_t^*}{W_t}-1\right) = E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{\theta_{t+k}} - (1+\eta^l\zeta) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{\Pi_t^w}{\Pi^w}-1\right) +$$
$$+ (1+\eta^l\zeta) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{\Pi_{t+k}^w}{\Pi^w}-1\right) + \zeta E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k L_{t+k} - E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{\psi_{t+k}} -$$
$$- E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{1}{1+\gamma-h} \widehat{C^{uc}_{t+k-1}} + E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{1+g}{1+\gamma-h} \widehat{C^{uc}_{t+k}} - E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{W^r_{t+k}} +$$
$$+ E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{1}{1-\eta^l} \widehat{\eta^l_{t+k}} + E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{h}{1+\gamma^z-h} \frac{\gamma^z}{1+\gamma^z} \widehat{\gamma^z_{t+k}}$$

Let's multiply both sides by $\frac{1-\beta\theta_w}{1+\eta^l\zeta}$ and then take terms in k=0 period out separately from the sum expressions, we get:

$$\left(\frac{W_t^*}{W_t}-1\right) =$$
$$= \frac{1-\beta\theta_w}{1+\eta^l\zeta}(\widehat{\theta}_t - \widehat{\psi}_t) - \left(\frac{\Pi_t^w}{\Pi^w}-1\right) + (1-\beta\theta_w)\left(\frac{\Pi_t^w}{\Pi^w}-1\right) + \frac{\zeta(1-\beta\theta_w)}{1+\eta^l\zeta} L_t -$$
$$- \frac{1-\beta\theta_w}{1+\eta^l\zeta} \frac{1}{1+\gamma^z-h} \widehat{C^{uc}_{t-1}} + \frac{1-\beta\theta_w}{1+\eta^l\zeta} \frac{1+\gamma^z}{1+\gamma^z-h} \widehat{C^{uc}_t} - \frac{1-\beta\theta_w}{1+\eta^l\zeta} \widehat{W^k_t} -$$
$$+ \frac{1-\beta\theta_w}{1+\eta^l\zeta} \frac{1}{1-\eta^l} \widehat{\eta^l_t} + \frac{1-\beta\theta_w}{1+\eta^l\zeta} \frac{h}{1+\gamma^z-h} \widehat{\gamma^z_t} +$$
$$+ \frac{1-\beta\theta_w}{1+\eta^l\zeta}(\beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{\theta_{t+k+1}} - \frac{1-\beta\theta_w}{1+\eta^l\zeta}(\beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{\psi_{t+k+1}} +$$
$$+ (1-\beta\theta_w)(\beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{\Pi_{t+k+1}^w}{\Pi^w}-1\right) + \frac{\zeta(1-\beta\theta_w)}{1+\eta^l\zeta}(\beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k L_{t+k+1} -$$
$$- \frac{1-\beta\theta_w}{1+\eta^l\zeta} \frac{1}{1+\gamma^z-h}(\beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{C^{uc}_{t+k}} + \frac{1-\beta\theta_w}{1+\eta^l\zeta}(\beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{C^{uc}_{t+k+1}} -$$
$$- \frac{1-\beta\theta_w}{1+\eta^l\zeta}(\beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{W^r_{t+k}} - \frac{1-\beta\theta_w}{1+\eta^l\zeta}(\beta\theta_w) \frac{1}{1-\eta^l} E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{\eta^l_{t+k+1}} +$$
$$+ \frac{1-\beta\theta_w}{1+\eta^l\zeta} \frac{h}{1+\gamma^z-h} \frac{\gamma^z}{1+\gamma^z}(\beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \widehat{\gamma^z_{t+k}} - (\beta\theta_w)\left(\frac{\Pi_{t+1}^w}{\Pi^w}-1\right) +$$
$$+ (\beta\theta_w)\left(\frac{\Pi_{t+1}^w}{\Pi^w}-1\right) \tag{A.3.11}$$

The equation can be written as:

$$
\begin{aligned}
&\left(\frac{W_t^*}{W_t}-1\right)= \\
&\quad=\frac{1-\beta\theta_w}{1+\eta^l\zeta}(\widehat{\theta}_t-\widehat{\psi}_t)-(\beta\theta_w)\left(\frac{\Pi_t^w}{\Pi^w}-1\right)+\frac{\zeta(1+\beta\theta_w)}{1+\eta^l\zeta}L_t- \\
&\quad-\frac{1-\beta\theta_w}{1+\eta^l\zeta}\left(\frac{1}{1+\gamma^z-h}\widehat{C_{t-1}^{uc}}-\frac{1+\gamma^z}{1+\gamma^z-h}\widehat{C_t^{uc}}\right)-\frac{1-\beta\theta_w}{1+\eta^l\zeta}\widehat{W_{t+k}^r}+ \\
&\quad+\frac{1-\beta\theta_w}{1+\eta^l\zeta}\frac{1}{1-\eta^l}\widehat{\eta_t^l}+\frac{1-\beta\theta_w}{1+\eta^l\zeta}\frac{h}{1+\gamma^z-h}\widehat{\gamma_t^z} \\
&\quad+\beta\theta_w E_t\left(\frac{\Pi_t^w}{\Pi^w}-1\right)+\beta\theta_w E_t\left(\frac{W_{t+1}^*}{W_{t+1}}-1\right)
\end{aligned}
\tag{A.3.12}
$$

If we substitute $\left(\frac{W_t^*}{W_t}-1\right)$ with the equation for aggregate wage dynamics, we can write:

$$
\begin{aligned}
&\frac{\theta_w}{1-\theta_w}\frac{1}{\Pi_w}\left(\Pi_t^w-\Pi_{t-1}^w\right)= \\
&\quad=\frac{1-\beta\theta_w}{1+\eta_t^l\zeta}(\widehat{\theta}_t-\widehat{\psi}_t)-\frac{\beta\theta_w}{\Pi^w}\Pi_t^w-\frac{1-\beta\theta_w}{1+\eta_t^l\zeta}\widehat{W_t^r}+ \\
&\quad+\frac{1-\beta\theta_w}{1+\eta_t^l\zeta}\left(\zeta L_t-\frac{1}{1+\gamma^z-h}\widehat{C_{t-1}^{uc}}+\frac{1+\gamma^z}{1+\gamma^z-h}\widehat{C_t^{uc}}\right) \\
&\quad+\frac{1-\beta\theta_w}{1+\eta^l\zeta}\frac{1}{1-\eta^l}\widehat{\eta_t^l}+\frac{1-\beta\theta_w}{1+\eta^l\zeta}\frac{h}{1+\gamma^z-h}\widehat{\gamma_t^z}+ \\
&\quad+\frac{\beta\theta_w}{\Pi^w}E_t\Pi_{t+1}^w+\frac{\beta{\theta_w}^2}{\Pi^w(1-\theta_w)}E_t(\Pi_{t+1}^w-\Pi_t^w)
\end{aligned}
\tag{A.3.13}
$$

After multiplying the equation by Π^w

$$
\begin{aligned}
&\frac{\theta_w}{1-\theta_w}\left(\pi_t^w-\pi_{t-1}^w\right)= \\
&\quad=\frac{1-\beta\theta_w}{1+\eta_t^l\zeta}\Pi^w(\widehat{\theta}_t-\widehat{\psi}_t)-\frac{1-\beta\theta_w}{1+\eta_t^l\zeta}\Pi^w\widehat{W_t^r}+ \\
&\quad+\frac{1-\beta\theta_w}{1+\eta_t^l\zeta}\Pi^w\left(\zeta\widehat{L}_t-\frac{1}{1+\gamma^z-h}\widehat{C_{t-1}^{uc}}+\frac{1+\gamma^z}{1+\gamma^z-h}\widehat{C_t^{uc}}+\frac{h}{1+\gamma^z-h}\widehat{\gamma_t^z}\right)- \\
&\quad+\frac{1-\beta\theta_w}{1+\eta^l\zeta}\frac{\Pi^w}{1-\eta^l}\widehat{\eta_t^l}+\beta\theta_w E_t(\pi_{t+1}^w-\pi_t^w)+\frac{\beta\theta_w^2}{\Pi^w(1-\theta_w)}E_t(\pi_{t+1}^w-\pi_t^w)
\end{aligned}
\tag{A.3.14}
$$

Finally, after collecting the same terms and recalling the equation for the gap of the marginal rate of substitution the linear form of the wage Phillips curve is given by:

$$\pi_t^w = \frac{1}{1+\beta}\pi_{t-1}^w + \frac{\beta}{1+\beta}E_t\pi_{t+1}^w + \frac{\Pi^w(1-\theta)(1-\beta\theta_w)}{\theta_w(1+\eta^l\zeta)}(\widehat{MRS}_t - \widehat{W}_t^r) - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\eta^l\zeta)}\frac{\Pi^w}{\eta^l-1}\widehat{\eta}_t^l \tag{A.3.15}$$

Note, the last term represents wage markup shock in the wage PC equation.

Appendix B Entrepreneurs

B.1 Functional Forms

Capital utilization cost function $\gamma(u)$ is convex, increasing and has the following form:

$$\gamma(u) = 0.5\sigma_a\sigma_b u^2 + \sigma_b(1-\sigma_a)u + \sigma_b\left(\frac{\sigma_a}{2} - 1\right) \tag{B.1.1}$$

where σ_a and σ_b are parameters governing the shape and curvature of the function. In steady state $u = 1, \gamma(1) = 0, \gamma'(1) = \sigma_b, \gamma''(1) = \sigma_b\sigma_a > 0$.

Investment adjustment cost function $\tilde{S}(x)$ as well as its derivatives have the following form[23]:

$$\begin{aligned}
\tilde{S}(x) &= \frac{1}{2}\left\{\exp\left[\sqrt{\tilde{S}''}\left(x - g^I\right)\right] + \exp\left[-\sqrt{\tilde{S}''}\left(x - g^I\right)\right] - 2\right\} \\
&= 0, \quad x = g^I && \text{(B.1.2)} \\
\tilde{S}'(x) &= \frac{1}{2}\sqrt{\tilde{S}''}\left\{\exp\left[\sqrt{\tilde{S}''}\left(x - g^I\right)\right] - \exp\left[-\sqrt{\tilde{S}''}\left(x - g^I\right)\right]\right\} \\
&= 0, \quad x = g^I && \text{(B.1.3)} \\
\tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}\left(x - g^I\right)\right] + \exp\left[-\sqrt{\tilde{S}''}\left(x - g^I\right)\right]\right\} \\
&= \tilde{S}'', \quad x = g^I && \text{(B.1.4)}
\end{aligned}$$

B.2 First Order Conditions and Linearization

The Lagrangian of the representative entrepreneur's problem 18 will be:

[23] In our model $x = \frac{I_t}{I_{t-1}}$

$$\mathcal{L}\left(u_t, I_t, \overline{K}_{t+1}, \lambda_t^e\right) = E_0 \sum_{t=0}^{\infty} \tilde{M}_t \Bigg[R_t^k \overline{K}_t u_t - \gamma(u_t)\overline{K}_t P_t^i - I_t P_t^i \\ + \lambda_t^e \left((1-\delta)\, \overline{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t - \overline{K}_{t+1} \right) \Bigg] \tag{B.2.1}$$

FOCs:

$$[u_t]: \qquad \frac{\partial \mathcal{L}}{\partial u_t} = 0 \Rightarrow \tilde{M}_t \big[R_t^k \overline{K}_t - \gamma'(u_t)\overline{K}_t P_t^i \big] = 0$$

$$\Rightarrow R_t^k = \gamma'\left(u_t\right) P_t^i \tag{B.2.2}$$

$$[I_t]: \qquad \frac{\partial \mathcal{L}}{\partial I_t} = 0 \Rightarrow \tilde{M}_t \Bigg[-P_t^i + \lambda_t^e \Bigg(1 - \tilde{S}\Bigg(\frac{I_t}{I_{t-1}}\Bigg) + I_t \Bigg(-\tilde{S}'\Bigg(\frac{I_t}{I_{t-1}}\Bigg) \frac{1}{I_{t-1}} \Bigg) \Bigg) \Bigg] \\ + E_t \Bigg[\tilde{M}_{t+1} \lambda_{t+1}^e I_{t+1} \Bigg(-\tilde{S}'\Bigg(\frac{I_{t+1}}{I_t}\Bigg) \frac{I_{t+1}}{I_t^2}(-1) \Bigg) \Bigg] = 0$$

$$\Rightarrow P_t^i = \lambda_t^e \Bigg(1 - \tilde{S}\Bigg(\frac{I_t}{I_{t-1}}\Bigg) - \tilde{S}'\Bigg(\frac{I_t}{I_{t-1}}\Bigg) \frac{I_t}{I_{t-1}} \Bigg) \\ + E_t \Bigg[\frac{\tilde{M}_{t+1}}{\tilde{M}_t} \lambda_{t+1}^e \tilde{S}'\Bigg(\frac{I_{t+1}}{I_t}\Bigg) \frac{I_{t+1}^2}{I_t^2} \Bigg] \tag{B.2.3}$$

$$\left[\overline{K}_{t+1}\right]: \qquad \frac{\partial \mathcal{L}}{\partial \overline{K}_{t+1}} = 0 \Rightarrow \tilde{M}_t(-\lambda_t^e) \\ + E_t \Big[\tilde{M}_{t+1} \big(R_{t+1}^k u_{t+1} - \gamma(u_{t+1}) p_{t+1}^i + (1-\delta)\lambda_{t+1}^e \big) \Big] = 0$$

$$\Rightarrow \lambda_t^e = E_t \left[\frac{\tilde{M}_{t+1}}{\tilde{M}_t} \big(R_{t+1}^k u_{t+1} - \gamma(u_{t+1}) p_{t+1}^i \big) \right] \\ + (1-\delta) E_t \left[\frac{\tilde{M}_{t+1}}{\tilde{M}_t} \lambda_{t+1}^e \right] \tag{B.2.4}$$

$$[\lambda_t^e]: \qquad \frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \Rightarrow \overline{K}_{t+1} = (1-\delta)\overline{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \tag{B.2.5}$$

First, rewrite the first-order conditions in real terms by dividing both sides of equations by P_t^c. Letting $r_t^k = \frac{R_t^k}{P_t^c}, P_t^i = \frac{P_t^i}{P_t^c}, \tilde{\lambda}_t^e = \frac{\lambda_t^e}{P_t^c}$ and $\frac{\lambda_{t+1}^e}{P_t^c} = \frac{\lambda_{t+1}^e}{P_t^c}\frac{P_{t+1}^c}{P_{t+1}^c} = \frac{\lambda_{t+1}^e}{P_{t+1}^c}\frac{P_{t+1}^c}{P_t^c} = \tilde{\lambda}_{t+1}^e \Pi_{t+1}^c$[24] and noting that $\frac{\tilde{M}_{t+1}}{\tilde{M}_t} = \frac{\beta\psi_{t+1}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t \Pi_{t+1}^c (C_{t+1}^{uc} - hC_t^{uc})}$, FOCs will become:

[24] $\Pi_{t+1}^c = \frac{P_{t+1}^c}{P_t^c} = 1 + \pi_{t+1}^c$ as was the case in the Household's section 2.2

$$[u_t]: \qquad r_t^k = \gamma'(u_t)P_t^i \tag{B.2.6}$$

$$\begin{aligned}
[I_t]: \qquad P_t^i &= \tilde{\lambda}_t^e\left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) \\
&+ E_t\left[\frac{\beta\psi_{t+1}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t(C_{t+1}^{uc} - hC_t^{uc})}\tilde{\lambda}_{t+1}^e\tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\frac{I_{t+1}^2}{I_t^2}\right] \qquad \text{(B.2.7)}
\end{aligned}$$

$$\begin{aligned}
[\overline{K}_{t+1}]: \qquad \tilde{\lambda}_t^e &= E_t\left[\frac{\beta\psi_{t+1}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t(C_{t+1}^{uc} - hC_t^{uc})}\left(r_{t+1}^k u_{t+1} - \gamma(u_{t+1})p_{t+1}^i\right)\right] \\
&+ (1-\delta)E_t\left[\frac{\beta\psi_{t+1}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t(C_{t+1}^{uc} - hC_t^{uc})}\tilde{\lambda}_{t+1}^e\right] \qquad \text{(B.2.8)}
\end{aligned}$$

$$[\lambda_t]: \qquad \overline{K}_{t+1} = (1-\delta)\overline{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)I_t \tag{B.2.9}$$

In the next stage we log-linearize the stationary forms of FOCs around the steady state of the model.[25]

First, consider equation I.1.14 for capital utilization and take logs on both sides:

$$\ln(r_t^k) = \ln(\gamma'(u_t)P_t^i) \Rightarrow \ln(r_t^k) = \ln(\gamma'(u_t)) + \ln(P_t^i) \tag{B.2.10}$$

Now let's find partial derivatives of both sides of B.2.10 w.r.t each variable and find their values in steady state:

$$\frac{\mathrm{d}LLHS}{\mathrm{d}r_t^k} = \frac{1}{r_t^k} \overset{\text{in SS}}{=\!=\!=} \frac{1}{r^k} \tag{B.2.11}$$

$$\frac{\partial LRHS}{\partial u_t} = \frac{\gamma''(u_t)}{\gamma'(u_t)} \overset{\text{in SS}}{=\!=\!=} \frac{\gamma''(1)}{\gamma'(1)} = \frac{\sigma_a\sigma_b}{\sigma_b} = \sigma_a \tag{B.2.12}$$

$$\frac{\partial LRHS}{\partial P_t^i} = \frac{1}{P_t^i} \overset{\text{in SS}}{=\!=\!=} \frac{1}{p^I} \tag{B.2.13}$$

Using B.2.11 - B.2.13, the first order Taylor series expansion of B.2.10 around steady state will be:

[25] In the remaining of this section I will use *LHS* for the left-hand side and *RHS* for the right-hand side of the corresponding equation under consideration. *LLHS* and *LRHS* serve the same purpose but for the natural logarithm of the same equation.

$$\ln(r^k) + \frac{1}{r^k}\left(r^k_t - r^k\right) = \ln(\gamma'(1)) + \ln(p^I) + \sigma_a(u_t - 1) + \frac{1}{p^I}\left(P^i_t - p^I\right) \tag{B.2.14}$$

Using B.2.10 in steady state some terms cancel out in B.2.14 and we get:

$$\frac{r^k_t - r^k}{r^k} = \ln(\gamma'(1)) + \ln(p^I) + \sigma_a(u_t - 1) + \frac{P^i_t - p^I}{p^I} \tag{B.2.15}$$

Using hats we get:

$$\widehat{r^k_t} = r^k\sigma_a\widehat{u}_t + r^k\widehat{P^i_t} \tag{B.2.16}$$

where the hat above each variable denotes percentage deviation from its own steady state value except for the variables that are already expressed in percentage terms (for example $\hat{r}^k_t = r^k_t - r^k$).[26]

Rest part of the equations in the entrepreneurs' problem includes non-stationary variables, hence, before linearization, we need to make them stationary first which are given in the appendix I.2. Now consider the FOC in real terms w.r.t. $\widetilde{I}_t$ (equation I.1.15) and take natural logs on both sides:

$$\begin{aligned}\ln(P^i_t) = \ln\Bigg(&\tilde{\lambda}^e_t\left(1 - \tilde{S}\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right) - \tilde{S}'\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)(1+\gamma^z_t)\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)\\ &+E_t\left[\frac{\beta\psi_{t+1}(\widetilde{C^{uc}_t} - \frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}})}{\psi_t((1+\gamma^z_{t+1})\widetilde{C^{uc}_{t+1}} - h\widetilde{C^{uc}_t})}\tilde{\lambda}^e_{t+1}\tilde{S}'\left(\frac{\widetilde{I_{t+1}}}{\widetilde{I}_t}\right)(1+\gamma^z_{t+1})^2\frac{\widetilde{I_{t+1}}^2}{\widetilde{I}_t^2}\right]\Bigg)\end{aligned} \tag{B.2.17}$$

Then, find partial derivatives of both sides of B.2.17 w.r.t each variable and find their values in steady state:[27]

[26]This convention will be used through the remaining of the section.

[27] On the right-hand side of the equation B.2.17 in steady state it equals to $\tilde{\lambda}^e$.
Note also that $\frac{\beta\psi(\widetilde{C^{uc}} - \frac{h}{1+\gamma^z}\widetilde{C^{uc}})}{\psi((1+\gamma^z)\widetilde{C^{uc}} - h\widetilde{C^{uc}})} = \frac{\beta}{1+\gamma^z}$.

$$\frac{\mathrm{d}LLHS}{\mathrm{d}P^i_t} = \frac{1}{P^i_t} \overset{\text{in SS}}{=\!=\!=} \frac{1}{p^I} \tag{B.2.18}$$

$$\begin{aligned}\frac{\partial LRHS}{\partial \tilde{\lambda}^e_t} =& \frac{1}{RHS}\frac{\partial RHS}{\partial \tilde{\lambda}^e_t} = \frac{1}{RHS}\left(1 - \tilde{S}\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right) - \tilde{S}'\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)(1+\gamma^z_t)\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right) \overset{\text{in SS}}{=\!=\!=} \\ =& \frac{1}{\tilde{\lambda}^e}\left(1 - \tilde{S}\left(\frac{\widetilde{I}}{\widetilde{I}}\right) - \tilde{S}'\left(\frac{\widetilde{I}}{\widetilde{I}}\right)(1+\gamma^z)\frac{\widetilde{I}}{\widetilde{I}}\right) \\ \overset{\text{footnote 27}}{=\!=\!=\!=\!=}& \frac{1}{\tilde{\lambda}^e}\end{aligned} \tag{B.2.19}$$

$$\begin{aligned}\frac{\partial LRHS}{\partial \widetilde{I_{t-1}}} =& \frac{1}{RHS}\frac{\partial RHS}{\partial \widetilde{I_{t-1}}} = \\ =& \frac{1}{RHS}\tilde{\lambda}^e_t\left(\tilde{S}'\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)\frac{\widetilde{I}_t}{\widetilde{I^2_{t-1}}} + \tilde{S}'\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)(1+\gamma^z_t)\frac{\widetilde{I}_t}{\widetilde{I^2_{t-1}}} + \right. \\ & \left. + \tilde{S}''\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)(1+\gamma^z_t)\frac{\widetilde{I}_t^2}{\widetilde{I_{t-1}}^3}\right) \overset{\text{in SS}}{=\!=\!=} \\ =& \frac{1}{\tilde{\lambda}^e}\tilde{\lambda}^e\left(\tilde{S}'\left(\frac{\widetilde{I}}{\widetilde{I}}\right)\frac{\widetilde{I}}{\widetilde{I}^2} + \tilde{S}'\left(\frac{\widetilde{I}}{\widetilde{I}}\right)(1+\gamma^z)\frac{\widetilde{I}}{\widetilde{I}^2} + \tilde{S}''\left(\frac{\widetilde{I}}{\widetilde{I}}\right)(1+\gamma^z)\frac{\widetilde{I}^2}{\widetilde{I}^3}\right) \overset{\text{footnote 27}}{=\!=\!=\!=\!=} \\ =& \frac{(1+\gamma^z)\tilde{S}''}{I}\end{aligned} \tag{B.2.20}$$

$$\begin{aligned}\frac{\partial LRHS}{\partial \widetilde{I}_t} =& \frac{1}{RHS}\frac{\partial RHS}{\partial \widetilde{I}_t} = \\ =& \frac{1}{RHS}\tilde{\lambda}^e_t\left(-\tilde{S}'\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)\frac{1}{\widetilde{I_{t-1}}} - \tilde{S}'\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)(1+\gamma^z_t)\frac{1}{\widetilde{I_{t-1}}} - \right. \\ & \left. - \tilde{S}''\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)(1+\gamma^z_t)\frac{\widetilde{I}_t}{\widetilde{I^2_{t-1}}}\right) + \\ & + \frac{1}{RHS}E_t\left[\frac{\beta\psi_{t+1}(\widetilde{C^{uc}_t} - \frac{h}{1+\gamma^z_t}\widetilde{C^{uc}_{t-1}})}{\psi_t((1+\gamma^z_{t+1})\widetilde{C^{uc}_{t+1}} - h\widetilde{C^{uc}_t})}\tilde{\lambda}^e_{t+1}\left(-\tilde{S}''\left(\frac{\widetilde{I_{t+1}}}{\widetilde{I}_t}\right)(1+\gamma^z_{t+1})^2\frac{\widetilde{I^3_{t+1}}}{\widetilde{I}^4_t} - \right.\right. \\ & \left.\left. - 2\tilde{S}'\left(\frac{\widetilde{I_{t+1}}}{\widetilde{I}_t}\right)(1+\gamma^z_{t+1})^2\frac{\widetilde{I^2_{t+1}}}{\widetilde{I}^3_t}\right)\right] \\ \overset{\text{in SS}}{=\!=\!=}& \frac{1}{\tilde{\lambda}^e}\tilde{\lambda}^e\left(-\tilde{S}'\left(\frac{\widetilde{I}}{\widetilde{I}}\right)\frac{1}{\widetilde{I}} - \tilde{S}'\left(\frac{\widetilde{I}}{\widetilde{I}}\right)(1+\gamma^z)\frac{1}{\widetilde{I}} - \tilde{S}''\left(\frac{\widetilde{I}}{\widetilde{I}}\right)(1+\gamma^z)\frac{\widetilde{I}}{\widetilde{I}^2}\right) + \\ & + \frac{1}{\tilde{\lambda}^e}\left(\frac{\beta\psi(\widetilde{C^{uc}} - \frac{h}{1+\gamma^z}\widetilde{C^{uc}})}{\psi((1+\gamma^z)\widetilde{C^{uc}} - h\widetilde{C^{uc}})}\tilde{\lambda}^e\left(-\tilde{S}''\left(\frac{\widetilde{I}}{\widetilde{I}}\right)(1+\gamma^z)^2\frac{\widetilde{I}^3}{\widetilde{I}^4} - 2\tilde{S}'\left(\frac{\widetilde{I}}{\widetilde{I}}\right)(1+\gamma^z)^2\frac{\widetilde{I}^2}{\widetilde{I}^3}\right)\right) \\ \overset{\text{footnote 27}}{=\!=\!=\!=\!=}& -\frac{(1+\gamma^z)(1+\beta)\tilde{S}''}{\widetilde{I}}\end{aligned} \tag{B.2.21}$$

Similarly, we will log linearize the stationary FOC w.r.t γ_t^z, γ_{t+1}^z, $\widetilde{I_{t+1}}$, $\tilde{\lambda}_{t+1}^e$, $\widetilde{C_{t-1}^{uc}}$, $\widetilde{C_t^{uc}}$, $\widetilde{C_{t+1}^{uc}}$, ψ_t, ψ_{t+1}.

Note also that in SS derivatives of LRHS w.r.t $\tilde{\lambda}_{t+1}^e$, $\widetilde{C_{t-1}^{uc}}$, $\widetilde{C_t^{uc}}$, $\widetilde{C_{t+1}^{uc}}$, ψ_t, ψ_{t+1},γ_t^z, γ_{t+1}^z equals 0. Using B.2.18 - B.2.21 the first order Taylor series expansion of B.2.17 around steady state will be:

$$\begin{aligned}\ln\left(p^I\right) + \frac{P_t^i - p^I}{p^I} =& \ln\left(\tilde{\lambda}^e\right) + \frac{1}{\tilde{\lambda}^e}\left(\tilde{\lambda}_t^e - \tilde{\lambda}^e\right) + \frac{(1+\gamma^z)\tilde{S}''}{\widetilde{I}}\left(\widetilde{I_{t-1}} - \widetilde{I}\right)\\ &-\frac{\tilde{S}''(1+\gamma^z)(1+\beta)}{\widetilde{I}}\left(\widetilde{I_t} - \widetilde{I}\right) + \frac{\beta(1+\gamma^z)\tilde{S}''}{\widetilde{I}}E_t\left(\widetilde{I_{t+1}} - \widetilde{I}\right) \quad \text{(B.2.22)}\end{aligned}$$

Using B.2.17 in steady state some terms cancel out in B.2.22 and we get:

$$\begin{aligned}\widehat{P_t^i} =& \widehat{\tilde{\lambda}_t^e} + (1+\gamma^z)\tilde{S}''\widehat{I_{t-1}}\\ &-\tilde{S}''(1+\gamma^z)(1+\beta)\widehat{I_t} + \tilde{S}''\beta(1+\gamma^z)E_t\widehat{I_{t+1}} \quad \text{(B.2.23)}\end{aligned}$$

Now consider equation I.1.16 and take natural logs on both sides:

$$\begin{aligned}\ln\left(\tilde{\lambda}_t^e\right) =& \ln\left(E_t\left[\frac{\beta\psi_{t+1}(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}})}{\psi_t((1+\gamma_{t+1}^z)\widetilde{C_{t+1}^{uc}} - h\widetilde{C_t^{uc}})}\left(r_{t+1}^k u_{t+1} - \gamma(u_{t+1})p_{t+1}^i\right)\right] + \right.\\ &\left. + (1-\delta)E_t\left[\frac{\beta\psi_{t+1}(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}})}{\psi_t((1+\gamma_{t+1}^z)\widetilde{C_{t+1}^{uc}} - h\widetilde{C_t^{uc}})}\tilde{\lambda}_{t+1}^e\right]\right) \quad \text{(B.2.24)}\end{aligned}$$

Next, calculate partial derivatives of both sides of B.2.24 w.r.t. each variable and find their values in steady state.[28] We are showing derivatives of RHS wrt. $\tilde{\lambda}_t$,r_{t+1}^k and $\widetilde{C_t^{uc}}$ to demonstrate the idea and the rest part of derivations are not given here to save the space.

$$\frac{\partial LLHS}{\partial\tilde{\lambda}_t^e} = \frac{1}{\tilde{\lambda}_t^e} \overset{\text{in SS}}{=\!=\!=} \frac{1}{\tilde{\lambda}^e} \quad \text{(B.2.25)}$$

$$\frac{\partial LRHS}{\partial r_{t+1}^k} = \frac{1}{RHS}\frac{\partial RHS}{\partial r_{t+1}^k} = \frac{1}{RHS}E_t\left[\frac{\beta\psi_{t+1}(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}})}{\psi_t((1+\gamma_{t+1}^z)\widetilde{C_{t+1}^{uc}} - h\widetilde{C_t^{uc}})}u_{t+1}\right]$$

[28] We will use the following information: $u^{ss} = 1, \gamma(1) = 0, \gamma'(1) = \sigma_b$ and $RHS^{ss} = \tilde{\lambda}^{ss}$.

$$\xlongequal{\text{in SS}} \frac{1}{\tilde{\lambda}^e} \frac{\beta}{1+\gamma^z} \tag{B.2.26}$$

$$\frac{\partial LRHS}{\partial C_t^{uc}} = \frac{1}{RHS} \frac{\partial RHS}{\partial C_t^{uc}} =$$

$$= \frac{1}{RHS} E_t \Bigg[\frac{\beta \psi_{t+1} \psi_t \left((1+\gamma_{t+1}^z) \widetilde{C_t^{uc}} - h \widetilde{C_t^{uc}} \right) + \beta h \psi_t \psi_{t+1} \left(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z} \widetilde{C_t^{uc}} \right)}{\psi_t^2 \left((1+\gamma_{t+1}^z) \widetilde{C_t^{uc}} - h \widetilde{C_t^{uc}} \right)^2} \times$$

$$\times \left(r_{t+1}^k u_{t+1} - \gamma(u_{t+1}) P_t^i \right) + (1-\delta) \lambda_{t+1} \times$$

$$\times \frac{\beta \psi_{t+1} \psi_t \left((1+\gamma_{t+1}^z) \widetilde{C_t^{uc}} - h \widetilde{C_t^{uc}} \right) + \beta h \psi_t \psi_{t+1} \left(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z} \widetilde{C_t^{uc}} \right)}{\psi_t^2 \left((1+\gamma_{t+1}^z) \widetilde{C_t^{uc}} - h \widetilde{C_t^{uc}} \right)^2} \Bigg]$$

$$\xlongequal{\text{in SS}} \frac{1+\gamma^z+h}{1+\gamma^z-h} \frac{1}{\widetilde{C^{uc}}} \tag{B.2.27}$$

Using B.2.25 - B.2.27 the first order Taylor series expansion of B.2.24 around steady state will be:

$$\begin{aligned}
\ln(\tilde{\lambda}^e) + \frac{\tilde{\lambda}_t^e - \tilde{\lambda}^e}{\tilde{\lambda}^e} &= \ln\left(\frac{\beta}{1+\gamma^z} \left(r^k + (1-\delta)\lambda^e \right) \right) - \frac{\psi_t - \psi}{\psi} + E_t \frac{\psi_{t+1} - \psi}{\psi} - \\
&- \frac{h}{1+\gamma^z-h} \frac{\widetilde{C^{uc}}_{t-1} - \widetilde{C^{uc}}}{\widetilde{C^{uc}}} + \frac{1+\gamma^z+h}{1+\gamma^z-h} \frac{\widetilde{C_t^{uc}} - \widetilde{C^{uc}}}{\widetilde{C^{uc}}} - \frac{1+\gamma^z}{1+\gamma^z-h} E_t \frac{\widetilde{C^{uc}}_{t+1} - \widetilde{C^{uc}}}{\widetilde{C^{uc}}} + \\
&+ \frac{h\gamma^z}{(1+\gamma^z-h)(1+\gamma^z)} \frac{\gamma_t^z - \gamma^z}{\gamma^z} - \frac{\gamma^z}{1+\gamma^z-h} E_t \frac{\gamma_{t+1}^z - \gamma^z}{\gamma^z} + \frac{r^k}{r^k + (1-\delta)\lambda^e} E_t \frac{r_{t+1}^k - r^k}{r^k} + \\
&+ \frac{1}{r^k + (1-\delta)\tilde{\lambda}^e} (r^k - \gamma'(u) p^I) E_t (u_{t+1} - 1) + \frac{1-\delta}{r^k + (1-\delta)\tilde{\lambda}^e} E_t \frac{\lambda_{t+1}^{\tilde{e}} - \tilde{\lambda}^e}{\tilde{\lambda}^e}
\end{aligned} \tag{B.2.28}$$

In terms of gaps it could be rewritten as:

$$\begin{aligned}
\widehat{\tilde{\lambda}_t^e} &= -\widehat{\psi}_t + \widehat{\psi_{t+1}} - \frac{h}{1+\gamma^z-h} \widehat{C_{t-1}^{uc}} + \frac{1+\gamma^z+h}{1+\gamma^z-h} \widehat{C_t^{uc}} - \frac{1+\gamma^z}{1+\gamma^z-h} E_t \widehat{C_{t+1}^{uc}} + \\
&+ \frac{h\gamma^z}{(1+\gamma^z-h)(1+\gamma^z)} \widehat{\gamma_t^z} - \frac{\gamma^z}{1+\gamma^z-h} E_t \widehat{\gamma_{t+1}^z} + \frac{r^k}{r^k + (1-\delta)\lambda^e} E_t \widehat{r_{t+1}^k} + \\
&+ \frac{1}{r^k + (1-\delta)\tilde{\lambda}^e} (r^k - \gamma'(u) p^I) E_t \widehat{u_{t+1}} + \frac{1-\delta}{r^k + (1-\delta)\tilde{\lambda}^e} E_t \widehat{\lambda_{t+1}^e}
\end{aligned} \tag{B.2.29}$$

Lastly, we are left with the stationary form of I.1.17. Again, take natural logs and we get:

$$\ln\left((1+\gamma^z_{t+1})\widetilde{\overline{K}_{t+1}}\right) = \ln\left((1-\delta)\widetilde{\overline{K}}_t + \left(1-\tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)\widetilde{I}_t\right) \tag{B.2.30}$$

Then, calculate partial derivatives of both sides of B.2.30 w.r.t. each variable and find their values in steady state.[29]

$$\frac{\partial LLHS}{\partial \widetilde{\overline{K}_{t+1}}} = \frac{1+\gamma^z_{t+1}}{(1+\gamma^z_{t+1})\widetilde{\overline{K}_{t+1}}} \xlongequal{\text{in SS}} \frac{1}{\widetilde{\overline{K}}} \tag{B.2.31}$$

$$\frac{\partial LLHS}{\partial \gamma^z_{t+1}} = \frac{\widetilde{\overline{K}}_{t+1}}{(1+\gamma^z_{t+1})\widetilde{\overline{K}_{t+1}}} \xlongequal{\text{in SS}} \frac{1}{(1+\gamma^z)} \tag{B.2.32}$$

$$\frac{\partial LRHS}{\partial \widetilde{\overline{K}}_t} = \frac{1}{RHS}\frac{\partial RHS}{\partial \widetilde{\overline{K}}_t} = \frac{1}{RHS}(1-\delta) \xlongequal{\text{in SS}} (1-\delta)\frac{1}{(1+\gamma^z)\widetilde{\overline{K}}}$$

$$\frac{\partial LRHS}{\partial \widetilde{I_{t-1}}} = \frac{1}{RHS}\frac{\partial RHS}{\partial \widetilde{I_{t-1}}} = \frac{1}{RHS}\widetilde{I}_t\left(\tilde{S}'\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)\frac{\widetilde{I}^2_t}{\widetilde{I^2_{t-1}}}\right) \xlongequal{\text{in SS}} 0 \tag{B.2.33}$$

$$\frac{\partial LRHS}{\partial \widetilde{I}_t} = \frac{1}{RHS}\frac{\partial RHS}{\partial \widetilde{I}_t} = \frac{1}{RHS}\left(1-\tilde{S}\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right) - \tilde{S}'\left(\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right)$$

$$\xlongequal{\text{in SS}} \frac{1}{(1+\gamma^z)\widetilde{\overline{K}}} \tag{B.2.34}$$

Using B.2.31 - B.2.34 the first order Taylor series expansion of B.2.30 around steady state will be:

$$\begin{aligned}
\ln\left((1+\gamma^z)\widetilde{\overline{K}}\right) + \frac{\widetilde{\overline{K}}_{t+1} - \widetilde{\overline{K}}}{\widetilde{\overline{K}}} + \\
+ \frac{1}{(1+\gamma^z)}\left(\gamma^z_{t+1} - \gamma^z\right) = \ln\left((1-\delta)\widetilde{\overline{K}} + \left(1-\tilde{S}\left(\frac{\widetilde{I}}{\widetilde{\overline{I}}}\right)\right)\widetilde{I}\right) + \\
+ (1-\delta)\frac{1}{(1+\gamma^z)\widetilde{\overline{K}}}\left(\widetilde{\overline{K}}_t - \overline{K}\right) + 0\left(\widetilde{I_{t-1}} - \widetilde{I}\right) + \frac{1}{(1+\gamma^z)\widetilde{\overline{K}}}\left(\widetilde{\overline{I}}_t - \widetilde{\overline{I}}\right)
\end{aligned} \tag{B.2.35}$$

Using B.2.30 in steady state some terms cancel out in B.2.35 and we get FOC for λ_t

[29] We will use the following information: $RHS = (1+\gamma^z)\widetilde{\overline{K}}$.

in terms of linear approximation around steady state of the model:[30]

$$\frac{\widetilde{\widetilde{K}_{t+1}} - \widetilde{\widetilde{K}}}{\widetilde{\widetilde{K}}} + \frac{\gamma^z}{1+\gamma^z}\frac{\gamma^z_{t+1} - \gamma^z}{\gamma^z} = (1-\delta)\frac{1}{(1+\gamma^z)}\frac{\left(\widetilde{\widetilde{K}}_t - \widetilde{\widetilde{K}}\right)}{\widetilde{\widetilde{K}}} + \frac{\widetilde{I}}{(1+\gamma^z)\widetilde{\widetilde{K}}}\frac{\left(\widetilde{I}_t - \widetilde{I}\right)}{\widetilde{I}} \tag{B.2.36}$$

Using hats we have:

$$\begin{aligned}\widehat{\widetilde{\widetilde{K}}_{t+1}} + \frac{\gamma^z}{1+\gamma^z}\widehat{\gamma^z_{t+1}} &= \frac{(1-\delta)}{(1+\gamma^z)}\widehat{\widetilde{\widetilde{K}}}_t + \frac{\widetilde{I}}{(1+\gamma^z)\widetilde{\widetilde{K}}}\widehat{I} \\ \xrightarrow{\text{using footnote 30}} \widehat{\widetilde{\widetilde{K}}_{t+1}} &= \frac{1-\delta}{1+\gamma^z}\widehat{\widetilde{\widetilde{K}}}_t + \frac{\gamma^z+\delta}{1+\gamma^z}\widehat{I}_t - \frac{\gamma^z}{1+\gamma^z}\widehat{\gamma^z_t} \end{aligned} \tag{B.2.37}$$

Appendix C Production of the Domestic Differentiated Inputs

Cost minimization problem of i^{th} intermediate good producer reads:

$$\underset{K_t(i), L_t(i), Y^m_t(i)}{\text{minimize}} \quad R^k_t K_t(i) + W_t L_t(i) + P^m_t Y^m_t(i) \tag{C.1a}$$

$$\text{subject to} \quad Y^d_t \leq (z_t L_t(i))^{\alpha_1}\gamma_t K_t(i)^{\alpha_2}\left(\frac{Y^m_t(i)}{a^x_t}\right)^{1-\alpha_1-\alpha_2} - F^d_t \tag{C.1b}$$

Lagrangian:

$$\mathcal{L} = R^k_t K_t(i) + W_t L_t(i) + P^m_t Y^m_t(i) - \lambda_t(i)\left[(z_t L_t(i))^{\alpha_1}\gamma_t K_t(i)^{\alpha_2}\left(\frac{Y^m_t(i)}{a^x_t}\right)^{1-\alpha_1-\alpha_2} - F^d_t - Y^d_t\right] \tag{C.2}$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial K_t(i)} = 0 \Longleftrightarrow R^k_t = \lambda_t(i)\alpha_2(z_t L_t(i))^{\alpha_1}\gamma_t K_t(i)^{\alpha_2-1}\left(\frac{Y^m_t(i)}{a^x_t}\right)^{1-\alpha_1-\alpha_2} \tag{C.3}$$

[30] We can show that in ss $\widetilde{I} = (\gamma^z + \delta)\widetilde{\widetilde{K}}$

$$\frac{\partial \mathcal{L}}{\partial L_t(i)} = 0 \Longleftrightarrow W_t = \lambda_t(i)\alpha_1 {z_t}^{\alpha_1}(L_t(i))^{\alpha_1 - 1}\gamma_t K_t(i)^{\alpha_2}\left(\frac{Y_t^m(i)}{a_t^x}\right)^{1-\alpha_1-\alpha_2} \tag{C.4}$$

$$\frac{\partial \mathcal{L}}{\partial Y_t^m(i)} = 0 \Longleftrightarrow P_t^m = \lambda_t(i)(1-\alpha_1-\alpha_2)(z_t L_t(i))^{\alpha_1}\gamma_t K_t(i)^{\alpha_2} Y_t^m(i)^{-\alpha_1-\alpha_2}\left(\frac{1}{a_t^x}\right)^{1-\alpha_1-\alpha_2} \tag{C.5}$$

Lets divide (C.3) by (C.4)

$$\frac{(10)}{(11)} \Rightarrow \frac{R_t^k}{W_t} = \frac{\lambda_t(i)\alpha_2(z_t L_t(i))^{\alpha_1}\gamma_t K_t(i)^{\alpha_2 - 1}\left(\frac{Y_t^m(i)}{a_t^x}\right)^{1-\alpha_1-\alpha_2}}{\lambda_t(i)\alpha_1 {z_t}^{\alpha_1}(L_t(i))^{\alpha_1-1}\gamma_t K_t(i)^{\alpha_2}\left(\frac{Y_t^m(i)}{a_t^x}\right)^{1-\alpha_1-\alpha_2}} \tag{C.6}$$

From (C.6) we can obtain:

$$K_t(i) = \frac{\alpha_2 W_t}{\alpha_1 R_t^k} L_t(i) \tag{C.7}$$

Now, lets divide (C.5) by (C.4)

$$\frac{(12)}{(11)} \Rightarrow \frac{P_t^m}{W_t} = \frac{\lambda_t(i)(1-\alpha_1-\alpha_2)(z_t L_t(i))^{\alpha_1}\gamma_t K_t(i)^{\alpha_2} Y_t^m(i)^{-\alpha_1-\alpha_2}\left(\frac{1}{a_t^x}\right)^{1-\alpha_1-\alpha_2}}{\lambda_t(i)\alpha_1 {z_t}^{\alpha_1}(L_t(i))^{\alpha_1-1}\gamma_t K_t(i)^{\alpha_2}\left(\frac{Y_t^m(i)}{a_t^x}\right)^{1-\alpha_1-\alpha_2}} \tag{C.8}$$

From (C.8) we can obtain:

$$Y_t^m(i) = \frac{(1-\alpha_1-\alpha_2)W_t}{\alpha_1 P_t^m} L_t(i) \tag{C.9}$$

Plug (C.7) and (C.9) into production function (2.4.1.29), and we will get:

$$Y_t(i) = (z_t L_t(i))^{\alpha_1}\gamma_t\left(\frac{\alpha_2 W_t}{\alpha_1 R_t^k} L_t(i)\right)^{\alpha_2}\left(\frac{(1-\alpha_1-\alpha_2)W_t}{\alpha_1 P_t^m} L_t(i)\right)^{1-\alpha_1-\alpha_2}\left(\frac{1}{a_t^x}\right)^{1-\alpha_1-\alpha_2} - F_t^d \tag{C.10}$$

From (C.10) we can obtain the demand function for labor input $L_t(i)$ as a function of $Y_t(i)$:

$$L_t(i) = \frac{{a_t^x}^{1-\alpha_1-\alpha_2}}{\gamma_t {z_t}^{\alpha_1}}\left(\frac{{\alpha_1}^{1-\alpha_1}}{{\alpha_2}^{\alpha_2}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}}\right)\left(\frac{{R_t^k}^{\alpha_2}{P_t^m}^{1-\alpha_1-\alpha_2}}{W_t^{1-\alpha_1}}\right)\left(Y_t(i) + F_t^d\right) \tag{C.11}$$

We need to apply same transformation for the capital and imported inputs. From the eq (C.7):

$$L_t(i) = \frac{\alpha_1 R_t^k}{\alpha_2 W_t} K_t(i) \tag{C.12}$$

Now, lets divide (C.3) by (C.5)

$$\frac{(10)}{(12)} \Rightarrow \frac{R_t^k}{P_t^m} = \frac{\lambda_t(i)\alpha_2 (z_t L_t(i))^{\alpha_1} \gamma_t K_t(i)^{\alpha_2 - 1} Y_t^m(i)^{1-\alpha_1-\alpha_2}}{\lambda_t(i)(1-\alpha_1-\alpha_2)(z_t L_t(i))^{\alpha_1} \gamma_t K_t(i)^{\alpha_2} Y_t^m(i)^{-\alpha_1-\alpha_2} \left(\frac{1}{a_t^x}\right)^{1-\alpha_1-\alpha_2}} \tag{C.13}$$

From eq (C.13) we get

$$Y_t^m(i) = \frac{R_t^k(1-\alpha_1-\alpha_2)}{P_t^m \alpha_2} K_t(i) \tag{C.14}$$

Plug eq (C.12) and eq (C.14) in the production function (2.4.1.29). We'll get:

$$Y_t(i) = {z_t}^{\alpha_1} \left(\frac{\alpha_1 R_t^k}{\alpha_2 W_t} K_t(i)\right)^{\alpha_1} \gamma_t K_t(i)^{\alpha_2} \left(\frac{R_t^k(1-\alpha_1-\alpha_2)}{P_t^m \alpha_2} K_t(i)\right)^{1-\alpha_1-\alpha_2} \left(\frac{1}{a_t^x}\right)^{1-\alpha_1-\alpha_2} - F_t^d \tag{C.15}$$

From eq (C.15) we can get the demand for capital $K_t(i)$ as a function of $Y_t(i)$:

$$K_t(i) = \frac{{a_t^x}^{1-\alpha_1-\alpha_2}}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{{\alpha_2}^{1-\alpha_2}}{{\alpha_1}^{\alpha_1}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}}\right) \left(\frac{W_t^{\alpha_1} {P_t^m}^{1-\alpha_1-\alpha_2}}{{R_t^k}^{1-\alpha_2}}\right) \left(Y_t(i) + F_t^d\right) \tag{C.16}$$

Now, let's do the same thing for import. From eq (C.9) and eq (C.14) we can get:

$$L_t(i) = \frac{P_t^m \alpha_1}{W_t(1-\alpha_1-\alpha_2)} Y_t^m(i) \tag{C.17}$$

$$K_t(i) = \frac{P_t^m \alpha_2}{R_t^k(1-\alpha_1-\alpha_2)} Y_t^m(i) \tag{C.18}$$

After plugging eq (C.17) and eq (C.18) in the production function (2.4.1.29) we'll get:

$$Y_t(i) = {z_t}^{\alpha_1} \left(\frac{\alpha_1 P_t^m}{(1-\alpha_1-\alpha_2)W_t} Y_t^m(i)\right)^{\alpha_1} \gamma_t \left(\frac{\alpha_2 P_t^m}{(1-\alpha_1-\alpha_2)R_t^k} Y_t^m(i)\right)^{\alpha_2} \left(\frac{Y_t^m(i)}{a_t^x}\right)^{1-\alpha_1-\alpha_2} - F_t^d \tag{C.19}$$

From (C.19) we can get the demand for imported input $Y_t^m(i)$ as a function of $Y_t(i)$:

$$Y_t^m(i) = \frac{{a_t^x}^{1-\alpha_1-\alpha_2}}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{(1-\alpha_1-\alpha_2)^{\alpha_1+\alpha_2}}{{\alpha_1}^{\alpha_1}{\alpha_2}^{\alpha_2}}\right) \left(\frac{W_t^{\alpha_1} {R_t^k}^{\alpha_2}}{{P_t^m}^{\alpha_1+\alpha_2}}\right) \left(Y_t(i) + F_t^d\right) \tag{C.20}$$

The next step is to plug (C.11), (I.1.32) and (I.1.33) in the total cost function:

$$
\begin{aligned}
TC_t(i) &= W_t L_t(i) + R_t^k K_t(i) + P_t^m Y_t^m(i) = \\
&= W_t \left[\frac{a_t^{x\,1-\alpha_1-\alpha_2}}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{{\alpha_1}^{1-\alpha_1}}{{\alpha_2}^{\alpha_2}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}} \right) \left(\frac{R_t^{k\,\alpha_2} P_t^{m\,1-\alpha_1-\alpha_2}}{W_t^{1-\alpha_1}} \right) \left(Y_t(i) + F_t^d \right) \right] \\
&+ R_t^k \left[\frac{a_t^{x\,1-\alpha_1-\alpha_2}}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{{\alpha_2}^{1-\alpha_2}}{{\alpha_1}^{\alpha_1}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}} \right) \left(\frac{W_t^{\alpha_1} P_t^{m\,1-\alpha_1-\alpha_2}}{R_t^{k\,1-\alpha_2}} \right) \left(Y_t(i) + F_t^d \right) \right] \\
&+ P_t^m \left[\frac{a_t^{x\,1-\alpha_1-\alpha_2}}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{(1-\alpha_1-\alpha_2)^{\alpha_1+\alpha_2}}{{\alpha_1}^{\alpha_1}{\alpha_2}^{\alpha_2}} \right) \left(\frac{W_t^{\alpha_1} R_t^{k\,\alpha_2}}{P_t^{m\,\alpha_1+\alpha_2}} \right) \left(Y_t(i) + F_t^d \right) \right] \qquad \text{(C.21)}
\end{aligned}
$$

It follows that the marginal cost function is given by:

$$
MC_t = \frac{\partial TC_t(i)}{\partial Y_t(i)} = \frac{1}{\alpha_1^{\alpha_1} \alpha_2^{\alpha_2} (1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}} \frac{a_t^{x\,1-\alpha_1-\alpha_2}}{\gamma_t {z_t}^{\alpha_1}} W_t^{\alpha_1} R_t^{k\,\alpha_2} P_t^{m\,1-\alpha_1-\alpha_2} \qquad \text{(C.22)}
$$

C.1 Price Indexation in Domestic Differentiated Input Sector

Let's consider the price-setting problem of the firm. The nominal profit of firm i can be written as:

$$
\Pi_t^d(i) = P_t(i) Y_t(i) - W_t L_t(i) - R_t^k K_t(i) - P_t^m Y_t^m(i) \qquad \text{(C.1.1)}
$$

Note here, that from the cost minimization problem above, $\lambda_t(i)$ and (C.2) reflects the marginal cost (nominal) and we can replace it by MC_t. Then we can write:

$$
(C.3) \Rightarrow R_t^k K_t(i) = MC_t \alpha_2 (z_t L_t(i))^{\alpha_1} \gamma_t K_t(i)^{\alpha_2} \left(\frac{Y_t^m(i)}{a_t^x} \right)^{1-\alpha_1-\alpha_2} \qquad \text{(C.1.2)}
$$

$$
(C.4) \Rightarrow W_t L_t(i) = MC_t \alpha_1 (z_t L_t(i))^{\alpha_1} \gamma_{tK_t(i)}^{\alpha_2} \left(\frac{Y_t^m(i)}{a_t^x} \right)^{1-\alpha_1-\alpha_2} \qquad \text{(C.1.3)}
$$

$$
(C.5) \Rightarrow P_t^m Y_t^m(i) = MC_t (1-\alpha_1-\alpha_2) (z_t L_t(i))^{\alpha_1} \gamma_t K_t(i)^{\alpha_2} \left(\frac{Y_t^m(i)}{a_t^x} \right)^{1-\alpha_1-\alpha_2} \qquad \text{(C.1.4)}
$$

So if we sum up LHS and RHS of the equations (C.1.2), (C.1.3) and (C.1.4) we'll get:

$$
R_t^k K_t(i) + W_t L_t(i) + P_t^m Y_t^m(i) = MC_t \left(Y_t(i) + F_t^d \right) \qquad \text{(C.1.5)}
$$

And (C.1.1) could be rewritten as:

$$\Pi_t(i) = P_t(i)Y_t(i) - MC_t\left(Y_t(i) + F_t^d\right) \tag{C.1.6}$$

Firms are operating on monopolistic competitive market and a random sample of firms have power to set prices (la Calvo), the rest are updating their prices linked to the previous period inflation:

$$P_t(i) = \begin{cases} P_t^{*d}(i) & \text{if } \; P_t(i) \text{ is chosen optimally} \\ \Pi_{t+k-1,t-1}^d P_{t-1}(i) & \text{if otherwise} \end{cases} \tag{C.1.7}$$

The gross inflation from $t-1$ to $t+k-1$ could be defined as:

$$\Pi_{t+k-1,t-1}^d = \frac{P_{t+k-1}^d}{P_{t-1}^d} \tag{C.1.8}$$

The firm i solves the profit maximization problem wrt. optimal price in period t by taking into account that the firm might be unable to reset price in the next k periods.

$$\underset{P_t^{*d}(i)}{\text{maximize}} \quad E_t \sum_{k=0}^{\infty} \theta_d^k Q_{t+k,t}\left(P_t^{*d}(i)\Pi_{t+k-1,t-1}^d Y_{t+k}(i) - MC_{t+k}\left(Y_{t+k}(i) + F_{t+k}^d\right)\right) \tag{C.1.9a}$$

$$\text{subject to} \quad Y_{t+k}(i) = \left(\frac{P_t^{*d}(i)\Pi_{t+k-1,t-1}^d}{P_{t+k}^d}\right)^{-\eta_{t+k}^d} Y_{t+k}^d \tag{C.1.9b}$$

Let's substitute the demand constraint into the objective function and, move $P_t(i)$ inside the right brackets so that we can re-write maximization problem as:

$$\begin{aligned} \max_{P_t^{*d}(i)} E_t \sum_{k=0}^{\infty} \theta_d^k Q_{t+k,t} \Bigg\{ & P_t(i)^{*d^{1-\eta_{t+k}^d}} \left(\frac{\Pi_{t+k-1|t-1}^d}{P_{t+k}^d}\right)^{1-\eta_{t+k}^d} P_{t+k}^d Y_{t+k} - \\ & - MC_{t+k} P_t(i)^{*d^{-\eta_{t+k}^d}} \left(\frac{\Pi_{t+k-1|t-1}^d}{P_{t+k}^d}\right)^{-\eta_{t+k}^d} Y_{t+k} + MC_{t+k}F_{t+k}^d \Bigg\} \end{aligned} \tag{C.1.10}$$

The first-order condition can be written as:

$$E_t \sum_{k=0}^{\infty} \theta_d^k Q_{t+k,t} \Bigg\{ (1-\eta_{t+k}^d) P_t(i)^{*d-\eta_{t+k}^d} \left(\frac{\Pi_{t+k-1|t-1}^d}{P_{t+k}^d} \right)^{1-\eta_{t+k}^d} P_{t+k}^d Y_{t+k} +$$

$$+ \eta_{t+k}^d MC_{t+k} P_t(i)^{*d-\eta_{t+k}^d-1} \left(\frac{\Pi_{t+k-1|t-1}^d}{P_{t+k}^d} \right)^{-\eta_{t+k}^d} Y_{t+k} \Bigg\} = 0 \tag{C.1.11}$$

In order to write the equation in a recursive way, we have to simplify it before, by treating η_{t+k}^d as a parameter. After that, the optimal price inflation could be written as:

$$\frac{P_t^{*d}(i)}{P_t^d} = \frac{\eta^d}{\eta^d - 1} \frac{E_t \sum_{k=0}^{\infty} \theta_d^k Q_{t+k,t} \left(\frac{\Pi_t^d}{\Pi_{t+k}^d} \right)^{1-\eta^d} Y_{t+k}^d MC_{t+k}}{E_t \sum_{k=0}^{\infty} \theta_d^k Q_{t+k,t} \left(\frac{\Pi_t^d}{\Pi_{t+k}^d} \right)^{1-\eta^d} P_{t+k}^d Y_{t+k}^d} \tag{C.1.12}$$

Now it could be rewritten (C.1.12) in recursive form. Since nothing on the right-hand side depends on i, firms who are allowed to update are setting same prices P_t^{*d}. Lets divide both sides of eq (C.1.12) by P_t^d:

$$\frac{P_t^{*d}}{P_t^d} = \frac{\eta^d}{\eta^d - 1} \frac{E_t \sum_{k=0}^{\infty} \theta_d^k Q_{t+k,t} \left(\frac{P_{t+k-1}^d P_t^d}{P_{t+k}^d P_{t-1}^d} \right)^{-\eta_t^d} Y_{t+k}^d MC_{t+k}}{E_t \sum_{k=0}^{\infty} \theta_d^k Q_{t+k,t} \left(\frac{P_{t+k-1}^d P_t^d}{P_{t+k}^d P_{t-1}^d} \right)^{1-\eta_t^d} P_{t+k}^d Y_{t+k}^d} \tag{C.1.13}$$

We note that:

$$Q_{t+k,t} = \beta^k \frac{\psi_{t+k}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t(C_{t+k}^{uc} - hC_{t+k-1}^{uc})\Pi_{t+k|t}^C} \tag{C.1.14}$$

Then the optimality condition will be given by:

$$\frac{P_t^{*d}}{P_t^d} = \frac{\eta^d}{\eta^d - 1} \frac{E_t \sum_{k=0}^{\infty} \theta_d^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})P_{t+k}^c} \left(\frac{\Pi_{t+k}^d}{\Pi_t^d} \right)^{\eta_t^d} Y_{t+k}^d MC_{t+k}}{E_t \sum_{k=0}^{\infty} \theta_d^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})P_{t+k}^c} \left(\frac{\Pi_{t+k}^d}{\Pi_t^d} \right)^{\eta_t^d - 1} P_{t+k}^d Y_{t+k}^d} \tag{C.1.15}$$

Let's define:

$$D_{1,t} \equiv E_t \sum_{k=0}^{\infty} \theta_d^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})P_{t+k}^c} \left(\frac{\Pi_{t+k}^d}{\Pi_t^d} \right)^{\eta_t^d} MC_{t+k}^{r^d} P_{t+k}^d Y_{t+k}^d =$$

$$= \beta^0 \theta_d^0 \frac{\psi_t}{(C_t^{uc} - hC_{t-1}^{uc})P_t^c} MC_t^{r^d} P_t^d Y_t^d +$$

$$+\beta\theta_d\left(\frac{\Pi^d_{t+1}}{\Pi^d_t}\right)^{\eta^d_t}E_t\sum_{k=0}^{\infty}\theta_d^k\frac{\psi_{t+k+1}}{(C^{uc}_{t+k+1}-hC^{uc}_{t+k})P^c_{t+k+1}}\left(\frac{\Pi^d_{t+k+1}}{\Pi^d_{t+1}}\right)^{\eta^d_t}MC^{r^d}_{t+k+1}P^d_{t+k+1}Y^d_{t+k+1}$$
(C.1.16)

$$D_{1,t+1}=E_t\sum_{k=0}^{\infty}\theta_d^k\frac{\psi_{t+k+1}}{(C^{uc}_{t+k+1}-hC^{uc}_{t+k})P^c_{t+k+1}}\left(\frac{\Pi^d_{t+k+1}}{\Pi^d_{t+1}}\right)^{\eta^d_t}MC^{r^d}_{t+k+1}P^d_{t+k+1}Y^d_{t+k+1} \tag{C.1.17}$$

Therefore, we can write:

$$D_{1,t}=\frac{\psi_t}{(C^{uc}_t-hC^{uc}_{t-1})P^c_t}MC^{r^d}_tP^d_tY^d_t+\beta\theta_dE_t\left(\frac{\Pi^d_{t+1}}{\Pi^d_t}\right)^{\eta^d_t}D_{1,t+1} \tag{C.1.18}$$

Similarly let's define $D_{2,t}$ (the denominator of eq (C.1.12)).

$$D_{2,t}\equiv E_t\sum_{k=0}^{\infty}\theta_d^k\beta^k\frac{\psi_{t+k}}{(C^{uc}_{t+k}-hC^{uc}_{t+k-1})P^c_{t+k}}\left(\frac{\Pi^d_{t+k}}{\Pi^d_t}\right)^{\eta^d_t-1}P^d_{t+k}Y^d_{t+k} \tag{C.1.19}$$

Then after applying the same steps of transformation as in the case of D_{1t} we get:

$$D_{2,t}=\frac{\psi_t}{(C^{uc}_t-hC^{uc}_{t-1})P^c_t}P^d_tY^d_t+\beta\theta_dE_t\left(\frac{\Pi^d_{t+1}}{\Pi^d_t}\right)^{\eta^d_t-1}D_{2,t+1} \tag{C.1.20}$$

Finally, we can write the relative optimal price of domestic differentiated inputs as :

$$\frac{P^{*d}_t}{P^d_t}=\frac{\eta^d}{\eta^d-1}\frac{D_{1,t}}{D_{2,t}} \tag{C.1.21}$$

After deriving the optimal price equation in the recursive form, we could return η^d_t as a variable in the above equation.

C.2 Inflation Dynamics in Domestic Differentiated Input Sector

The aggregate price index in the domestic intermediate goods sector is given by:

$$P^d_t=\left[\int_0^1(P_t(i))^{1-\eta^d_t}di\right]^{\frac{1}{1-\eta^d_t}} \tag{C.2.1}$$

$(1-\theta_d)$ share of the firms updates its price, while the rest part (θ_d) uses the price indexation rule defined in eq (2.4.1.32). Hence, we can re-write the previous equation (C.2.1) as:

$$P_t^d = \left[\int_0^{\theta_d} \left[(\Pi_{t-1,t-2}^d)P_{t-1}(i)\right]^{1-\eta_t^d} di + \int_{\theta_d}^1 \left(P_t^{*d}\right)^{1-\eta_t^d}\right]^{\frac{1}{1-\eta_t^d}} \tag{C.2.2}$$

Where, P_t^{*d} is the optimal price of firms who get to update their price in period t. By assuming that optimizer firms are random selection out of a continuum of firms in period t, we can re-write eq (C.2.2) as:

$$P_t^d = \left[\theta_d \left[\Pi_{t-1,t-2}^d P_{t-1}^d\right]^{1-\eta_t^d} + (1-\theta_d)\left(P_t^{*d}\right)^{1-\eta_t^d}\right]^{\frac{1}{1-\eta_t^d}} \tag{C.2.3}$$

Divide both sides by P_t^d and we'll get:

$$1 = \left[\theta_d \left[\frac{P_{t-1}^d}{P_t^d}\Pi_{t-1,t-2}^d\right]^{1-\eta_t^d} + (1-\theta_d)\left(\frac{P_t^{*d}}{P_t^d}\right)^{1-\eta_t^d}\right]^{\frac{1}{1-\eta_t^d}} \tag{C.2.4}$$

Since, $\frac{P_{t-1}^d}{P_t^d} = \frac{1}{\Pi_t^d}$ we can simply eliminate $\frac{1}{1-\eta_t^d}$ power on the right hand side:

$$1 = \theta_d \left[\frac{1}{\Pi_t^d}\Pi_{t-1,t-2}^d\right]^{1-\eta_t^d} + (1-\theta_d)\left(\frac{P_t^{*d}}{P_t^d}\right)^{1-\eta_t^d} \tag{C.2.5}$$

Multiplying both sides by $\left[\Pi_t^d\right]^{1-\eta_t^d}$ we'll get:

$$\left[\Pi_t^d\right]^{1-\eta_t^d} = \theta_d \left[\Pi_{t-1,t-2}^d\right]^{1-\eta_t^d} + (1-\theta_d)\left[\Pi_t^d\right]^{1-\eta_t^d}\left(\frac{P_t^{*d}}{P_t^d}\right)^{1-\eta_t^d} \tag{C.2.6}$$

Now, we can do the first order approximation of eq (C.2.6) around the balanced growth path equilibrium where $\frac{P_t^{*d}}{P_t^d} = 1$.

$$\begin{aligned}(\Pi^d)^{1-\eta_t^d} + \left(1-\eta_t^d\right)(\Pi^d)^{-\eta_t^d}\left(\Pi_t^d - \Pi^d\right) = {} & \theta_d(\Pi^d)^{1-\eta_t^d} + \theta_d\left(1-\eta_t^d\right)(\Pi^d)^{-\eta_t^d}\left(\Pi_{t-1,t-2}^d - \Pi^d\right) \\ & +(1-\theta_d)(\Pi^d)^{1-\eta_t^d} + (1-\theta_d)\left(1-\eta_t^d\right)(\Pi^d)^{-\eta_t^d}\left(\Pi_t^d - \Pi^d\right) \\ & +(1-\theta_d)\left(1-\eta_t^d\right)(\Pi^d)^{1-\eta_t^d}\left(\frac{P_t^{*d}}{P_t^d} - 1\right)\end{aligned} \tag{C.2.7}$$

From eq (C.2.6) we know that $\left(\Pi^d\right)^{1-\eta_t^d} = \theta_d \left(\Pi^d\right)^{1-\eta_t^d} + (1-\theta_d) \left(\Pi^d\right)^{1-\eta_t^d}$. Also, divide both sides of eq (C.2.7) by $(1-\eta_t^d)$. As a result we'll get:

$$\begin{aligned}(\Pi^d)^{-\eta_t^d} \left(\Pi_t^d - \Pi^d\right) = \theta_d (\Pi^d)^{-\eta_t^d} \left(\Pi_{t-1,t-2}^d - \Pi^d\right) + (1-\theta_d)(\Pi^d)^{-\eta_t^d} \left(\Pi_t^d - \Pi^d\right) \\ +(1-\theta_d)(\Pi^d)^{1-\eta_t^d} \left(\frac{P_t^{*d}}{P_t^d} - 1\right)\end{aligned} \tag{C.2.8}$$

Lets combine LHS of eq (C.2.8) and middle part of the RHS in eq (C.2.8):

$$\theta_d (\Pi^d)^{-\eta_t^d} \left(\Pi_t^d - \Pi^d\right) = \theta_d (\Pi^d)^{-\eta_t^d} \left(\Pi_{t-1,t-2}^d - \Pi^d\right) + (1-\theta_d)(\Pi^d)^{1-\eta_t^d} \left(\frac{P_t^{*d}}{P_t^d} - 1\right) \tag{C.2.9}$$

Divide both sides of eq (C.2.9) by $\theta_d (\Pi^d)^{-\eta_t^d}$ and we'll get:

$$\left(\Pi_t^d - \Pi^d\right) = \left(\Pi_{t-1,t-2}^d - \Pi^d\right) + \frac{1-\theta_d}{\theta_d} \Pi^d \left(\frac{P_t^{*d}}{P_t^d} - 1\right) \tag{C.2.10}$$

And finally, note that $\Pi_{t-1,t-2}^d = \Pi_{t-1}^d$

$$\Pi_t^d = \Pi_{t-1}^d + \frac{1-\theta_d}{\theta_d} \Pi^d \left(\frac{P_t^{*d}}{P_t^d} - 1\right) \tag{C.2.11}$$

The steps to linearize the optimal price equation in the domestic intermediate goods sector are similar to an import sector which is shown in Appendix E.4. However, note that the real marginal cost in the case of domestic intermediate input producers (mc_t^{rd}) is given by $\frac{MC_t}{P_t^d}$, and the gap of the real marginal cost in the domestic intermediate goods sector could be given as:

$$\widehat{mc_t^{rd}} = ln(mc_t^{rd}) - ln(mc^{rd}) = ln(MC_t) - ln(P_t^d) - ln(mc^{rd}) \tag{C.2.12}$$

Appendix D Final Goods Sector Derivations

D.1 Consumption Retailers

The consumption retailer solves the profit maximization problem:

$$\underset{C_t^d, C_t^m}{\text{maximize}} \quad P_t^c C_t - \left(P_t^d C_t^d + P_t^{mG} C_t^m\right) \tag{D.1.1a}$$

$$\text{subject to} \quad C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} C_t^{d\,\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(\frac{C_t^m}{a_t^x}\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}} \tag{D.1.1b}$$

Substitute D.1.1b in D.1.1a and the optimization problem looks:

$$\underset{C_t^d, C_t^m}{\text{maximize}} \quad P_t^c \left[(1-\omega_c)^{\frac{1}{\eta_c}} C_t^{d\,\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(\frac{C_t^m}{a_t^x}\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}} - \left(P_t^d C_t^d + P_t^{mG} C_t^m\right) \tag{D.1.2}$$

Taking the first-order conditions yields:

$\left[\partial C_t^d\right]$:

$$P_t^c \frac{\eta_c}{\eta_c - 1} \left[(1-\omega_c)^{\frac{1}{\eta_c}} C_t^{d\,\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(\frac{C_t^m}{a_t^x}\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}-1} \frac{\eta_c - 1}{\eta_c} (1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d\right)^{\frac{\eta_c-1}{\eta_c}-1} - P_t^d = 0 \tag{D.1.3}$$

$\left[\partial C_t^m\right]$:

$$P_t^c \frac{\eta_c}{\eta_c - 1} \left[(1-\omega_c)^{\frac{1}{\eta_c}} C_t^{d\,\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(\frac{C_t^m}{a_t^x}\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}-1} \frac{\eta_c - 1}{\eta_c} \omega_c^{\frac{1}{\eta_c}} \left(\frac{C_t^m}{a_t^x}\right)^{\frac{\eta_c-1}{\eta_c}-1} \frac{1}{a_t^x} - P_t^{mG} = 0 \tag{D.1.4}$$

From (D.1.3) we can obtain:

$$C_t^{\frac{1}{\eta_c}} (1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d\right)^{-\frac{1}{\eta_c}} = \frac{P_t^d}{P_t^c} \tag{D.1.5}$$

And from (D.1.5) we will get the demand function of C_t^d, which reads:

$$C_t^d = (1-\omega_c) \left(\frac{P_t^d}{P_t^c}\right)^{-\eta_c} C_t \tag{D.1.6}$$

Similarly from (D.1.4) we can obtain demand function for imported consumption good C_t^m:

$$\frac{C_t^m}{a_t^x} = \omega_c \left(\frac{P_t^{mG} a_t^x}{P_t^c} \right)^{-\eta_c} C_t \tag{D.1.7}$$

The next step is to derive the aggregate price index of the consumption good C_t. Plug the optimal demand functions in the (2.4.2.1) and we'll have:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left((1-\omega_c) \left(\frac{P_t^d}{P_t^c} \right)^{-\eta_c} C_t \right)^{\frac{\eta_c - 1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(\omega_c \left(\frac{P_t^{mG} a_t^x}{P_t^c} \right)^{-\eta_c} C_t \right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}} \tag{D.1.8}$$

$$1 = (1-\omega_c)^{\frac{1}{\eta_c}} \left((1-\omega_c) \left(\frac{P_t^d}{P_t^c} \right)^{-\eta_c} \right)^{\frac{\eta_c - 1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(\omega_c \left(\frac{P_t^{mG} a_t^x}{P_t^c} \right)^{-\eta_c} \right)^{\frac{\eta_c - 1}{\eta_c}} \tag{D.1.9}$$

After simplifying (D.1.9), the aggregate price of consumption goods is given as:

$$P_t^c = \left[(1-\omega_c) \left(P_t^d \right)^{1-\eta_c} + \omega_c \left(P_t^{mG} a_t^x \right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \tag{D.1.10}$$

D.2 Final Investments Goods Production

The investment goods producer solves the following profit maximization problem:

$$\underset{I_t^d,\, I_t^m}{\text{maximize}} \quad P_t^i I_t - \left(P_t^d I_t^d + P_t^{mG} I_t^m \right) \tag{D.2.1a}$$

$$\text{subject to} \quad I_t = \left[(1-\omega_i)^{\frac{1}{\eta_i}} I_t^{d\,\frac{\eta_i - 1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(\frac{I_t^m}{a_t^x} \right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}} \tag{D.2.1b}$$

Substitute D.2.1b in D.2.1a and the optimization problem becomes:

$$\underset{I_t^d,\, I_t^m}{\text{maximize}} \quad P_t^i \left[(1-\omega_i)^{\frac{1}{\eta_i}} I_t^{d\,\frac{\eta_i - 1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(\frac{I_t^m}{a_t^x} \right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}} - \left(P_t^d I_t^d + P_t^{mG} I_t^m \right) \tag{D.2.2}$$

After doing similar steps as in the previous section we can obtain the following demand functions of the domestic and imported investment goods:

$$I_t^d = (1-\omega_i)\left(\frac{P_t^d}{P_t^i}\right)^{-\eta_i} I_t \tag{D.2.3}$$

$$\frac{I_t^m}{a_t^x} = \omega_i\left(\frac{P_t^{mG}a_t^x}{P_t^i}\right)^{-\eta_i} I_t \tag{D.2.4}$$

The aggregate price index of the investment good is:

$$P_t^i = \left[(1-\omega_i)\left(P_t^d\right)^{1-\eta_i} + \omega_i\left(P_t^{mG}a_t^x\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} \tag{D.2.5}$$

Appendix E Import Sector

E.1 Aggregate Price Index in Import Sector

As said, θ_m part of firms update their price based on the price index in the t period, while the $(1-\theta_m)$ set the optimal price in that period. We assume that the information set available for price optimizers are same in the t period, which implies that optimal prices set by them in the t period are the same as well. Therefore, we can write the aggregate price index in the following way:

$$P_t^{mf} = \left[\int_0^1 (P_t(i)^{mf})^{1-\varepsilon_t^m} di\right]^{\frac{1}{1-\varepsilon_t^m}} = \left[\int_0^{\theta_m}\left[P_{t-1}^{mf}(i)\Pi_{t-1}^{mf}\right]^{1-\varepsilon_t^m} di + \int_{\theta_m}^1 (P_t^{*mf})^{1-\varepsilon_t^m} di\right]^{\frac{1}{1-\varepsilon_t^m}} \tag{E.1.1}$$

Here, we use the assumption by Calvo, that if the subset of firms who set prices optimally are a random selection from the entire continuum of firms, then the aggregate price of some subset of firms will be the same as the aggregate price of the entire set of firms. Then, we can write:

$$P_t^{mf} = \left[\theta_m\left[P_{t-1}^{mf}(\Pi_{t-1}^{mf})\right]^{1-\varepsilon_t^m} + (1-\theta_m)(P_t^{*mf})^{1-\varepsilon_t^m}\right]^{\frac{1}{1-\varepsilon_t^m}} \tag{E.1.2}$$

If we divide both sides of the equation (E.1.2) by P_t^{mf} , then we get:

$$1 = \left[\theta_m\left[\frac{P_{t-1}^{mf}}{P_t^{mf}}\Pi_{t-1}^{mf}\right]^{1-\varepsilon_t^m} + (1-\theta_m)\left[\frac{P_t^{*mf}}{P_t^{mf}}\right]^{1-\varepsilon_t^m}\right]^{\frac{1}{1-\varepsilon_t^m}} \tag{E.1.3}$$

As $\frac{P_{t-1}^{mf}}{P_t^{mf}} = \frac{1}{\Pi_t^{mf}}$, where Π_t^{mf} is the gross inflation of imported goods, we can write:

$$1 = \left[\theta_m \left[\frac{1}{\Pi_t^{mf}} \Pi_{t-1}^{mf}\right]^{1-\varepsilon_t^m} + (1-\theta_m)\left[\frac{P_t^{*mf}}{P_t^{mf}}\right]^{1-\varepsilon_t^m}\right]^{\frac{1}{1-\varepsilon_t^m}} \tag{E.1.4}$$

By multiplying both sides of the previous equation by Π_t^{mf} and then take in power $(1-\varepsilon_t^m)$ we get:

$$(\Pi_t^{mf})^{1-\varepsilon_t^m} = \theta_m \left[\Pi_{t-1}^{mf}\right]^{1-\varepsilon_t^m} + (1-\theta_m)(\Pi_t^{mf})^{1-\varepsilon_t^m} \left[\frac{P_t^{*mf}}{P_t^{mf}}\right]^{1-\varepsilon_t^m} \tag{E.1.5}$$

From the first order approximation of the equation (E.1.5) around the balanced growth path equilibrium where $\frac{P_t^{*mf}}{P_t^{mf}} = 1$, we get:

$$\begin{aligned}
{\Pi^{mf}}^{1-\varepsilon^m} &+ (1-\varepsilon^m)(\Pi^{mf})^{-\varepsilon^m}\left(\Pi_t^{mf} - \Pi^{mf}\right) = \theta_m(\Pi^{mf})^{1-\varepsilon^m} + \\
&+ \theta_m(1-\varepsilon^m)(\Pi^{mf})^{-\varepsilon^m}\left(\Pi_{t-1}^{mf} - \Pi^{mf}\right) + (1-\theta_m)(\Pi^{mf})^{1-\varepsilon^m} \\
&+ (1-\theta_m)(1-\varepsilon^m){\Pi^{mf}}^{1-\varepsilon^m}\left(\frac{P_t^{*mf}}{P_t^{mf}} - 1\right) \\
&+ (1-\theta_m)(1-\varepsilon^m)(\Pi^{mf})^{-\varepsilon^m}\left(\Pi_t^{mf} - \Pi^{mf}\right)
\end{aligned} \tag{E.1.6}$$

As ${\Pi^{mf}}^{1-\varepsilon^m} = \theta_m {\Pi^{mf}}^{1-\varepsilon^m} + (1-\theta_m){\Pi^{mf}}^{1-\varepsilon^m}$, let's divide both sides of the equation by $(1-\varepsilon^m)$, then we get:

$$\begin{aligned}
&(\Pi^{mf})^{-\varepsilon^m}\left(\Pi_t^{mf} - \Pi^{mf}\right) = \\
&= \theta_m(\Pi^{mf})^{-\varepsilon^m}\left(\Pi_{t-1}^{mf} - \Pi^{mf}\right) + (1-\theta_m)(\Pi^{mf})^{1-\varepsilon^m}\left(\frac{P_t^{*mf}}{P_t^{mf}} - 1\right) + \\
&+ (1-\theta_m)(\Pi^{mf})^{-\varepsilon_t^m}\left(\Pi_t^{mf} - \Pi^{mf}\right)
\end{aligned} \tag{E.1.7}$$

After rearranging the same terms in the equation, we get:

$$\theta_m(\Pi^{mf})^{-\varepsilon^m}\left(\Pi_t^{mf} - \Pi^{mf}\right) = \theta_m(\Pi^{mf})^{-\varepsilon^m}\left(\Pi_{t-1}^{mf} - \Pi^{mf}\right) + (1-\theta_m)(\Pi^{mf})^{1-\varepsilon^m}\left(\frac{P_t^{*mf}}{P_t^{mf}} - 1\right) \tag{E.1.8}$$

After dividing both sides of the equation (E.1.8) by $\theta_m(\Pi^{mf})^{-\varepsilon^m}$:

$$\Big(\Pi_t^{mf} - \Pi^{mf}\Big) = \Big(\Pi_{t-1}^{mf} - \Pi^{mf}\Big) + \frac{1-\theta_m}{\theta_m}\Pi^{mf}\Big(\frac{P_t^{*mf}}{P_t^{mf}} - 1\Big) \tag{E.1.9}$$

Finally,

$$\Pi_t^{mf} = \Pi_{t-1}^{mf} + \frac{1-\theta_m}{\theta_m}\Pi^{mf}\Big(\frac{P_t^{*mf}}{P_t^{mf}} - 1\Big) \tag{E.1.10}$$

E.2 Profit Maximization Problem of Imported Input Retailer

After putting the constraints into the profit function 2.4.5.6, the maximization problem of differentiated imported goods producer can be written as:

$$\begin{aligned}&\underset{P_t(i)^{*mf}}{\text{maximize}} \quad E_t\sum_{k=0}^{\infty}\Bigg\{\theta_m^k Q_{t,t+k}^f\Bigg[P_t(i)^{*mf}\Pi_{t+k-1|t-1}^{mf}\times\\ &\times\Bigg(\frac{P_t(i)^{*mf}\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}}\Bigg)^{-\epsilon_{t+k}^m} M_{t+k} - MC_{t+k}^m\Bigg(\frac{P_t(i)^{*mf}\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}}\Bigg)^{-\epsilon_{t+k}^m} M_{t+k}\Bigg]\Bigg\}\end{aligned} \tag{E.2.1}$$

We take $P_t^{*mf}(i)$ out of brackets, then:

$$\begin{aligned}&\underset{P_t(i)^{*mf}}{\text{maximize}} \quad E_t\sum_{k=0}^{\infty}\Bigg\{\theta_m^k Q_{t,t+k}^f P_{t+k}^{mf}\Bigg[P_t(i)^{*mf^{1-\varepsilon_{t+k}^m}}\times\\ &\times\Bigg(\frac{\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}}\Bigg)^{1-\varepsilon_{t+k}^m} M_{t+k} - \frac{MC_{t+k}^m}{P_{t+k}^{mf}}P_t(i)^{*mf^{-\varepsilon_{t+k}^m}}\Bigg(\frac{\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}}\Bigg)^{-\varepsilon_{t+k}^m} M_{t+k}\Bigg]\Bigg\}\end{aligned} \tag{E.2.2}$$

Previously, we define the $\frac{MC_{t+k}^m}{P_{t+k}^{mf}}$ as real marginal cost in import sector $MC_{t+k}^{m^r}$. We can apply a few more steps to express the real marginal cost as the function of the real effective exchange rate.

$$MC_t^{rm} = \frac{e_t^{D/R}P_t^R}{P_t^{mf}} = \frac{e_t^{D/R}P_t^R}{\frac{P_t^{mG}}{e_t^{Gel/D}}} = \frac{e_t^{Gel/D}P_t^R}{P_t^{mG}} = \frac{e_t^{Gel/R}P_t^R}{P_t^c}\frac{P_t^c}{P_t^{mG}} = REER_t\frac{P_t^c}{P_t^{mG}} \tag{E.2.3}$$

Where $REER_t$ is the real effective exchange rate of lari at the period t.

The FOC of the maximization problem can be written as:

$$\left[\partial P_t(i)^{*mf}\right]:$$

$$E_t\sum_{k=0}^{\infty}\left\{\theta_m^k Q_{t,t+k}^f P_{t+k}^{mf}\left[(1-\varepsilon_{t+k}^m)P_t(i)^{*mf-\varepsilon_{t+k}^m}\left(\frac{\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}}\right)^{1-\varepsilon_{t+k}^m}M_{t+k}+\right.\right.$$

$$\left.\left.+\,\varepsilon_{t+k}^m MC_{t+k}^{m^r}P_t(i)^{*mf-\varepsilon_{t+k}^m-1}\left(\frac{\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}}\right)^{-\varepsilon_{t+k}^m}M_{t+k}\right]\right\}=0 \tag{E.2.4}$$

E.3 Recursive Form of Optimal Price

The equation (E.2.4) can be rewritten in recursive form. However, to make it possible we need to simplify the problem beforehand. Here, we assume that the elasticity of substitution is constant and after deriving the equation in the recursive way, we reintroduce the elasticity of substitution coefficient as the variable again. Also, the profit optimization problem is symmetric for all i individuals, and the prices set by optimizer firms are the same across optimizers. Taking all of the conditions into account, the optimality condition could be written as (after dividing both sides by P_t^{mf}):

$$\frac{P_t^{*mf}}{P_t^{mf}}=\frac{\varepsilon^m}{\varepsilon^m-1}\frac{E_t\sum_{k=0}^{\infty}\left\{\theta_m^k Q_{t,t+k}^f P_{t+k}^{mf}\left(\frac{P_{t+k-1}^{mf}P_t^{mf}}{P_{t+k}^{mf}P_{t-1}^{mf}}\right)^{-\varepsilon^m}M_{t+k}MC_{t+k}^{m^r}\right\}}{E_t\sum_{k=0}^{\infty}E_t\left\{\theta_m^k Q_{t,t+k}^f P_{t+k}^{mf}\left(\frac{P_{t+k-1}^{mf}P_t^{mf}}{P_{t+k}^{mf}P_{t-1}^{mf}}\right)^{1-\varepsilon^m}M_{t+k}\right\}} \tag{E.3.1}$$

We define gross inflations as: $\Pi_{t+k}^{mf}\equiv\frac{P_{t+k}^{mf}}{P_{t+k-1}^{mf}}$ and $\Pi_t^{mf}\equiv\frac{P_t^{mf}}{P_{t-1}^{mf}}$. Then the optimality condition is given by:

$$\frac{P_t^{*mf}}{P_t^{mf}}=\frac{\varepsilon^m}{\varepsilon^m-1}\frac{E_t\sum_{k=0}^{\infty}\left\{\theta_m^k Q_{t,t+k}^f P_{t+k}^{mf}\left(\frac{\Pi_{t+k}^{mf}}{\Pi_t^{mf}}\right)^{\varepsilon^m}M_{t+k}MC_{t+k}^{m^r}\right\}}{E_t\sum_{k=0}^{\infty}\left\{\theta_m^k Q_{t,t+k}^f P_{t+k}^{mf}\left(\frac{\Pi_{t+k}^{mf}}{\Pi_t^{mf}}\right)^{\varepsilon^m-1}M_{t+k}\right\}} \tag{E.3.2}$$

Let's denote:

$$A_{1t}\equiv E_t\sum_{k=0}^{\infty}\left\{\theta_m^k Q_{t,t+k}^f P_{t+k}^{mf}\left(\frac{\Pi_{t+k}^{mf}}{\Pi_t^{mf}}\right)^{\varepsilon^m}M_{t+k}MC_{t+k}^{m^r}\right\} \tag{E.3.3}$$

And

$$A_{2t} \equiv E_t \sum_{k=0}^{\infty} \left\{ \theta_m^k Q_{t,t+k}^f P_{t+k}^{mf} \left(\frac{\Pi_{t+k}^{mf}}{\Pi_t^{mf}} \right)^{\varepsilon^m - 1} M_{t+k} \right\} \tag{E.3.4}$$

Then by taking definitions (E.3.3) and (E.3.4) into account, the equation (E.3.2) can be written as:

$$\frac{P_t^{*mf}}{P_t^{mf}} = \frac{\varepsilon^m}{\varepsilon^m - 1} \frac{A_{1t}}{A_{2t}} \tag{E.3.5}$$

We can write the A_{1t} and A_{2t} recursively, such as:

$$A_{1t} = \theta_m^0 Q_{t,t}^f P_t^{mf} M_t MC_t^{m^r} + E_t \sum_{k=1}^{\infty} \left\{ \theta_m^k Q_{t,t+k}^f P_{t+k}^{mf} \left(\frac{\Pi_{t+k}^{mf}}{\Pi_t^{mf}} \right)^{\varepsilon^m} M_{t+k} MC_{t+k}^{m^r} \right\} =$$

$$= P_t^{mf} M_t MC_t^{m^r} +$$

$$+ \theta_m E_t Q_{t,t+1}^f \left(\frac{\Pi_{t+1}^{mf}}{\Pi_t^{mf}} \right)^{\varepsilon^m} \sum_{k=0}^{\infty} \left\{ \theta_m^k Q_{t+1,t+k+1}^f P_{t+k+1}^{mf} \left(\frac{\Pi_{t+k+1}^{mf}}{\Pi_{t+1}^{mf}} \right)^{\varepsilon^m} M_{t+k+1} MC_{t+k+1}^{r^m} \right\} =$$

$$= P_t^{mf} M_t MC_t^{m^r} + \theta_m E_t Q_{t,t+1}^f \left(\frac{\Pi_{t+1}^{mf}}{\Pi_t^{mf}} \right)^{\varepsilon^m} A_{1t+1} \tag{E.3.6}$$

Applying the same modification for the equation (E.3.4), we can rewrite equation (E.3.2) in the recursive form:

$$\frac{P_t^{*mf}}{P_t^{mf}} = \frac{\varepsilon^m}{\varepsilon^m - 1} \frac{A_{1t}}{A_{2t}}$$

$$where$$

$$A_{1t} = P_t^{mf} M_t MC_t^{m^r} + \theta_m E_t Q_{t,t+1}^f \left(\frac{\Pi_{t+1}^{mf}}{\Pi_t^{mf}} \right)^{\varepsilon^m} A_{1t+1}$$

$$and$$

$$A_{2t} = P_t^{mf} M_t + \theta_m E_t Q_{t,t+1}^f \left(\frac{\Pi_{t+1}^{mf}}{\Pi_t^{mf}} \right)^{\varepsilon^m - 1} A_{2t+1} \tag{E.3.7}$$

E.4 Linear Transformation of Optimal Price Setting Problem in Import Sector

Getting the linear version of optimal price equation helps us to illustrate the drivers and dynamics of inflation process.

Note, that the optimality condition is given by the equation:

$$E_t \sum_{k=0}^{\infty} \left\{ \theta_m^k Q_{t,t+k}^f P_{t+k}^{mf} \left[(1-\varepsilon_{t+k}^m) P_t(i)^{*mf-\varepsilon_{t+k}^m} \left(\frac{\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}} \right)^{1-\varepsilon_{t+k}^m} M_{t+k} \right.\right.$$
$$\left.\left. + \varepsilon_{t+k}^m MC_{t+k}^{m^r} P_t(i)^{*mf-\varepsilon_{t+k}^m-1} \left(\frac{\Pi_{t+k-1|t-1}^{mf}}{P_{t+k}^{mf}} \right)^{-\varepsilon_{t+k}^m} M_{t+k} \right] \right\} = 0 \tag{E.4.1}$$

After dividing both sides by P_t^{mf}, it takes the form:

$$E_t \sum_{k=0}^{\infty} \left\{ \theta_m^k Q_{t,t+k}^f P_{t+k}^{mf} \left[(1-\varepsilon_{t+k}^m) \left(\frac{P_t(i)^{*mf}}{P_t^{mf}} \right)^{-\varepsilon_{t+k}^m} \left(\frac{\Pi_t^{mf}}{\Pi_{t+k}^{mf}} \right)^{1-\varepsilon_{t+k}^m} M_{t+k} \right.\right.$$
$$\left.\left. + \varepsilon_{t+k}^m MC_{t+k}^{m^r} \left(\frac{P_t(i)^{*mf}}{P_t^{mf}} \right)^{-\varepsilon_{t+k}^m-1} \left(\frac{\Pi_t^{mf}}{\Pi_{t+k}^{mf}} \right)^{-\varepsilon_{t+k}^m} M_{t+k} \right] \right\} = 0 \tag{E.4.2}$$

To save space we are introducing the following definitions:

$$LHS_{t+k} \equiv Q_{t,t+k}^f P_{t+k}^{mf} (1-\varepsilon_{t+k}^m) \left(\frac{\Pi_t^{mf}}{\Pi_{t+k}^{mf}} \right)^{1-\varepsilon_{t+k}^m} M_{t+k} \left(\frac{P_t(i)^{*mf}}{P_t^{mf}} \right)^{-\varepsilon_{t+k}^m} \tag{E.4.3}$$

And,

$$RHS_{t+k} \equiv Q_{t,t+k}^f P_{t+k}^{mf} \varepsilon_{t+k}^m \left(\frac{\Pi_t^{mf}}{\Pi_{t+k}^{mf}} \right)^{-\varepsilon_{t+k}^m} M_{t+k} MC_{t+k}^{m^r} \left(\frac{P_t(i)^{*mf}}{P_t^{mf}} \right)^{-\varepsilon_{t+k}^m-1} \tag{E.4.4}$$

We can show that $Q_{t,t+k}^f P_{t+k}^{mf} M_{t+k}$ is stationary process around the deterministic trend $(\beta R^\rho)^k$. Let's introduce the following definition:

$$M_{t+k}^* \equiv Q_{t,t+k}^f P_{t+k}^{mf} M_{t+k} \tag{E.4.5}$$

Then using the stationary variables the last two equations could be re-written as:

$$LHS_{t+k} = (\beta R^\rho)^k (1-\varepsilon_{t+k}^m) \left(\frac{\Pi_t^{mf}}{\Pi_{t+k}^{mf}} \right)^{1-\varepsilon_{t+k}^m} \left(\frac{P_t(i)^{*mf}}{P_t^{mf}} \right)^{-\varepsilon_{t+k}^m} \widetilde{M_{t+k}^*} \tag{E.4.6}$$

And,

$$RHS_{t+k} = (\beta R^{\rho})^k \varepsilon^m_{t+k} \left(\frac{\Pi^{mf}_t}{\Pi^{mf}_{t+k}}\right)^{-\varepsilon^m_{t+k}} \left(\frac{P_t(i)^{*mf}}{P^{mf}_t}\right)^{-\varepsilon^m_{t+k}-1} MC^{m^r}_{t+k}\widetilde{M^*_{t+k}} \tag{E.4.7}$$

Let's define the risk-adjusted discount rate as $\beta^* \equiv \beta R^{\rho}$. Before writing the linear version of the equation (E.4.1), as an example, we note that the first order derivative w.r.t. $\widetilde{M^*_{t+k}}$ of the left-hand side is:

$$\begin{aligned}\frac{\partial E_t \sum_{k=0}^{\infty} \theta^k_m LHS_{t+k}}{\partial \widetilde{M^*_{t+k}}} =& E_t \sum_{k=0}^{\infty} (\beta^*\theta_m)^k (1-\varepsilon^m_{t+k}) \left(\frac{\Pi^{mf}_t}{\Pi^{mf}_{t+k}}\right)^{1-\varepsilon^m_{t+k}} \left(\frac{P_t(i)^{*mf}}{P^{mf}_t}\right)^{-\varepsilon^m_{t+k}} = \\ =& E_t \sum_{k=0}^{\infty} (\beta^*\theta^k_m)^k \frac{LHS_{t+k}}{\widetilde{M^*_{t+k}}} \end{aligned} \tag{E.4.8}$$

And in SS:

$$\sum_{k=0}^{\infty} (\beta^*\theta^k_m)^k \frac{LHS_{t+k}}{\widetilde{M^*_{t+k}}} = \sum_{k=0}^{\infty} (\beta^*\theta^k_m)^k \frac{LHS}{\widetilde{M^*}} \tag{E.4.9}$$

Taking into account the last two equations (again, the same logic could be applied for deriving SS values of derivatives of LHS^m_{t+k} w.r.t the rest of the variables), the linear version of the left-hand side of the equation (E.4.1) is:

$$\begin{aligned} E_t \sum_{k=0}^{\infty} (\beta^*\theta_m)^k LHS^m_{t+k} \approx& \sum_{k=0}^{\infty} (\beta^*\theta_m)^k LHS^m + E_t \sum_{k=0}^{\infty} (\beta^*\theta_m)^k LHS^m \frac{\widetilde{M^*_{t+k}} - \widetilde{M^*}}{\widetilde{M^*}} + \\ &+ (\varepsilon^m - 1) E_t \sum_{k=0}^{\infty} (\beta^*\theta_m)^k LHS^m \frac{\Pi^{mf}_{t+k} - \Pi^{mf}}{\Pi^{mf}} - (\varepsilon^m - 1) E_t \sum_{k=0}^{\infty} (\beta^*\theta_m)^k LHS^m \frac{\Pi^{mf}_t - \Pi^{mf}}{\Pi^{mf}} \\ &- \frac{1}{1-\varepsilon^m} E_t \sum_{k=0}^{\infty} (\beta^*\theta_m)^k LHS^m \Pi^{mf} (\varepsilon^m_t - \varepsilon^m) - \varepsilon^m E_t \sum_{k=0}^{\infty} (\beta^*\theta_m)^k LHS^m \left(\frac{P^{*xf}_t}{P^{xf}_t} - 1\right) \end{aligned} \tag{E.4.10}$$

And the linear version of the right-hand side of the equation (E.4.1) can be written as:

$$\begin{aligned} \sum_{k=0}^{\infty} E_t \theta^k_m RHS^m \approx& \sum_{k=0}^{\infty} E_t (\beta^*\theta_m)^k RHS^m + \sum_{k=0}^{\infty} E_t (\beta^*\theta_m)^k RHS^m \frac{\widetilde{M^*_{t+k}} - \widetilde{M^*}}{\widetilde{M^*}} + \\ &+ \varepsilon^m E_t \sum_{k=0}^{\infty} E_t (\beta^*\theta_m)^k RHS^m \frac{\Pi^{mf}_{t+k} - \Pi^{mf}}{\Pi^{mf}} - \varepsilon^m E_t \sum_{k=0}^{\infty} E_t (\beta^*\theta_m)^k RHS^m \frac{\Pi^{mf}_t - \Pi^{mf}}{\Pi^{mf}} \end{aligned}$$

$$+ E_t\varepsilon^m \sum_{k=0}^{\infty}(\beta^*\theta_m)^k RHS^m(\varepsilon_t^m - \varepsilon^m) + E_t \sum_{k=0}^{\infty}(\beta^*\theta_m)^k RHS^m \frac{MC_{t+k}^{m\ \ r} - MC^{mr}}{MC^{mr}} -$$

$$-(\varepsilon^m - 1)E_t \sum_{k=0}^{\infty}(\beta^*\theta_m)^k RHS^m \left(\frac{P_t^{*mf}}{P_t^{mf}}\right) \tag{E.4.11}$$

As long as LHS_{t+k}^m and RHS_{t+k}^m in SS do not depend on k and are equal to each other, we can further simplify and combine the above equations. Resulting from this, the terms with the same colors in (E.4.10) and (E.4.11) will cancel each other, and by combining the rest of the parts of those equations we get:

$$\sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \left(\frac{P_t^{*mf}}{P_t^{mf}} - 1\right) = \sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \frac{\Pi_{t+k}^{mf} - \Pi^{mf}}{\Pi^{mf}}$$
$$-\sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \frac{\Pi_t^{mf} - \Pi^{mf}}{\Pi^{mf}} - \sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k RHS^m \frac{1}{\varepsilon^m - 1}\frac{\varepsilon_{t+k}^m - \varepsilon^m}{\varepsilon^m}$$
$$+\sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \frac{MC_{t+k}^{m^r} - MC^{r^m}}{MC^{r^m}} \tag{E.4.12}$$

Let's denote $\frac{MC_{t+k}^{m^r} - MC^{r^m}}{MC^{r^m}} \equiv \widehat{MC_{t+k}^{m^r}}$, as the gap of real marginal cost; also we can write that $\Pi^{mf} = 1 + \pi^{mf}$. Taking into account those facts, the equation (E.4.12) can be rewritten as:

$$\frac{1}{1-\beta^*\theta_m}\left(\frac{P_t^{*mf}}{P_t^{mG}} - 1\right) = \sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \frac{\Pi_{t+k}^{mf} - \Pi^{mf}}{\Pi^{mf}} - \sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \frac{\Pi_t^{mf} - \Pi^{mf}}{\Pi^{mf}} -$$
$$-\sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \frac{1}{\varepsilon^m - 1}\widehat{\varepsilon_{t+k}^m} + \sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \widehat{MC_{t+k}^{m^r}} \tag{E.4.13}$$

Or

$$\left(\frac{P_t^{*mf}}{P_t^{mG}} - 1\right) = (1-\beta^*\theta_m)\Bigg(\sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \frac{\Pi_{t+k}^{mf} - \Pi^{mf}}{\Pi^{mf}} - \frac{1}{1-\beta^*\theta_m}\frac{\Pi_t^{mf} - \Pi^{mf}}{\Pi^{mf}} +$$
$$-\sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \frac{1}{\varepsilon^m - 1}\widehat{\varepsilon_{t+k}^m} + \sum_{k=0}^{\infty} E_t(\beta^*\theta_m)^k \widehat{MC_{t+k}^{m^r}}\Bigg) \tag{E.4.14}$$

$\Longrightarrow$

$$\left(\frac{P_t^{*mf}}{P_t^{mG}}-1\right)=-\frac{\Pi_t^{mf}-\Pi^{mf}}{\Pi^{mf}}+(1-\beta^*\theta_m)\Bigg(\sum_{k=0}^{\infty}E_t(\beta^*\theta_m)^k\frac{\Pi_{t+k}^{mf}-\Pi^{xf}}{\Pi^{mf}}$$
$$-\sum_{k=0}^{\infty}E_t(\beta^*\theta_m)^k\frac{1}{\varepsilon^m-1}\widehat{\varepsilon_{t+k}^m}+\sum_{k=0}^{\infty}E_t(\beta^*\theta_m)^k\widehat{MC_{t+k}^{m^r}}\Bigg) \quad \text{(E.4.15)}$$

The right-hand side of the equation (E.4.15) can be written as:

$$\left(\frac{P_t^{*mf}}{P_t^{mG}}-1\right)=-\frac{\Pi_t^{mf}-\Pi^{mf}}{\Pi^{mf}}+(1-\beta^*\theta_m)\Bigg((\beta^*\theta_m)^0\frac{\Pi_t^{mf}-\Pi^{mf}}{\Pi^{mf}}-(\beta^*\theta_m)^0\frac{1}{\varepsilon^m-1}\widehat{\varepsilon_t^m}+$$
$$+(\beta^*\theta_m)^0\widehat{MC_t^r}+\sum_{k=1}^{\infty}E_t(\beta^*\theta_m)^k\frac{\Pi_{t+k}^{mf}-\Pi^{mf}}{\Pi^{mf}}-\sum_{k=1}^{\infty}E_t(\beta^*\theta_m)^k\frac{1}{\varepsilon^m-1}\widehat{\varepsilon_{t+k}^m}+\sum_{k=1}^{\infty}E_t(\beta^*\theta_m)^k\widehat{MC_{t+k}^{m^r}}\Bigg) \quad \text{(E.4.16)}$$

$\Longrightarrow$

$$\left(\frac{P_t^{*mf}}{P_t^{mG}}-1\right)=-\beta^*\theta_m\frac{\Pi_t^{mf}-\Pi^{mf}}{\Pi^{mf}}-\frac{1-\beta^*\theta_m}{\varepsilon^m-1}\widehat{\varepsilon_t^m}+(1-\beta^*\theta_m)\widehat{MC_t^r}+$$
$$+(1-\beta^*\theta_m)\Bigg(\sum_{k=1}^{\infty}E_t(\beta^*\theta_m)^k\frac{\Pi_{t+k}^{mf}-\Pi^{mf}}{\Pi^{mf}}-\sum_{k=0}^{\infty}E_t(\beta^*\theta_m)^k\frac{1}{\varepsilon^m-1}\widehat{\varepsilon_{t+k}^m}+\sum_{k=1}^{\infty}E_t(\beta^*\theta_m)^k\widehat{MC_{t+k}^{m^r}}\Bigg)$$

Let's add and subtract $(\beta^*\theta_m)^0\frac{E_t\Pi_{t+1}^{xf}-\Pi^{mf}}{\Pi^{mf}}$ in the last equation and start summation from the period k=0, we get:

$$\left(\frac{P_t^{*mf}}{P_t^{mG}}-1\right)=-\beta^*\theta_m\frac{\Pi_t^{mf}-\Pi^{mf}}{\Pi^{mf}}-\frac{1-\beta^*\theta_m}{\varepsilon^m-1}\widehat{\varepsilon_t^m}+(1-\beta^*\theta_m)\widehat{MC_t^r}+$$
$$+\beta^*\theta_m\frac{E_t\Pi_{t+1}^{mf}-\Pi^{mf}}{\Pi^{mf}}-\beta^*\theta_m\frac{E_t\Pi_{t+1}^{mf}-\Pi^{mf}}{\Pi^{mf}}+(1-\beta^*\theta_m)\Bigg(\beta^*\theta_m\sum_{k=0}^{\infty}E_t(\beta^*\theta_m)^k\frac{\Pi_{t+k+1}^{mf}-\Pi^{mf}}{\Pi^{mf}}-$$
$$-\beta^*\theta_m\sum_{k=0}^{\infty}E_t(\beta^*\theta_m)^k\frac{1}{\varepsilon^m-1}\widehat{\varepsilon_{t+k+1}^m}+\beta^*\theta_m\sum_{k=0}^{\infty}E_t(\beta^*\theta_m)^k\widehat{MC_{t+k+1}^{r^m}}\Bigg) \quad \text{(E.4.17)}$$

We can note that the terms in blue color in the equation (E.4.17) is $\beta^*\theta_m E_t\left(\frac{P_{t+1}^{*mf}}{P_{t+1}^{mf}}-1\right)$ Hence,

$$\left(\frac{P_t^{*mf}}{P_t^{mG}}-1\right)=-\beta^*\theta_m\frac{\Pi_t^{mf}-\Pi^{mf}}{\Pi^{mf}}-\frac{1-\beta^*\theta_m}{\varepsilon^m-1}\widehat{\varepsilon_t^m}+(1-\beta^*\theta_m)\widehat{MC_t^{r^m}}+$$

$$+\beta^*\theta_m \frac{E_t\Pi^{mf}_{t+1}-\Pi^{mf}}{\Pi^{mf}} + \beta^*\theta_m E_t\left(\frac{P^{*mf}_{t+1}}{P^{mf}_{t+1}}-1\right) \tag{E.4.18}$$

Now, recall that the aggregate gross inflation in the import sector is given by the equation (E.1.10): $\Pi^{mf}_t = \Pi^{mf}_{t-1} + \frac{1-\theta_m}{\theta_m}\Pi^{mf}\left(\frac{P^{*mf}_t}{P^{mG}_t}-1\right)$, that can be written as:

$$\left(\frac{P^{*mf}_t}{P^{mf}_t}-1\right) = \frac{\theta_m}{1-\theta_m}\left(\frac{\Pi^{mf}_t}{\Pi^{mf}}-\frac{\Pi^{mf}_{t-1}}{\Pi^{mf}}\right) \tag{E.4.19}$$

If we shift the equation (E.4.19) one period forward and put in the equation (E.4.18) we get:

$$\begin{aligned}\frac{\theta_m}{1-\theta_m}\left(\frac{\Pi^{mf}_t}{\Pi^{mf}}-\frac{\Pi^{mf}_{t-1}}{\Pi^{mf}}\right) &= -\beta^*\theta_m\frac{\Pi^{mf}_t-\Pi^{mf}}{\Pi^{mf}} - \frac{1-\beta^*\theta_m}{\varepsilon^m-1}\widehat{\varepsilon^m_t} + (1-\beta^*\theta_m)\widehat{MC^{rm}_t} + \\ &+\beta^*\theta_m E_t\frac{\Pi^{mf}_{t+1}-\Pi^{mf}}{\Pi^{mf}} + \beta^*\theta_m\frac{\theta_m}{1-\theta_m}E_t\left(\frac{\Pi^{mf}_{t+1}}{\Pi^{mf}}-\frac{\Pi^{mf}_t}{\Pi^{mf}}\right)\end{aligned} \tag{E.4.20}$$

After multiplying both sides of the equation (E.4.20) by Π^{mf} and taking into account the definition that the capital letter Π expresses gross inflation, we can write:

$$\begin{aligned}&\frac{\theta_m}{1-\theta_m}\left(1+\pi^{mf}_t-1-\pi^{mf}_{t-1}\right) = \beta^*\theta_m E_t\left(1+\pi^{mf}_{t+1}-1-\pi^{mf}_t\right)\\ &-\frac{(1-\beta^*\theta_m)(1+\pi^{mf})}{\varepsilon^m-1}\widehat{\varepsilon^m_t} + (1-\beta^*\theta_m)(1+\pi^{mf})\widehat{MC^{rm}_t} + \beta^*\theta_m\frac{\theta_m}{1-\theta_m}E_t\left(1+\pi^{mf}_{t+1}-1-\pi^{mf}_t\right)\end{aligned} \tag{E.4.21}$$

By collecting the same terms we get:

$$\begin{aligned}\frac{\theta_m+\beta^*\theta_m-\beta^*\theta_m^2+\beta^*\theta_m^2}{1-\theta_m}\pi^{mf}_t &= \frac{\theta_m}{1-\theta_m}\pi^{mf}_{t-1} - \frac{(1-\beta^*\theta_m)(1+\pi^{mf})}{\varepsilon^m-1}\widehat{\varepsilon^m_t}\\ +(1-\beta^*\theta_m)(1+\pi^{mf})\widehat{MC^{rm}_t} &+ \frac{\beta^*\theta_m-\beta^*\theta_m^2+\beta^*\theta_m^2}{1-\theta_m}E_t\pi^{mf}_{t+1}\end{aligned} \tag{E.4.22}$$

$\Longrightarrow$

$$\frac{\theta_m(1+\beta^*)}{1-\theta_m}\pi^{mf}_t =$$

$$=\frac{\theta_m}{1-\theta_m}\pi^{mf}_{t-1}-\frac{(1-\beta^*\theta_m)(1+\pi^{mf})}{\varepsilon^m-1}\widehat{\varepsilon^m_t}+(1-\beta^*\theta_m)(1+\pi^{mf})\widehat{MC^{r^m}_t}+\frac{\beta^*\theta_m}{1-\theta_m}E_t\pi^{mf}_{t+1} \tag{E.4.23}$$

Finally,

$$\pi^{mf}_t=\frac{1}{1+\beta^*}\pi^{mf}_{t-1}+\frac{\beta^*}{1+\beta^*}E_t\pi^{mf}_{t+1}+\frac{(1-\beta^*\theta_m)(1-\theta_m)(1+\pi^{mf})}{\theta_m(1+\beta^*)}(\widehat{MC^{r^m}_t}-\frac{1}{\varepsilon^m-1}\widehat{\varepsilon^m_t}) \tag{E.4.24}$$

Note, that $\widehat{\varepsilon^m_t}$ express the mark down shock to prices.

Appendix F Exported Goods Sector Derivations

F.1 Aggregate Price Index and Inflation Dynamic in Export Sector

Homogeneous exported goods producers maximize its profit subject to the CES production technology (used to aggregate differentiated exported goods), the firm makes a decision on the optimal combination of differentiated exported goods used in its production process:

$$\underset{X_t(i)}{\text{maximize}} \quad P^{xf}_t\left(\int_0^1 X_t(i)^{\frac{\varepsilon^x_t-1}{\varepsilon^x_t}}di\right)^{\frac{\varepsilon^x_t}{\varepsilon^x_t-1}}-\int_0^1 P^{xf}_t(i)X_t(i)di \tag{F.1.1}$$

F.O.C.

$$[\partial X_t(i)]: \qquad P^{xf}_t\frac{\varepsilon^x_t}{\varepsilon^x_t-1}\left(\int_0^1 X_t(i)^{\frac{\varepsilon^x_t-1}{\varepsilon^x_t}}di\right)^{\frac{1}{\varepsilon^x_t-1}}\frac{\varepsilon^x_t-1}{\varepsilon^x_t}X_t(i)^{\frac{-1}{\varepsilon^x_t}}=P_t(i)^{xf} \tag{F.1.2}$$

We note that $\left(\int_0^1 X_t(i)^{\frac{\varepsilon^x_t-1}{\varepsilon^x_t}}di\right)^{\frac{1}{\varepsilon^x_t-1}}=X_t^{\frac{1}{\varepsilon^x_t}}$; then from the equation (F.1.2) we get:

$$X_t(i)^{\frac{-1}{\varepsilon^x_t}}=\left(\frac{P^{xf}_t(i)}{P^{xf}_t}\right)X_t^{\frac{-1}{\varepsilon^x_t}} \tag{F.1.3}$$

Finally:

$$X_t(i)=\left(\frac{P^{xf}_t(i)}{P^{xf}_t}\right)^{-\varepsilon^x_t}X_t \tag{F.1.4}$$

The equation (F.1.4) determines the demand for goods produced by differentiated exported goods producer.

Putting (F.1.4) into the CES aggregation function yields the following aggregate price index in the export sector (in USD):

$$P_t^{xf} = \left(\int_0^1 \left(P_t^{xf}(i)\right)^{1-\varepsilon_t^x} di \right)^{\frac{1}{1-\varepsilon_t^x}} \tag{F.1.5}$$

$(1-\theta_x)$ share of firms set optimal price at P_t^{*xf} in period t, while θ_x part of firms update their price in line to the price index prevailed in the previous period:

$$P_t^{xf} = \left[\int_0^1 (P_t^{xf}(i))^{1-\varepsilon_t^x} di \right]^{\frac{1}{1-\varepsilon_t^x}} = \left[\int_0^{\theta_x} \left[(P_{t-1}(i)^{xf} \left(\Pi_{t-1}^{xf}\right) \right]^{1-\varepsilon_t^x} di + \int_{\theta_x}^1 (P_t^{*xf})^{1-\varepsilon_t^x} di \right]^{\frac{1}{1-\varepsilon_t^x}} \tag{F.1.6}$$

By applying the same assumptions used in the derivation of the aggregate price index in the import sector, the right-hand side of the equation (F.1.6) can be written as:

$$P_t^{xf} = \left[\theta_x \left[P_{t-1}^{xf} \left(\Pi_{t-1}^{xf}\right) \right]^{1-\varepsilon_t^x} + (1-\theta_x)(P_t^{*xf})^{1-\varepsilon_t^x} \right]^{\frac{1}{1-\varepsilon_t^x}} \tag{F.1.7}$$

If we divide both sides of equation (F.1.7) by P_t^{xf} , then we get:

$$1 = \left[\theta_x \left[\frac{P_{t-1}^{xf}}{P_t^{xf}} \Pi_{t-1}^{xf} \right]^{1-\varepsilon_t^x} + (1-\theta_x) \left[\frac{P_t^{*xf}}{P_t^{xf}} \right]^{1-\varepsilon_t^x} \right]^{\frac{1}{1-\varepsilon_t^x}} \tag{F.1.8}$$

As $\frac{P_{t-1}^{xf}}{P_t^{xf}} = \frac{1}{\Pi_t^{xf}}$, then we can write:

$$1 = \left[\theta_x \left[\frac{1}{\Pi_t^{xf}} \Pi_{t-1}^{xf} \right]^{1-\varepsilon_t^x} + (1-\theta_x) \left[\frac{P_t^{*xf}}{P_t^{xf}} \right]^{1-\varepsilon_t^x} \right]^{\frac{1}{1-\varepsilon_t^x}} \tag{F.1.9}$$

After multiplying both sides of the previous equation by Π_t^{xf}, and taking the both side of the equation in $(1-\varepsilon_t^x)$ power, we get:

$$(\Pi_t^{xf})^{1-\varepsilon_t^x} = \theta_x \left[\Pi_{t-1}^{xf} \right]^{1-\varepsilon_t^x} + (1-\theta_x)(\Pi_t^{xf})^{1-\varepsilon_t^x} \left[\frac{P_t^{*xf}}{P_t^{xf}} \right]^{1-\varepsilon_t^x} \tag{F.1.10}$$

From the first order approximation of the equation (F.1.10) around balanced growth path equilibrium where $\frac{P_t^{*xf}}{P_t^{xf}} = 1$, we get:

$$
\begin{aligned}
{\Pi^{xf}}^{1-\varepsilon^x} &+ (1-\varepsilon^x)(\Pi^{xf})^{-\varepsilon^x}\left(\Pi_t^{xf}-\Pi^{xf}\right) = \theta_x(\Pi^{xf})^{1-\varepsilon^x} + \theta_x(1-\varepsilon^x)(\Pi^{xf})^{-\varepsilon^x}\left(\Pi_{t-1}^{xf}-\Pi^{xf}\right) \\
&+(1-\theta_x)(\Pi^{xf})^{1-\varepsilon^x} + (1-\theta_x)(1-\varepsilon^x){\Pi^{xf}}^{1-\varepsilon^x}\Big(\frac{P_t^{*xf}}{P_t^{xf}}-1\Big) \\
&+(1-\theta_x)(1-\varepsilon^x)(\Pi^{xf})^{-\varepsilon^x}\left(\Pi_t^{xf}-\Pi^{xf}\right) \qquad \text{(F.1.11)}
\end{aligned}
$$

As $(\Pi^{xf})^{1-\varepsilon^x} = \theta_x(\Pi^{xf})^{1-\varepsilon^x} + (1-\theta_x)(\Pi^{xf})^{1-\varepsilon^x}$, and after dividing the both sides of the equation (F.1.11) by $(1-\varepsilon^x)$, we get:

$$
\begin{aligned}
(\Pi^{xf})^{-\varepsilon^x}\left(\Pi_t^{xf}-\Pi^{xf}\right) =& \theta_x(\Pi^{xf})^{-\varepsilon^x}\left(\Pi_{t-1}^{xf}-\Pi^{xf}\right)+ \\
&+(1-\theta_x)(\Pi_t^{xf})^{1-\varepsilon^x}\Big(\frac{P_t^{*xf}}{P_t^{xf}}-1\Big) + (1-\theta_x)(\Pi^{xf})^{-\varepsilon^x}\left(\Pi_t^{xf}-\Pi^{xf}\right) \\
&\qquad \text{(F.1.12)}
\end{aligned}
$$

By combining the left side of the equation (F.1.12) and the last part of the equation, we get:

$$
\theta_x(\Pi^{xf})^{-\varepsilon^x}\left(\Pi_t^{xf}-\Pi^{xf}\right) = \theta_x(\Pi^{xf})^{-\varepsilon^x}\left(\Pi_{t-1}^{xf}-\Pi^{xf}\right) + (1-\theta_x)(\Pi_t^{xf})^{1-\varepsilon^x}\Big(\frac{P_t^{*xf}}{P_t^{xf}}-1\Big) \qquad \text{(F.1.13)}
$$

After dividing both sides of the equation (F.1.13) by $\theta_x(\Pi^{xf})^{-\varepsilon^x}$:

$$
\left(\Pi_t^{xf}-\Pi^{xf}\right) = \left(\Pi_{t-1}^{xf}-\Pi^{xf}\right) + \frac{1-\theta_x}{\theta_x}\Pi_t^{xf}\Big(\frac{P_t^{*xf}}{P_t^{xf}}-1\Big) \qquad \text{(F.1.14)}
$$

Finally,

$$
\Pi_t^{xf} = \Pi_{t-1}^{xf} + \frac{1-\theta_x}{\theta_x}\Pi^{xf}\Big(\frac{P_t^{*xf}}{P_t^{xf}}-1\Big) \qquad \text{(F.1.15)}
$$

F.2 Marginal Cost Function of Differentiated Exported Goods Producer

Differentiated exported goods producer i use the domestic and imported inputs to produce differentiated exported goods using the CES production technology. We assume that production of exported goods is characterized with excess positive productivity compared to the rest of the economy. The positive excess trend productivity makes exported goods relatively cheaper, amid the real export of homogeneous goods increases faster than implied only by trade partner's demand. Also, the technology process a_t^x makes domestic input relatively more efficient in export production. The cost minimization problem of differentiated exported goods producer i can be written using the related Lagrange function:

$$\mathcal{L} = P_t^d X_t^d + P_t^{mG} X_t^m - \lambda(i) \left(a_t^{r\,\frac{2}{\eta_x - 1}} \left[\omega_x^{\frac{1}{\eta_x}} \left(X_t^d a_t^x \right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} X_t^{m\,\frac{\eta_x - 1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x - 1}} - F_t^x - X_t(i) \right) \tag{F.2.1}$$

Note, that fixed cost (F_t^x) is required to enter into the sector.

F.O.Cs.

$\left[\partial X_t^d \right]$:

$$\begin{aligned} P_t^d - \lambda_t(i) \Bigg(& \frac{\eta_x}{\eta_x - 1} a_t^{r\,\frac{2}{\eta_x - 1}} \left[\omega_x^{\frac{1}{\eta_x}} \left(X_t^d a_t^x \right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} X_t^{m\,\frac{\eta_x - 1}{\eta_x}} \right]^{\frac{1}{\eta_x - 1}} \times \\ & \times \frac{\eta_x - 1}{\eta_x} \omega_x^{\frac{1}{\eta_x}} \left(X_t^d a_t^x \right)^{\frac{-1}{\eta_x}} a_t^x \Bigg) = 0 \end{aligned} \tag{F.2.2}$$

$\left[\partial X_t^d \right]$:

$$\begin{aligned} P_t^{mG} - \lambda_t(i) \Bigg(& \frac{\eta_x}{\eta_x - 1} a_t^{r\,\frac{2}{\eta_x - 1}} \left[\omega_x^{\frac{1}{\eta_x}} \left(X_t^d a_t^x \right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} X_t^{m\,\frac{\eta_x - 1}{\eta_x}} \right]^{\frac{1}{\eta_x - 1}} \times \\ & \times \frac{\eta_x - 1}{\eta_x} (1-\omega_x)^{\frac{1}{\eta_x}} X_t^{m\,\frac{-1}{\eta_x}} \Bigg) = 0 \end{aligned} \tag{F.2.3}$$

$\lambda_t(i)$ can be interpreted as the shadow price of inputs, or as marginal cost. Hence, $\lambda_t(i) = MC_t^x(i)$.

Note that $a_t^{r\frac{2}{\eta_x(\eta_x-1)}}\left[\omega_x^{\frac{1}{\eta_x}}\left(X_t^d a_t^x\right)^{\frac{\eta_x-1}{\eta_x}}+(1-\omega_x)^{\frac{1}{\eta_x}}X_t^{m\frac{\eta_x-1}{\eta_x}}\right]^{\frac{1}{\eta_x-1}}=(X_t(i)+F_t^x)^{\frac{1}{\eta_x}}$. Then

$$\left(X_t(i)^d a_t^x\right)^{\frac{1}{\eta_x}}=\omega_x^{\frac{1}{\eta_x}}a_t^{r\frac{2}{\eta_x}}\frac{MC_t(i)^x}{P_t^d/a_t^x}\left(X_t(i)+F_t^x\right)^{\frac{1}{\eta_x}} \tag{F.2.4}$$

$$X_t(i)^{m\frac{1}{\eta_x}}=(1-\omega_x)^{\frac{1}{\eta_x}}a_t^{r\frac{2}{\eta_x}}\frac{MC_t(i)^x}{P_t^{mG}}\left(X_t(i)+F_t^x\right)^{\frac{1}{\eta_x}} \tag{F.2.5}$$

Finally:

$$X_t(i)^d a_t^x=\omega_x a_t^{r2}\left[\frac{MC_t(i)^x}{P_t^d/a_t^x}\right]^{\eta_x}\left(X_t(i)+F_t^x\right) \tag{F.2.6}$$

$$X_t(i)^m=(1-\omega_x)a_t^{r2}\left[\frac{MC_t(i)^x}{P_t^{mG}}\right]^{\eta_x}\left(X_t(i)+F_t^x\right) \tag{F.2.7}$$

By putting equations (F.2.6) and (F.2.7) into the production function, we get:

$$\begin{aligned}X_t(i)=&a_t^{r\frac{2}{\eta_x(\eta_x-1)}}\left[\omega_x^{\frac{1}{\eta_x}}\left(\omega_x a_t^{r2}\left[\frac{MC_t(i)^x}{P_t^d/a_t^x}\right]^{\eta_x}\left(X_t(i)+F_t^x\right)\right)^{\frac{\eta_x-1}{\eta_x}}+\right.\\&\left.+(1-\omega_x)^{\frac{1}{\eta_x}}\left((1-\omega_x)a_t^{r2}\left[\frac{MC_t(i)^x}{P_t^{mG}}\right]^{\eta_x}\left(X_t(i)+F_t^x\right)\right)^{\frac{\eta_x-1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x-1}}-F_t^x\end{aligned} \tag{F.2.8}$$

From (F.2.8):

$$X_t(i)+F_t^x=a_t^{r\frac{2\eta_x}{\eta_x-1}}\left(X_t(i)+F_t^x\right)\left[\omega_x MC_t(i)^{x^{\eta_x-1}}\left(\frac{P_t^d}{a_t^x}\right)^{1-\eta_x}+(1-\omega_x)MC_t(i)^{x^{\eta_x-1}}(P_t^{mG})^{1-\eta_x}\right]^{\frac{\eta_x}{\eta_x-1}} \tag{F.2.9}$$

The equation (F.2.9), also be written as:

$$1=a_t^{r\frac{2\eta_x}{\eta_x-1}}MC_t(i)^{x^{\eta_x}}\left[\omega_x\left(\frac{P_t^d}{a_t^x}\right)^{1-\eta_x}+(1-\omega_x)(P_t^{mG})^{1-\eta_x}\right]^{\frac{\eta_x}{\eta_x-1}} \tag{F.2.10}$$

Then from equation (F.2.10)

$$MC_t(i)^x=a_t^{r-\frac{2}{\eta_x-1}}\left[\omega_x\left(\frac{P_t^d}{a_t^x}\right)^{1-\eta_x}+(1-\omega_x)(P_t^{mG})^{1-\eta_x}\right]^{\frac{-\eta_x}{\eta_x-1}\frac{1}{\eta_x}} \tag{F.2.11}$$

Here, we use the fact that cost minimization problem is symmetric across firms, and subsequently, we can write:

$$MC_t^x = a_t^{r-\frac{2}{\eta_x-1}}\left[\omega_x\left(\frac{P_t^d}{a_t^x}\right)^{1-\eta_x} + (1-\omega_x)(P_t^{mG})^{1-\eta_x}\right]^{\frac{1}{1-\eta_x}} \tag{F.2.12}$$

F.3 Profit Maximization Problem of Differentiated Exported Goods Producer

After taking the constraints into profit maximization problem 2.4.6.15 of differentiated exported goods producer, we get:

$$\begin{aligned}\underset{P_t(i)^{*xf}}{\text{maximize}} \quad & E_t\sum_{k=0}^{\infty}\left\{\theta_x^k\beta^k\frac{U'(C_{t+k}^{uc})P_t^c}{U'(C_t^{uc})P_{t+k}^c}\left[e_{t+k}^{Gel/D}P_t(i)^{*xf}\Pi_{t+k-1|t-1}^{xf}\times\right.\right.\\ &\left.\left.\times\left(\frac{P_t(i)^{*xf}\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x}X_{t+k} - MC_{t+k}^x\left(\left(\frac{P_t(i)^{*xf}\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x}X_{t+k}+F_t^x\right)\right]\right\}\end{aligned} \tag{F.3.1}$$

We define export goods price in GEL as $P_t^{xG} \equiv e_t^{Gel/D}P_t^{xf}$. Then by multiplying and dividing the profit function by P_{t+k}^{xG} and, in addition, by substituting $U'(C_{t+k}^{uc})$ with its functional form : $U'(C_{t+k}^{uc}) = \frac{\psi_{t+k}}{(C_{t+k}^{uc}-hC_{t+k-1}^{uc})}$, we get:

$$\begin{aligned}\underset{P_t(i)^{*xf}}{\text{maximize}} \quad & E_t\sum_{k=0}^{\infty}\left\{\theta_x^k\beta^k\frac{\psi_{t+k}(C_t^{uc}-hC_{t-1}^{uc})}{\psi_t(C_{t+k}^{uc}-hC_{t+k-1}^{uc})\Pi_{t+k|t}^C}\frac{P_{t+k}^{xG}}{P_{t+k}^{xG}}\left[e_{t+k}^{Gel/D}P_t(i)^{*xf}\Pi_{t+k-1|t-1}^{xf}\times\right.\right.\\ &\left.\left.\times\left(\frac{P_t(i)^{*xf}\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x}X_{t+k} - MC_{t+k}^x\left(\frac{P_t(i)^{*xf}\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x}X_{t+k}\right]\right\}\end{aligned} \tag{F.3.2}$$

Because of the constant returns to scale feature in the production function, marginal costs are the same across firms. Therefore, when firms optimize their prices they have the same information and the optimal price is the same across firms as well. Hence,

$P_t(i)^{*xf}{=}P_t^{*xf}$, and as $P_t^{xG}{=}e_{t+k}^{Gel/D}P_t^{xf}$, we end up with:

$$\underset{P_t^{*xf}}{\text{maximize}} \quad E_t\sum_{k=0}^{\infty}\Bigg\{\theta_x^k\beta^k\frac{\psi_{t+k}(C_t^{uc}-hC_{t-1}^{uc})}{\psi_t(C_{t+k}^{uc}-hC_{t+k-1}^{uc})\Pi_{t+k|t}^{C}}P_{t+k}^{xG}\times$$
$$\times\Bigg[\frac{e_{t+k}^{Gel/D}P_t^{*xf}\Pi_{t+k-1|t-1}^{xf}}{e_{t+k}^{Gel/D}P_{t+k}^{xf}}\left(\frac{P_t^{*xf}\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x}X_{t+k}-\frac{MC_{t+k}^x}{e_{t+k}^{Gel/D}P_{t+k}^{xf}}\left(\frac{P_t^{*xf}\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x}X_{t+k}\Bigg]\Bigg\} \tag{F.3.3}$$

We can define real marginal cost as

$$MC_{t+k}^{x^r}\equiv\frac{MC_{t+k}^x}{e_{t+k}^{Gel/D}P_{t+k}^{xf}} \tag{F.3.4}$$

Then by collecting the same terms in (F.3.3) the resulted profit maximization problem is given by:

$$\underset{P_t^{*xf}}{\text{maximize}} \quad \sum_{k=0}^{\infty}E_t\Bigg\{\theta_x^k\beta^k\frac{\psi_{t+k}(C_t^{uc}-hC_{t-1}^{uc})}{\psi_t(C_{t+k}^{uc}-hC_{t+k-1}^{uc})\Pi_{t+k|t}^{C}}P_{t+k}^{xG}\times$$
$$\times\Bigg[(P_t^{*xf})^{1-\varepsilon_{t+k}^x}\left(\frac{\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{1-\varepsilon_{t+k}^x}X_{t+k}-MC_{t+k}^{x^r}\left(\frac{\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x}X_{t+k}(P_t^{*xf})^{-\varepsilon_{t+k}^x}\Bigg]\Bigg\} \tag{F.3.5}$$

The FOCs. w.r.t optimal price of the maximization problem reads:

$$\left[\partial P_t^{*xf}\right]:$$
$$E_t\sum_{k=0}^{\infty}\Bigg\{\theta_x^k\beta^k\frac{\psi_{t+k}(C_t^{uc}-hC_{t-1}^{uc})}{\psi_t(C_{t+k}^{uc}-hC_{t+k-1}^{uc})\Pi_{t+k|t}^{C}}P_{t+k}^{xG}\Bigg[(1-\varepsilon_{t+k}^x)(P_t^{*xf})^{-\varepsilon_{t+k}^x}\left(\frac{\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{1-\varepsilon_{t+k}^x}X_{t+k}$$
$$+\varepsilon_{t+k}^x MC_{t+k}^{x^r}\left(\frac{\Pi_{t+k-1|t-1}^{xf}}{P_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x}X_{t+k}(P_t^{*xf})^{-\varepsilon_{t+k}^x-1}\Bigg]\Bigg\}=0 \tag{F.3.6}$$

Now let's divide both side of the equation (F.3.6) by P_t^{xf}:

$$E_t\sum_{k=0}^{\infty}\Bigg\{\theta_x^k\beta^k\frac{\psi_{t+k}(C_t^{uc}-hC_{t-1}^{uc})}{\psi_t(C_{t+k}^{uc}-hC_{t+k-1}^{uc})\Pi_{t+k|t}^{C}}P_{t+k}^{xG}\Bigg[(1-\varepsilon_{t+k}^x)\left(\frac{P_t^{*xf}}{P_t^{xf}}\right)^{-\varepsilon_{t+k}^x}\left(\frac{\Pi_t^{xf}}{\Pi_{t+k}^{xf}}\right)^{1-\varepsilon_{t+k}^x}X_{t+k}$$
$$+\varepsilon_{t+k}^x MC_{t+k}^{x^r}\left(\frac{\Pi_t^{xf}}{\Pi_{t+k}^{xf}}\right)^{-\varepsilon_{t+k}^x}X_{t+k}\left(\frac{P_t^{*xf}}{P_t^{xf}}\right)^{-\varepsilon_{t+k}^x-1}\Bigg]\Bigg\}=0 \tag{F.3.7}$$

F.4 Recursive Form of Optimal Price

As it was in the case of differentiated imported goods producers, here, we have to simplify the equation a little bit, to make it possible to rewrite the equation in recursive form. In particular, we treat the time-varying elasticity of substitution as a parameter first and after deriving the equation we reintroduce it as a variable. Therefore, the optimal price equation given by F.3.7 can be rewritten as:

$$\frac{P_t^{*xf}}{P_t^{xf}} = \frac{\varepsilon^x}{\varepsilon^x - 1} \frac{E_t \sum_{k=0}^{\infty} \left\{ \theta_x^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})\Pi_{t+k|t}^C} P_{t+k}^{xG} \left(\frac{\Pi_{t+k}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon^x} X_{t+k} MC_{t+k}^{x^r} \right\}}{E_t \sum_{k=0}^{\infty} \left\{ \theta_x^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})\Pi_{t+k|t}^C} P_{t+k}^{xG} \left(\frac{\Pi_{t+k}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon^x - 1} X_{t+k} \right\}} \tag{F.4.1}$$

Where Π_{t+k}^{xf} and Π_t^{xf} are gross inflations in $t+k$ and t periods respectively.
If we take P_t^c out of the expectation operator, then the optimal relative price of exported goods can be written as:

$$\frac{P_t^{*xf}}{P_t^{xf}} = \frac{\varepsilon^x}{\varepsilon^x - 1} E_t \frac{\sum_{k=0}^{\infty} \left\{ \theta_x^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})P_{t+k}^c} P_{t+k}^{xG} \left(\frac{\Pi_{t+k}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon^x} X_{t+k} MC_{t+k}^{x^r} \right\}}{\sum_{k=0}^{\infty} \left\{ \theta_x^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})P_{t+k}^c} P_{t+k}^{xG} \left(\frac{\Pi_{t+k}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon^x - 1} X_{t+k} \right\}} \tag{F.4.2}$$

Let's denote:

$$B_{1t} \equiv E_t \sum_{k=0}^{\infty} \left\{ \theta_x^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})P_{t+k}^c} P_{t+k}^{xG} \left(\frac{\Pi_{t+k}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon^x} X_{t+k} MC_{t+k}^{x^r} \right\} \tag{F.4.3}$$

And

$$B_{2t} \equiv E_t \sum_{k=0}^{\infty} \left\{ \theta_x^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})P_{t+k}^c} P_{t+k}^{xG} \left(\frac{\Pi_{t+k}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon^x - 1} X_{t+k} \right\} \tag{F.4.4}$$

Then by taking definitions (F.4.3) and (F.4.4) into account, the equation (F.4.2) can be written as:

$$P_t^{*xf} = \frac{\varepsilon^x}{\varepsilon^x - 1} \frac{B_{1t}}{B_{2t}} \tag{F.4.5}$$

B_{1t} and B_{2t} can be written recursively, for example:

$$B_{1t} = \theta_x^0 \beta^0 \frac{\psi_t}{(C_t^{uc} - hC_{t-1}^{uc})P_t^c} P_t^{xG} X_t MC_t^{x^r} +$$
$$+E_t \sum_{k=1}^{\infty} \left\{ \theta_x^k \beta^k \frac{\psi_{t+k}}{(C_{t+k}^{uc} - hC_{t+k-1}^{uc})P_{t+k}^c} P_{t+k}^{xG} \left(\frac{\Pi_{t+k}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon^x} X_{t+k} MC_{t+k}^{x^r} \right\} =$$
$$= \frac{\psi_t}{(C_t^{uc} - hC_{t-1}^{uc})P_t^c} P_t^{xG} X_t MC_t^{x^r} +$$
$$+\theta_x \beta E_t \left(\frac{\Pi_{t+1}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon^x} \sum_{k=0}^{\infty} \left\{ \theta_x^k \beta^k \frac{\psi_{t+k+1}}{(C_{t+k+1}^{uc} - hC_{t+k}^{uc})P_{t+k+1}^C} P_{t+k+1}^{xG} \left(\frac{\Pi_{t+k+1}^{xf}}{\Pi_{t+1}^{xf}} \right)^{\varepsilon^x} X_{t+k+1} MC_{t+k+1}^{x^r} \right\} =$$
$$= \frac{\psi_t}{(C_t^{uc} - hC_{t-1}^{uc})P_t^c} P_t^{xG} X_t MC_t^{x^r} + \theta_x \beta E_t \left(\frac{\Pi_{t+1}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon^x} B_{1t+1} \tag{F.4.6}$$

Applying the same modification for the equation (F.4.4), we can write the equation (F.4.5) in the recursive form as well. Finally, by returning back ε^x as the variable, the equation could be rewritten as:

$$\frac{P_t^{*xf}}{P_t^{xf}} = \frac{\varepsilon_t^x}{\varepsilon_t^x - 1} \frac{B_{1t}}{B_{2t}}$$

$$where$$

$$B_{1t} = \frac{\psi_t}{(C_t^{uc} - hC_{t-1}^{uc})P_t^c} P_t^{xG} X_t MC_t^{x^r} + \theta_x \beta E_t \left(\frac{\Pi_{t+1}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon_t^x} B_{1t+1}$$

$$and$$

$$B_{1t} = \frac{\psi_t}{(C_t^{uc} - hC_{t-1}^{uc})P_t^c} P_t^{xG} X_t + \theta_x \beta E_t \left(\frac{\Pi_{t+1}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon_t^x - 1} B_{2t+1} \tag{F.4.7}$$

F.5 Linear Transformation of Optimal Price Setting Problem

We can show that in the optimal price equation:

$$E_t \sum_{k=0}^{\infty} \left\{ \theta_x^k \beta^k \frac{\psi_{t+k}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t(C_{t+k}^{uc} - hC_{t+k-1}^{uc})\Pi_{t+k|t}^C} P_{t+k}^{xG} \left[(1 - \varepsilon_{t+k}^x) \left(\frac{P_t^{*xf}}{P_t^{xf}} \right)^{-\varepsilon_{t+k}^x} \left(\frac{\Pi_t^{xf}}{\Pi_{t+k}^{xf}} \right)^{1-\varepsilon_{t+k}^x} X_{t+k} \right.\right.$$
$$\left.\left. + \varepsilon_{t+k}^x MC_{t+k}^{x^r} \left(\frac{\Pi_t^{xf}}{\Pi_{t+k}^{xf}} \right)^{-\varepsilon_{t+k}^x} X_{t+k} \left(\frac{P_t^{*xf}}{P_t^{xf}} \right)^{-\varepsilon_{t+k}^x - 1} \right] \right\} = 0 \tag{F.5.1}$$

the combination of the following variables $\frac{P^{xG}_{t+k}X_{t+k}}{z_{t+k}P^{c}_{t+k}}$ is jointly stationary, then, let's denote it as:

$$X^*_{t+k} \equiv \frac{P^{xG}_{t+k}X_{t+k}}{z_{t+k}P^{c}_{t+k}} \tag{F.5.2}$$

After substituting with a stationary form of the equation, to write it more compactly, we make the following definitions:

$$LHS^x_{t+k} \equiv (\varepsilon^x_{t+k}-1)\frac{\psi_{t+k}}{(\widetilde{C^{uc}_{t+k}}-\frac{h}{1+\gamma^z_{t+k}}\widetilde{C^{uc}_{t+k-1}})}\left(\frac{\Pi^{xf}_{t+k}}{\Pi^{xf}_{t}}\right)^{1-\varepsilon^x_{t+k}}\left(\frac{P^{*xf}_{t}}{P^{xf}_{t}}\right)^{-\varepsilon^x_{t+k}}X^*_{t+k} \tag{F.5.3}$$

And,

$$RHS^x_{t+k} \equiv \varepsilon^x_{t+k}\frac{\psi_{t+k}}{(\widetilde{C^{uc}_{t+k}}-\frac{h}{1+\gamma^z_{t+k}}\widetilde{C^{uc}_{t+k-1}})}\left(\frac{\Pi^{xf}_{t+k}}{\Pi^{xf}_{t}}\right)^{\varepsilon^x_{t+k}}\left(\frac{P^{*xf}_{t}}{P^{xf}_{t}}\right)^{-\varepsilon^x_{t+k}-1}X^*_{t+k}MC^{x^r}_{t+k} \tag{F.5.4}$$

The steady-state values of the equation are given by:

$$LHS^x = (\varepsilon^x-1)\frac{\psi(1+\gamma^z)}{(1+\gamma^z-h)\widetilde{C^{uc}}}X^* \tag{F.5.5}$$

And

$$RHS^x = \varepsilon^x\frac{\psi(1+\gamma^z)}{(1+\gamma^z-h)\widetilde{C^{uc}}}X^*MC^{x^r} \tag{F.5.6}$$

As an example we note that the first order derivative w.r.t. C^{uc}_{t+k} of the left hand side is:

$$\frac{\partial E_t\sum_{k=0}^{\infty}\theta^k_x\beta^k LHS^x_{t+k}}{\partial\widetilde{C^{uc}_{t+k}}} =$$

$$= -E_t\sum_{k=0}^{\infty}\left\{\theta^k_x\beta^k(\varepsilon^x_{t+k}-1)\frac{\psi_{t+k}}{(\widetilde{C^{uc}_{t+k}}-\frac{h}{1+\gamma^z_{t+k}}\widetilde{C^{uc}_{t+k-1}})^2}\left(\frac{\Pi^{xf}_{t+k}}{\Pi^{xf}_{t}}\right)^{\varepsilon^x_{t+k}-1}\left(\frac{P^{*xf}_{t}}{P^{xf}_{t}}\right)^{-\varepsilon^x_{t+k}}X^*_{t+k}\right\} \tag{F.5.7}$$

And the equation (F.5.7) in SS can be written as:

$$\frac{\partial \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k LHS^x}{\partial \widetilde{C^{uc}}} = -\sum_{k=0}^{\infty} \theta_x^k \beta^k \frac{\psi(1+\gamma^z)^2}{(1+\gamma^z-h)^2 \widetilde{C^{uc}}} \frac{X^*}{\widetilde{C^{uc}}} \tag{F.5.8}$$

Finally,

$$\frac{\partial \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k LHS^x}{\partial \widetilde{C^{uc}}} = -\sum_{k=0}^{\infty} \theta_x^k \beta^k LHS^x \frac{\psi(1+\gamma^z)}{(1+\gamma^z-h)} \frac{1}{\widetilde{C^{uc}}} \tag{F.5.9}$$

Taking into account (F.5.10), the linear version of the left-hand side of the equation (F.5.3) is:

$$\begin{aligned}
\sum_{k=0}^{\infty} E_t \theta_x^k \beta^k LHS_{t+k}^x \approx & \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k LHS^x + \frac{h}{1+\gamma^z-h} \sum_{k=0}^{\infty} E_t \theta_m^k \beta^k LHS^x \frac{\widetilde{C_{t+k-1}^{uc}} - \widetilde{C^{uc}}}{\widetilde{C^{uc}}} - \\
& - \frac{1+\gamma^z}{1+\gamma^z-h} \sum_{k=0}^{\infty} E_t \theta_m^k \beta^k LHS^x \frac{\widetilde{C_{t+k}^{uc}} - \widetilde{C^{uc}}}{\widetilde{C^{uc}}} + \sum_{k=0}^{\infty} E_t \theta_m^k \beta^k LHS^x \frac{\psi_{t+k} - \psi}{\psi} \\
& + (\varepsilon^x - 1) \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k LHS^x \frac{\Pi_{t+k}^{xf} - \Pi^{xf}}{\Pi^{xf}} - (\varepsilon^x - 1) \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k LHS \frac{\Pi^{xf} - \Pi^{xf}}{\Pi^{xf}} \\
& + \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k LHS^x \frac{\varepsilon^x}{\varepsilon^x - 1} \frac{\varepsilon_{t+k}^x - \varepsilon^x}{\varepsilon^x} - \varepsilon^x \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k LHS^x \left(\frac{P_t^{*xf}}{P_t^{xf}} - 1 \right)
\end{aligned} \tag{F.5.10}$$

And the linear version of the right hand side of the equation (F.5.3) can be written as:

$$\begin{aligned}
\sum_{k=0}^{\infty} E_t \theta_x^k \beta^k RHS_{t+k}^x \approx & \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k RHS^x + \frac{h}{1+\gamma^z-h} \sum_{k=0}^{\infty} E_t \theta_m^k \beta^k LHS^x \frac{\widetilde{C_{t+k-1}^{uc}} - \widetilde{C^{uc}}}{\widetilde{C^{uc}}} - \\
& - \frac{1+\gamma^z}{1+\gamma^z-h} \sum_{k=0}^{\infty} E_t \theta_m^k \beta^k RHS^x \frac{\widetilde{C_{t+k}^{uc}} - \widetilde{C^{uc}}}{\widetilde{C^{uc}}} + \sum_{k=0}^{\infty} E_t \theta_m^k \beta^k RHS^x \frac{\psi_{t+k} - \psi}{\psi} \\
& + \varepsilon^x \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k RHS^x \frac{\Pi_{t+k}^{xf} - \Pi^{xf}}{\Pi^{xf}} - \varepsilon^x \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k RHS \frac{\Pi_t^{xf} - \Pi^{xf}}{\Pi^{xf}} \\
& + \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k RHS^x \frac{\varepsilon_{t+k}^x - \varepsilon^x}{\varepsilon^x} + \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k RHS^x \frac{MC_{t+k}^{x^r} - MC^{x^r}}{MC^{x^r}} \\
& - (1+\varepsilon^x) \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k RHS^x \left(\frac{P_t^{*xf}}{P_t^{xf}} - 1 \right)
\end{aligned} \tag{F.5.11}$$

Resulting from this, the terms with the same colors in (F.5.10) and (F.5.11) will cancel each other and by combining the rest of the parts of these equations we get:

$$\sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \left(\frac{P_t^{*xf}}{P_t^{xf}} - 1 \right) = \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \frac{\Pi_{t+k}^{xf} - \Pi^{xf}}{\Pi^{xf}} - \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \frac{\Pi_t^{xf} - \Pi^{xf}}{\Pi^{xf}} + \\ - \frac{1}{\varepsilon^x - 1} \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k LHS^x \frac{\varepsilon_{t+k}^x - \varepsilon^x}{\varepsilon^x} + \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \frac{MC_{t+k}^{x^r} - MC^{x^r}}{MC^{x^r}} \tag{F.5.12}$$

We denote $\frac{MC_{t+k}^{x^r} - MC^{x^r}}{MC^{x^r}} \equiv \widehat{MC_{t+k}^{x^r}}$, as the gap of real marginal cost; also we note that in equilibrium $\Pi_{t+k}^{xf} = \Pi_t^{xf} = \Pi^{xf} = 1 + \pi^{xf}$. Taking into account those facts the equation (F.5.12) can be rewritten as:

$$\frac{1}{1-\theta_x \beta} \left(\frac{P_t^{*xf}}{P_t^{xf}} - 1 \right) = \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \frac{\Pi_{t+k}^{xf} - \Pi^{xf}}{\Pi^{xf}} - \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \frac{\Pi_t^{xf} - \Pi^{xf}}{\Pi^{xf}} - \\ - \frac{1}{\varepsilon^x - 1} \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \widehat{\varepsilon_t^x} + \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \widehat{MC_{t+k}^{x^r}} \tag{F.5.13}$$

Or

$$\left(\frac{P_t^{*xf}}{P_t^{xf}} - 1 \right) = (1-\theta_x \beta) \left(\sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \frac{\Pi_{t+k}^{xf} - \Pi^{xf}}{\Pi^{xf}} - \frac{1}{1-\theta_x \beta} \frac{\Pi_t^{xf} - \Pi^{xf}}{\Pi^{xf}} - \\ - \frac{1}{\varepsilon^x - 1} \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \widehat{\varepsilon_{t+k}^x} + \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \widehat{MC_{t+k}^{x^r}} \right) \tag{F.5.14}$$

$\Longrightarrow$

$$\left(\frac{P_t^{*xf}}{P_t^{xf}} - 1 \right) = - \frac{\Pi_t^{xf} - \Pi^{xf}}{\Pi^{xf}} + (1-\theta_x \beta) \left(\sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \frac{\Pi_{t+k}^{xf} - \Pi^{xf}}{\Pi^{xf}} \\ - \frac{1}{\varepsilon^x - 1} \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \widehat{\varepsilon_{t+k}^x} + \sum_{k=0}^{\infty} E_t \theta_x^k \beta^k \widehat{MC_{t+k}^{x^r}} \right) \tag{F.5.15}$$

The right-hand side of the equation (F.5.15) can be written as:

$$\left(\frac{P_t^{*xf}}{P_t^{xf}} - 1 \right) = - \frac{\Pi_t^{xf} - \Pi^{xf}}{\Pi^{xf}} + (1-\theta_x \beta) \Bigg(\theta_x^0 \beta^0 \frac{\Pi_t^{xf} - \Pi^{xf}}{\Pi^{xf}} - \theta_x^0 \beta^0 \frac{1}{\varepsilon^x - 1} \widehat{\varepsilon_t^x} + \theta_x^0 \beta^0 \widehat{MC_t^{x^r}}$$

$$+\sum_{k=1}^{\infty}E_t\theta_x^k\beta^k\frac{\Pi_{t+k}^{xf}-\Pi^{xf}}{\Pi^{xf}}-\frac{1}{\varepsilon^x-1}\sum_{k=0}^{\infty}E_t\theta_x^k\beta^k\widehat{\varepsilon_{t+k}^x}+\sum_{k=1}^{\infty}E_t\theta_x^k\beta^k\widehat{MC_{t+k}^{x^r}}\Bigg) \tag{F.5.16}$$

$\Longrightarrow$

$$\left(\frac{P_t^{*xf}}{P_t^{xf}}-1\right)=-\theta_x\beta\frac{\Pi_t^{xf}-\Pi^{xf}}{\Pi^{xf}}-\frac{(1-\theta_x\beta)}{\varepsilon^x-1}\widehat{\varepsilon_t^x}+(1-\theta_x\beta)\widehat{MC_t^{x^r}}+$$
$$+(1-\theta_x\beta)\left(\sum_{k=1}^{\infty}E_t\theta_x^k\beta^k\frac{\Pi_{t+k}^{xf}-\Pi^{xf}}{\Pi^{xf}}-\frac{1}{\varepsilon^x-1}\sum_{k=1}^{\infty}E_t\theta_x^k\beta^k\widehat{\varepsilon_{t+k}^x}-\sum_{k=1}^{\infty}E_t\theta_x^k\beta^k\widehat{MC_{t+k}^{x^r}}\right) \tag{F.5.17}$$

Let's add and subtract $\theta\beta\frac{\Pi_{t+1}^{xf}-\Pi^{xf}}{\Pi^{xf}}$ in the equation (F.5.17) and start summation from period k=0, we get:

$$\left(\frac{P_t^{*xf}}{P_t^{xf}}-1\right)=-\theta_x\beta\frac{\Pi_t^{xf}-\Pi^{xf}}{\Pi^{xf}}-\frac{(1-\theta_x\beta)}{\varepsilon^x-1}\widehat{\varepsilon_t^x}+(1-\theta_x\beta)\widehat{MC_t^{x^r}}+\theta_x\beta\frac{\Pi_{t+1}^{xf}-\Pi^{xf}}{\Pi^{xf}}-\theta_x\beta\frac{\Pi_{t+1}^{xf}-\Pi^{xf}}{\Pi^{xf}}$$
$$+(1-\theta_x\beta)\left(\theta_x\beta\sum_{k=0}^{\infty}E_t\theta_x^k\beta^k\frac{\Pi_{t+k+1}^{xf}-\Pi^{xf}}{\Pi^{xf}}-\theta_x\beta\frac{1}{\varepsilon^x-1}\sum_{k=0}^{\infty}E_t\theta_x^k\beta^k\widehat{\varepsilon_{t+k+1}^x}+\theta_x\beta\sum_{k=0}^{\infty}E_t\theta_x^k\beta^k\widehat{MC_{t+k+1}^{x^r}}\right) \tag{F.5.18}$$

We can note that the terms in blue color in the equation (F.5.18) is $\theta_x\beta E_t\left(\frac{P_{t+1}^{*xf}}{P_{t+1}^{xf}}-1\right)$ Hence,

$$\left(\frac{P_t^{*xf}}{P_t^{xf}}-1\right)=-\theta_x\beta\frac{\Pi_t^{xf}-\Pi^{xf}}{\Pi^{xf}}-\frac{(1-\theta_x\beta)}{\varepsilon^x-1}\widehat{\varepsilon_t^x}+(1-\theta_x\beta)\widehat{MC_t^{x^r}}+\theta_x\beta\frac{\Pi_{t+1}^{xf}-\Pi^{xf}}{\Pi^{xf}}+$$
$$+\theta_x\beta E_t\left(\frac{P_{t+1}^{*xf}}{P_{t+1}^{xf}}-1\right) \tag{F.5.19}$$

Now, recall that the aggregate gross inflation in the export sector is given by the equation (F.1.15) $\Pi_t^{xf}=\Pi_{t-1}^{xf}+\frac{1-\theta_x}{\theta_x}\Pi^{xf}\left(\frac{P_t^{*xf}}{P_t^{xf}}-1\right)$, that can be written as:

$$\left(\frac{P_t^{*xf}}{P_t^{xf}}-1\right)=\frac{\theta_x}{1-\theta_x}\left(\frac{\Pi_t^{xf}}{\Pi^{xf}}-\frac{\Pi_{t-1}^{xf}}{\Pi^{xf}}\right) \tag{F.5.20}$$

If we shift the equation (F.5.20) one period forward and put in the equation (F.5.19) we get:

$$\frac{\theta_x}{1-\theta_x}\left(\frac{\Pi_t^{xf}}{\Pi^{xf}}-\frac{\Pi_{t-1}^{xf}}{\Pi^{xf}}\right)=-\theta_x\beta\frac{\Pi_t^{xf}-\Pi^{xf}}{\Pi^{xf}}-\frac{(1-\theta_x\beta)}{\varepsilon^x-1}\widehat{\varepsilon_t^x}+(1-\theta_x\beta)\widehat{MC_t^{x^r}}+\theta_x\beta E_t\frac{\Pi_{t+1}^{xf}-\Pi^{xf}}{\Pi^{xf}}+$$
$$+\theta_x\beta E_t\frac{\theta_x}{1-\theta_x}\left(\frac{\Pi_{t+1}^{xf}}{\Pi^{xf}}-\frac{\Pi_t^{xf}}{\Pi^{xf}}\right) \tag{F.5.21}$$

After multiplying both sides of the equation (F.5.21) by Π^{xf} and taking into account that the capital letter Π expresses gross inflation, we can write:

$$\frac{\theta_x}{1-\theta_x}\left(1+\pi_t^{xf}-1-\pi_{t-1}^{xf}\right)=\theta_x\beta E_t\left(1+\pi_{t+1}^{xf}-1-\pi_t^{xf}\right)$$
$$-\frac{(1-\theta_x\beta)}{\varepsilon^x-1}\widehat{\varepsilon_t^x}+(1-\theta_x\beta)(1+\pi^{xf})\widehat{MC_t^{x^r}}+\theta_x\beta\frac{\theta_x}{1-\theta_x}E_t\left(1+\pi_{t+1}^{xf}-1-\pi_t^{xf}\right) \tag{F.5.22}$$

By collecting the same terms we get:

$$\frac{\theta_x+\theta_x\beta-\theta_x^2\beta+\theta_x^2\beta}{1-\theta_x}\pi_t^{xf}=\frac{\theta_x}{1-\theta_x}\pi_{t-1}-\frac{(1-\theta_x\beta)}{\varepsilon^x-1}\widehat{\varepsilon_t^x}$$
$$+(1-\theta_x\beta)(1+\pi^{xf})\widehat{MC_t^{x^r}}+\frac{\theta_x\beta-\theta_x^2\beta+\theta_x^2\beta}{1-\theta_x}E_t\pi_{t+1}^{xf} \tag{F.5.23}$$

$\Longrightarrow$

$$\frac{\theta_x(1+\beta)}{1-\theta_x}\pi_t^{xf}=\frac{\theta_x}{1-\theta_x}\pi_{t-1}^{xf}-\frac{(1-\theta_x\beta)(1+\pi^{xf})}{\varepsilon^x-1}\widehat{\varepsilon_t^x}+$$
$$+(1-\theta_x\beta)(1+\pi^{xf})\widehat{MC_t^{x^r}}+\frac{\theta_x\beta}{1-\theta_x}E_t\pi_{t+1}^{xf} \tag{F.5.24}$$

Finally,

$$\pi_t^{xf}=\frac{1}{1+\beta}\pi_{t-1}^{xf}+\frac{\beta}{1+\beta}E_t\pi_{t+1}^{xf}+\frac{(1-\theta_x\beta)(1-\theta_x)(1+\pi^{xf})}{\theta_x(1+\beta)}\widehat{MC_t^{x^r}}-$$
$$-\frac{(1-\theta_x\beta)(1-\theta_x)(1+\pi^{xf})}{\theta_x(1+\beta)(\varepsilon^x-1)}\widehat{\varepsilon_t^x} \tag{F.5.25}$$

The real marginal cost in the export sector is given by the equation (F.3.4). After the log-linear transformation of the equation, the real marginal cost gap in the export sector is given by:

$$\widehat{MC_t^{x^r}} = ln(MC_t^{x^r}) - ln(MC^{x^r}) = ln(MC_t^{x}) - ln(e_t^{Gel/D}) - ln(P_t^{xf}) - ln(MC^{x^r}) \tag{F.5.26}$$

Where, MC_t^x is nominal marginal cost, and $e_t^{Gel/D}$ is exchange rate of local currency against USD. While P_t^{xf} is the price of export goods in USD (something that is sticky in this sector).

Appendix G Law of Motions of Price Dispersion

As an example, we derive the law of motion of price dispersion in the export sector, since derivations are similar to other sectors and the same transformation could be applied to wage dispersion. As mentioned above (see, 2.10.15) the price dispersion in the export sector is given by:

$$\begin{aligned} d_t^x =& \int_0^1 \left(\frac{P_t(i)^{xf}}{P_t^{xf}}\right)^{-\varepsilon_t^x} di = \int_0^{\theta_x} \left(\frac{P_{t-1}(i)^{xf}\Pi_{t-1}^{xf}}{P_t^{xf}}\right)^{-\varepsilon_t^x} di + \int_{\theta_x}^1 \left(\frac{P_t(i)^{*xf}}{P_t^{xf}}\right)^{-\varepsilon_t^x} di = \\ =& \int_0^{\theta_x} \left(\frac{P_{t-1}(i)^{xf}P_{t-1}^{xf}\Pi_{t-1}^{xf}}{P_{t-1}^{xf}P_t^{xf}}\right)^{-\varepsilon_t^x} di + (1-\theta_x)\left(\frac{P_t(i)^{*xf}}{P_t^{xf}}\right)^{-\varepsilon_t^x} = \\ =&(1-\theta_x)\left(\frac{P_t^{*xf}}{P_t^{xf}}\right)^{-\varepsilon_t^x} + \theta_x {\Pi_{t-1}^{xf}}^{-\varepsilon_t^x} {\Pi_t^{xf}}^{\varepsilon_t^x} d_{t-1}^{xf} \end{aligned} \tag{G.0.1}$$

At the first stage of the derivations, we apply assumption that the optimaizers are random sample from the continuum of firms.

Appendix H Derivation of Modified UIP

As shown in (2.5.1), the problem of the forex dealers is:

$$\max_{B_t^f} E_0 \sum_{t=0}^{\infty} B_t^f \left\{ \lambda_{t+1} e_{t+1}^{Gel/D} R_t^f R_t^\rho exp\left(-\xi_{dl}\left(b_t^f - b^{f^{ss}}\right) - \xi^{fp}\left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1\right)\right) - \lambda_t e_t^{Gel/D} \right\} \tag{H.1}$$

FOC of which yields:

$$E_0 \lambda_{t+1} e_{t+1}^{Gel/D} R_t^f R_t^\rho exp\left(-\xi_{dl} \left(b_t^f - b^{f^{ss}} \right) - \xi^{fp} \left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1 \right) \right) = \lambda_t e_t^{Gel/D} \tag{H.2}$$

By dividing both sides of the equation on $e_t^{Gel/D}$ and λ_t additionally noting that $\frac{e_t^{Gel/D}}{e_{t-1}^{Gel/D}} = (1 + \gamma_t^{e^{Gel/D}})$ and $\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{R_t}$, we write:

$$R_t = E_0 (1 + \gamma_{t+1}^{e^{Gel/D}}) R_t^f R_t^\rho exp\left(-\xi^{dl} (b_t^f - b^f) - \xi^{fp} \left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1 \right) \right) \tag{H.3}$$

Now let's log linearize the equilibrium condition of the forex dealer's maximization problem:

$$\exp^{ln\left(\frac{E_0 \left(1+\gamma_{t+1}^{e^{Gel/D}}\right) R_t^f R_t^\rho exp\left(-\xi^{dl} (b_t^f - b^f) - \xi^{fp} \left(\left(1+\gamma_{t+1}^{e^{Gel/D}}\right) \left(1+\gamma_t^{e^{Gel/D}}\right) - 1 \right) \right)}{R_t} \right)} \approx$$

$$\approx 1 + i_t^f - i_t + i_t^\rho + E_0 \gamma_{t+1}^{e^{Gel/D}} - \xi^{dl} (b_t^f - b^f) - \xi^{fp} \left(E_0 \gamma_{t+1}^{e^{Gel/D}} + \gamma_t^{e^{Gel/D}} \right) \tag{H.4}$$

where, i_t^ρ is the net risk premium. The previous expression equals to 1 in SS, then the up to first order the UIP condition could be written as:

$$i_t = i_t^f + i_t^\rho + E_0 \gamma_{t+1}^{e^{Gel/D}} - \xi^{fp} \left(E_0 \gamma_{t+1}^{e^{Gel/D}} + \gamma_t^{e^{Gel/D}} \right) - \xi^{dl} (b_t^f - b^f) \tag{H.5}$$

The linear version of modified UIP condition involves one more term relative to standard UIP condition (with debt elastic risk premium). The deviation implies that the excess return could be earned going long a higher yielding currency.

Appendix I Full model economy

I.1 Non-linear Equilibrium Conditions

Household sector. *Euler equation*

$$R_t = \frac{E_t \psi_t (C_{t+1}^{uc} - h C_t^{uc}) \Pi_{t+1}^c}{E_t \beta \psi_{t+1} (C_t^{uc} - h C_{t-1}^{uc})} \tag{I.1.1}$$

Final consumption goods inflation

$$\Pi_t^c = \frac{P_t^c}{P_{t-1}^c} \tag{I.1.2}$$

Budget constraint of constrained HHs.

$$(1+\tau^c)P_t^c C_t^c = (1-\tau^w)W_t L_t + T_t^c \tag{I.1.3}$$

Aggregate consumption

$$C_t = (1-\lambda)C_t^{uc} + \lambda C_t^c \tag{I.1.4}$$

Optimal wage

$$\left(\frac{W_t^*}{W_t}\right)^{-(1+\eta_t^l\zeta)} = \frac{(\eta_t^l - 1)(1-\tau^w)}{\eta_t^l(1+\tau^c)}\frac{C_{1t}}{C_{2t}} \tag{I.1.5}$$

$$C_{1t} = \frac{\psi_t}{C_t^{uc} - hC_{t-1}^{uc}} W_t^r L_t + \beta\theta_w E_t \left(\frac{\Pi_t^w}{\Pi_{t+1}^w}\right)^{1-\eta_t^l} C_{1t+1} \tag{I.1.6}$$

$$C_{2t} = \chi\theta_t L_t^{1+\zeta} + \beta\theta_w E_t \left(\frac{\Pi_t^w}{\Pi_{t+1}^w}\right)^{-\eta_t^l(1+\zeta)} C_{2t+1} \tag{I.1.7}$$

Aggregate wage dynamic

$$\Pi_t^w = \Pi_{t-1}^w + \frac{1-\theta_w}{\theta_w}\Pi_w \left(\frac{W_t^*}{W_t} - 1\right) \tag{I.1.8}$$

Wage inflation

$$\Pi_t^w = \frac{W_t}{W_{t-1}} \tag{I.1.9}$$

The real wage

$$W_t^r = \frac{W_t}{P_t^c} \tag{I.1.10}$$

Preference, labor supply and elasticity of substitution shocks

$$\psi_t = (1-\rho_\psi)\psi + \rho_\psi \psi_{t-1} + \varepsilon_t^\psi \tag{I.1.11}$$

$$\theta_t = (1+\rho_\theta)\theta + \rho_\theta \theta_{t-1} + \varepsilon_t^\theta \tag{I.1.12}$$

$$\eta_t^l = (1-\rho^{\eta^l})\eta^l + \rho^{\eta^l} \eta_t^l + \varepsilon_t^{\eta^l} \tag{I.1.13}$$

Entrepreneurs. *Equilibrium conditions*

$$
\begin{aligned}
R_t^k &= \gamma'(u_t) P_t^i && \text{(I.1.14)}\\
P_t^i &= \lambda_t^e \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right)\\
&\quad + E_t\left[\frac{\beta\psi_{t+1}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t \Pi_{t+1}^c (C_{t+1}^{uc} - hC_t^{uc})}\lambda_{t+1}^e \tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\frac{I_{t+1}^2}{I_t^2}\right] && \text{(I.1.15)}\\
\lambda_t^e &= E_t\left[\frac{\beta\psi_{t+1}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t \Pi_{t+1}^c (C_{t+1}^{uc} - hC_t^{uc})}\left(R_{t+1}^k u_{t+1} - \gamma(u_{t+1}) p_{t+1}^i\right)\right]\\
&\quad + (1-\delta) E_t\left[\frac{\beta\psi_{t+1}(C_t^{uc} - hC_{t-1}^{uc})}{\psi_t \Pi_{t+1}^c (C_{t+1}^{uc} - hC_t^{uc})}\lambda_{t+1}^e\right] && \text{(I.1.16)}\\
\overline{K}_{t+1} &= (1-\delta)\overline{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t && \text{(I.1.17)}
\end{aligned}
$$

Functional forms of *capital utilization and investment adjustment costs*

$$\gamma(u_t) = 0.5\sigma_a\sigma_b u_t^2 + \sigma_b(1-\sigma_a)u_t + \sigma_b\left(\frac{\sigma_a}{2} - 1\right) \tag{I.1.18}$$

i.e. its first-order derivative is given by:

$$\gamma\prime(u_t) = \sigma_a\sigma_b u_t + \sigma_b(1-\sigma_a) \tag{I.1.19}$$

$$
\begin{aligned}
\tilde{S}(x) &= \frac{1}{2}\left\{\exp\left[\sqrt{\tilde{S}''}\left(x - g^I\right)\right] + \exp\left[-\sqrt{\tilde{S}''}\left(x - g^I\right)\right] - 2\right\}\\
&= 0, \quad x = g^I && \text{(I.1.20)}\\
\tilde{S}'(x) &= \frac{1}{2}\sqrt{\tilde{S}''}\left\{\exp\left[\sqrt{\tilde{S}''}\left(x - g^I\right)\right] + \exp\left[-\sqrt{\tilde{S}''}\left(x - g^I\right)\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= 0, \quad x = g^I && \text{(I.1.21)}\\
\tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}\left(x-g^I\right)\right]+\exp\left[-\sqrt{\tilde{S}''}\left(x-g^I\right)\right]\right\}\\
&= \tilde{S}'', \quad x = g^I && \text{(I.1.22)}
\end{aligned}
$$

where,

$$
x = \frac{I_t}{I_{t-1}} \tag{I.1.23}
$$

and g^I is ss value of x, i.e. gross growth rate of investment goods.

Domestic intermediate goods producers. *Optimal price of domestic intermediate goods*

$$
\frac{P_t^{d*}}{P_t^d} = \frac{\eta_t^d}{\eta_t^d - 1}\frac{D_{1t}}{D_{2t}} \tag{I.1.24}
$$

$$
D_{1t} = \frac{\psi_t}{\left(C_t^{uc} - hC_{t-1}^{uc}\right)P_t^c}P_t^d Y_t^d MC_t^{r^d} + \theta_d \beta E_t\left(\frac{\Pi_t^d}{\Pi_{t+1}^d}\right)^{-\eta_t^d} D_{1t+1} \tag{I.1.25}
$$

$$
D_{2t} = \frac{\psi_t}{\left(C_t^{uc} - hC_{t-1}^{uc}\right)P_t^c}P_t^d Y_t^d + \theta_d \beta E_t\left(\frac{\Pi_t^d}{\Pi_{t+1}^d}\right)^{1-\eta_t^d} D_{2t+1} \tag{I.1.26}
$$

Aggregate price index

$$
\Pi_t^d = \Pi_{t-1}^d + \frac{1-\theta_d}{\theta_d}\Pi^d\left(\frac{P_t^{*d}}{P_t^d} - 1\right) \tag{I.1.27}
$$

Domestic inflation

$$
\Pi_t^d = \frac{P_t^d}{P_{t-1}^d} \tag{I.1.28}
$$

Marginal cost function

$$
MC_t^d = \frac{1}{\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}}\frac{1}{\gamma_t z_t^{\alpha_1}}W_t^{\alpha_1}R_t^{k\alpha_2}\left(a_t^x P_t^{mG}\right)^{1-\alpha_1-\alpha_2} \tag{I.1.29}
$$

$$
MC_t^{d^r} = \frac{MC_t^d}{P_t^d} \tag{I.1.30}
$$

Demand on labor input

$$L_t = \frac{1}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{{\alpha_1}^{1-\alpha_1}}{{\alpha_2}^{\alpha_2}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}} \right) \left(\frac{{R_t^k}^{\alpha_2} \left(a_t^x P_t^{mG} \right)^{1-\alpha_1-\alpha_2}}{W_t^{1-\alpha_1}} \right) \left(Y_t + F_t^d \right) \tag{I.1.31}$$

Demand on capital input

$$K_t = \frac{1}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{{\alpha_2}^{1-\alpha_2}}{{\alpha_1}^{\alpha_1}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}} \right) \left(\frac{W_t^{\alpha_1} \left(a_t^x P_t^{mG} \right)^{1-\alpha_1-\alpha_2}}{{R_t^k}^{1-\alpha_2}} \right) \left(Y_t + F_t^d \right) \tag{I.1.32}$$

Demand on imported intermediate input

$$Y_t^m = \frac{a_t^x}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{(1-\alpha_1-\alpha_2)^{\alpha_1+\alpha_2}}{{\alpha_1}^{\alpha_1}{\alpha_2}^{\alpha_2}} \right) \left(\frac{W_t^{\alpha_1} {R_t^k}^{\alpha_2}}{(a_t^x P_t^{mG})^{\alpha_1+\beta}} \right) \left(Y_t + F_t^d \right) \tag{I.1.33}$$

Exogenous TFP process

$$\gamma_t = (1-\rho^\gamma)\gamma + \rho^\gamma \gamma_{t-1} + \varepsilon_t^\gamma \tag{I.1.34}$$

Labor augmented productivity process

$$1 + \gamma_t^z = \frac{z_t}{z_{t-1}} \tag{I.1.35}$$

$$\gamma_t^z = (1-\rho_{\gamma^z})\gamma^z + \rho_{\gamma^z}\gamma_{t-1}^z + \varepsilon_t^z \tag{I.1.36}$$

Relative Inefficiency Technology of Imported Inputs

$$1 + \gamma_t^{a^x} = \frac{a_t^x}{a_{t-1}^x} \tag{I.1.37}$$

$$\gamma_t^{a^x} = (1-\rho_{\gamma^{a^x}})\gamma^{a^x} + \rho_{\gamma^{a^x}}\gamma_{t-1}^{a^x} + \varepsilon_t^{\gamma^{a^x}} \tag{I.1.38}$$

Elasticity of substitution

$$\eta_t^d = (1-\rho^{\eta^d})\eta^d + \rho^{\eta^d}\eta_{t-1}^d + \varepsilon_t^{\eta^d} \tag{I.1.39}$$

Final consumption goods production *Demand on domestic and imported inputs in the final consumption goods production*

$$C_t^d = (1-\omega_c)\left(\frac{P_t^d}{P_t^c}\right)^{-\eta_c} C_t \tag{I.1.40}$$

$$\frac{C_t^m}{a_t^x} = \omega_c \left(\frac{P_t^{mG} a_t^x}{P_t^c}\right)^{-\eta_c} C_t \tag{I.1.41}$$

Price index of final consumption goods

$$P_t^c = \left[(1-\omega_c)\, P_t^{d\,1-\eta_c} + \omega_c \left(P_t^{mG} a_t^x\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}} \tag{I.1.42}$$

Final investment goods production *Demand on domestic and imported investment goods*

$$I_t^d = (1-\omega_i)\left(\frac{P_t^d}{P_t^i}\right)^{-\eta_i} I_t \tag{I.1.43}$$

$$\frac{I_t^m}{a_t^x} = \omega_i \left(\frac{P_t^{mG} a_t^x}{P_t^i}\right)^{-\eta_i} I_t \tag{I.1.44}$$

Price index of final investment goods

$$P_t^i = \left[(1-\omega_i)\, P_t^{d\,1-\eta_i} + \omega_i \left(P_t^{mG} a_t^x\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} \tag{I.1.45}$$

Import sector *Optimal price index of imported goods*

$$\frac{P_t^{*mf}}{P_t^{mf}} = \frac{\varepsilon_t^m}{\varepsilon_t^m - 1}\frac{A_{1t}}{A_{2t}} \tag{I.1.46}$$

where

$$A_{1t} = P_t^{mf} M_t MC_t^{m^r} + \theta_m E_t \left(\frac{\Pi_t^{mf}}{\Pi_{t+1}^{mf}}\right)^{-\varepsilon_t^m} Q_{t,t+1}^f A_{1t+1} \tag{I.1.47}$$

and

$$A_{2t} = P_t^{mf} M_t + \theta_m E_t \left(\frac{\Pi_t^{mf}}{\Pi_{t+1}^{mf}}\right)^{1-\varepsilon_t^m} Q_{t,t+1}^f A_{2t+1} \tag{I.1.48}$$

Foreign discount factor

$$Q^f_{t,t+1} = \frac{1}{R^f_{t+1}} \tag{I.1.49}$$

Aggregate price index

$$\Pi^{mf}_t = \Pi^{mf}_{t-1} + \frac{1-\theta_m}{\theta_m}\Pi^{mf}\Big(\frac{P^{*mf}_t}{P^{mf}_t} - 1\Big) \tag{I.1.50}$$

Dollar price inflation of imported goods

$$\Pi^{mf}_t = \frac{P^{mf}_t}{P^{mf}_{t-1}} \tag{I.1.51}$$

Real marginal cost of imported goods

$$MC^{m^r}_t = \frac{P^c_t}{P^{mG}_t} REER_t \tag{I.1.52}$$

Real effective exchange rate

$$REER_t = \frac{e^{Gel/R}_t P^R_t}{P^c_t} \tag{I.1.53}$$

Nominal effective exchange rate

$$e^{Gel/R}_t = e^{Gel/D}_t e^{D/R}_t \tag{I.1.54}$$

Aggregate price index of imported goods in currency units

$$P^{mG}_t = e^{Gel/D}_t P^{mf}_t \tag{I.1.55}$$

Elasticity of substitution in import sector

$$\varepsilon^m_t = (1-\rho^{\varepsilon^m})\varepsilon^m + \rho^{\varepsilon^m}\varepsilon^m_{t-1} + \varepsilon^{\varepsilon^m}_t \tag{I.1.56}$$

Exported goods sector *Optimal price index of exported goods*

$$\frac{P^{*xf}_t}{P^{xf}_t} = \frac{\varepsilon^x_t}{\varepsilon^x_t - 1}\frac{B_{1t}}{B_{2t}} \tag{I.1.57}$$

where

$$B_{1t} = \frac{\psi_t}{(C_t^{uc} - hC_{t-1}^{uc})P_t^c} P_t^{xG} X_t MC_t^{x^r} + \theta_x \beta E_t \left(\frac{\Pi_t^{xf}}{\Pi_{t+1}^{xf}} \right)^{-\varepsilon_t^x} B_{1t+1} \tag{I.1.58}$$

and

$$B_{1t} = \frac{\psi_t}{(C_t^{uc} - hC_{t-1}^{uc})P_t^c} P_t^{xG} X_t + \theta_x \beta E_t \left(\frac{\Pi_t^{xf}}{\Pi_{t+1}^{xf}} \right)^{1-\varepsilon_t^x} B_{2t+1} \tag{I.1.59}$$

Marginal cost in exported goods sector

$$MC_t^x = a_t^{r - \frac{2}{\eta_x - 1}} \left[\omega_x \left(\frac{P_t^d}{a_t^x} \right)^{1-\eta_x} + (1-\omega_x)(P_t^{mG})^{1-\eta_x} \right]^{\frac{1}{1-\eta_x}} \tag{I.1.60}$$

Real marginal cost

$$MC_t^{x^r} = \frac{MC_t^x}{P_t^{xG}} \tag{I.1.61}$$

Price index of exported goods in domestic currency

$$P_t^{xG} = P_t^{xf} e_t^{Gel/D} \tag{I.1.62}$$

Export-specific technology

$$1 + \gamma_t^{a^r} = \frac{a_t^r}{a_{t-1}^r} \tag{I.1.63}$$

$$\gamma_t^{a^r} = (1 - \rho_{\gamma^{a^r}})\gamma^{a^r} + \rho_{\gamma^{a^r}} \gamma_{t-1}^{a^r} + \varepsilon_t^{\gamma^{a^r}} \tag{I.1.64}$$

Aggregate price index in exported goods sector

$$\Pi_t^{xf} = \Pi_{t-1}^{xf} + \frac{1-\theta_x}{\theta_x} \Pi^{xf} \Big(\frac{P_t^{*xf}}{P_t^{xf}} - 1 \Big) \tag{I.1.65}$$

Exported goods inflation

$$\Pi_t^{xf} = \frac{P_t^{xf}}{P_{t-1}^{xf}} \tag{I.1.66}$$

Foreign demand on exported goods

$$X_t = \omega_w \alpha_t \left(\frac{P_t^{xf}}{P_t^*} \right)^{-\epsilon_x} Y_t^* \tag{I.1.67}$$

Demand on domestic and imported inputs in exported goods production

$$X_t^d a_t^x = (1-\omega_x) a_t^{r2} \left(\frac{P_t^d / a_t^x}{MC_t^x} \right)^{-\eta_x} X_t \tag{I.1.68}$$

$$X_t^m = \omega_x a_t^{r2} \left(\frac{P_t^{mG}}{MC_t^x} \right)^{-\eta_x} X_t \tag{I.1.69}$$

Foreigners' preference on exported goods

$$\alpha_t = (1-\rho_\alpha)\alpha + \rho_\alpha \alpha_{t-1} + \varepsilon_t^\alpha \tag{I.1.70}$$

Elasticity of substitution

$$\varepsilon_t^x = (1+\rho^{\varepsilon^x})\varepsilon^x + \rho^{\varepsilon^x} \varepsilon_{t-1}^x + \varepsilon_t^{\varepsilon^x} \tag{I.1.71}$$

UIP condition: *USD vs GEL*

$$R_t = E_0 R_t^f R_t^\rho (1+\gamma_{t+1}^{e^{Gel/D}}) exp \left(-\xi^{dl}(b_t^f - b^{f^{ss}}) - \xi^{fp} \left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1 \right) \right) \tag{I.1.72}$$

USD vs ROW

$$\frac{(1+i_t^{rw})(1+\gamma_t^{e^{R/D}})}{1+i_t^f} = \exp(\rho^{rwuip}((1+\gamma_t^{R/D})(1+E_t\gamma_{t+1}^{R/D}) - 1)) \tag{I.1.73}$$

Exogenous risk premium

$$R_t^\rho = (1-\rho_{prem})R^\rho + \rho_{prem} R_{t-1}^\rho + \eta_t \tag{I.1.74}$$

Fiscal sector *Primary balance rule*

$$gb_t = \rho_b gb_{t-1} + \phi\,(d_t - d) + u_t^g \tag{I.1.75}$$

$$gb_t = \frac{1}{P_t^d Y_t}\left(T_t - G_t - TR_t\right) \tag{I.1.76}$$

Law of motion of public debt (budget constraint of the government)

$$d_t = (1 + i_{t-1})\frac{1}{\Pi_t^d(1+\gamma_t^y)}d_{t-1} - gb_t \tag{I.1.77}$$

Tax revenue

$$T_t = \tau^c P_t^c C_t + \tau^w W_t L_t + \tau^{\pi r}\pi r_t^T \tag{I.1.78}$$

Public goods production

$$P_t^g Y_t^g = G_t \tag{I.1.79}$$

Demand on inputs in public goods production

$$G_t^d = (1-\omega_g)\left(\frac{P_t^d}{P_t^g}\right)^{-\eta_g} Y_t^G \tag{I.1.80}$$

$$\frac{G_t^m}{a_t^x} = \omega_g\left(\frac{P_t^{mG}a_t^x}{P_t^g}\right)^{-\eta_g} Y_t^G \tag{I.1.81}$$

Aggregate price index of public goods

$$P_t^g = \left((1-\omega_g)P_t^{d\,1-\eta_g} + \omega_g\left(P_t^{mG}a_t^x\right)^{1-\eta_g}\right)^{\frac{1}{1-\eta_g}} \tag{I.1.82}$$

Government spending shock

$$u_t^g = \rho_u u_{t-1}^g + \varepsilon_t^{u^g} \tag{I.1.83}$$

Monetary policy *Monetary policy rule*

$$i_t = \delta_1 i_{t-1} + (1-\delta_1)\left[i_t^N + \delta_2 E_t\left(\pi_{4.t+4} - \pi_{t+4}^{tar}\right)\right] + \epsilon_t^i \tag{I.1.84}$$

$$R_t = \frac{1}{1+i_t} \tag{I.1.85}$$

The intermediate target of monetary policy

$$\pi_{4,t} = 4(\Pi_t^c - 1) \tag{I.1.86}$$

Real neutral interest rate

$$1 + r_t^{nut} = \rho^r(1 + r_{t-1}^{nut}) + (1 - \rho^r)\frac{1}{1 + \gamma_t^{ax}}(1 + r_t^{fnut})R_t^{\rho^{nut}} + \varepsilon_t^{rnut} \tag{I.1.87}$$

Nominal neutral interest rate

$$1 + i_t^{nut} = E_t(1 + \pi_t^{exp})(1 + r_t^{nut}) \tag{I.1.88}$$

The expected inflation

$$\begin{aligned}\pi_t^{exp} =& \rho^{exp1}\pi_{t-1}^{exp} + (1 - \rho^{exp1})(\omega^\pi \pi_{t-1}^c + (1 - \omega^\pi)(\rho^{exp2}\pi_{t+1}^c + \\ &+(1 - \rho^{exp2})\pi_t^{tar})) + \varepsilon_t^{exp}\end{aligned} \tag{I.1.89}$$

Trend component of sovereign risk premium

$$R_t^{\rho^{nut}} = \rho^{\rho nut}R_{t-1}^{\rho^{nut}} + (1 - \rho^{\rho nut})R^{\rho^{nut}} + \varepsilon_t^{\rho nut} \tag{I.1.90}$$

Total sovereign risk premium

$$R_t^\rho = R_t^{\rho^{nut}}\widehat{R_t^\rho} \tag{I.1.91}$$

Inflation target

$$\pi_t^{tar} = \pi_{t-1}^{tar} + \epsilon_t^{tar} \tag{I.1.92}$$

Monetary policy shock

$$\epsilon_t^i = \rho_i \epsilon_{t-1}^i + \varepsilon_t^i \tag{I.1.93}$$

Balance of payment *Balance of payment identity*

$$B_t^f = CA_t + R_t^f R_t^\rho exp\left(-\xi^{dl}(b_t^f - b^f) - \xi^{fp}\left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1\right)\right)B_{t-1}^f \tag{I.1.94}$$

Definition of current account balance

$$CA_t = P_t^{xf}X_t - P_t^{mf}M_t \tag{I.1.95}$$

Foreign block *Definition of foreign inflation in trade partners currency units*

$$\Pi_t^R = \frac{P_t^R}{P_{t-1}^R} \tag{I.1.96}$$

Foreign inflation dynamic

$$\Pi_t^R = (1-\rho_{\Pi^R})\,\Pi^R + \rho_{\Pi^R}\Pi_{t-1}^R + \varepsilon_t^{\Pi^R} \tag{I.1.97}$$

Definition of foreign inflation in USD

$$\Pi_t^f = \frac{P_t^f}{P_{t-1}^f} \tag{I.1.98}$$

Foreign inflation (in USD)

$$\Pi_t^f = \rho_{\Pi^f}\Pi_{t-1}^f + (1-\rho_{\Pi^f})((1+\gamma_t^{e^{D/R}})\Pi_t^R) + \varepsilon_t^{\Pi^f} \tag{I.1.99}$$

Foreign interest rate (USD)

$$R_t^f = \frac{1}{1+i_t^f} \tag{I.1.100}$$

$$i_t^f = (1-\rho_{i^f})i^f + \rho_{i^f} i_{t-1}^f + \varepsilon_t^{i^f} \tag{I.1.101}$$

Foreign interest rate (ROW)

$$i_t^{rw} = \rho_{irw} i_{t-1}^{rw} + (1-\rho_{irw})i^{rw} + \varepsilon_t^{irw} \tag{I.1.102}$$

Foreign real neutral rate

$$r_t^{fnut} = \rho^{fnut} r_{t-1}^{fnut} + (1-\rho^{fnut})r^{fnut} + \varepsilon_t^{fnut} \tag{I.1.103}$$

foreign real rate

$$r_t^f = i_t^f - E_t\pi_{t+1}^f \tag{I.1.104}$$

The foreign real interest rate gap

$$r_t^f = r_t^{fnut} + \widehat{r_t^f} \tag{I.1.105}$$

Definition of foreign economic growth

$$(1 + \gamma_t^{Y^*}) = \frac{Y_t^*}{Y_{t-1}^*} \tag{I.1.106}$$

Foreign economy growth rate

$$\gamma_t^{Y^*} = (1 - \rho_{\gamma^{Y^*}})\gamma^{Y^*} + \rho_{\gamma^{Y^*}}\gamma_{t-1}^{Y^*} + \varepsilon_t^{\gamma^{Y^*}} \tag{I.1.107}$$

Rate of change of Dollar effective exchange rate

$$1 + \gamma_t^{e^{D/R}} = \frac{1 + \gamma_t^{Gel/R}}{1 + \gamma_t^{e^{Gel/D}}} \tag{I.1.108}$$

Market clearing and aggregated equilibrium conditions *Domestic intermediate goods market clears*

$$Y_t = d_t^d Y_t^d \tag{I.1.109}$$

Law of motion of domestic price dispersion

$$d_t^d = (1 - \theta_d)\left(\frac{P_t^{*d}}{P_t^d}\right)^{-\eta_t^d} + \theta_d {\Pi_{t-1}^d}^{-\eta_t^d} {\Pi_t^d}^{\eta_t^d} d_{t-1}^d \tag{I.1.110}$$

Labor market clears

$$L_t^s = d_t^w L_t \tag{I.1.111}$$

Law of motion of wage dispersion

$$d_t^w = (1 - \theta_w)\left(\frac{W_t^*}{W_t}\right)^{-\eta_l} + \theta_w {\Pi_{t-1}^w}^{-\eta_l} {\Pi_t^w}^{\eta_l} d_{t-1}^w \tag{I.1.112}$$

Wage of effective labor

$$w_t z_t = W_t \tag{I.1.113}$$

Capital market clears

$$K_t = u_t \overline{K}_t \tag{I.1.114}$$

Profit in domestic intermediate goods production

$$\pi r_t^d = P_t^d Y_t^d - R_t^k K_t - w_t(z_t L_t) - P_t^{mG} Y_t^m \tag{I.1.115}$$

Profit in export goods sector

$$\pi r_t^x = e_t^{Gel/D} P_t^{xf} X_t - P_t^d X_t^d - P_t^{mG} X_t^m \tag{I.1.116}$$

Aggregate profit functions of entrepreneurs, forex dealers, final consumption, investment and government goods producers:

$$\pi r_t^e = R_t^k \overline{K}_t u_t - \gamma(u_t) \overline{K}_t P_t^i - P_t^i I_t \tag{I.1.117}$$

$$\pi r_t^{fx} = e_t^{Gel/D} B_{t-1}^f R_t^f R_t^\rho \exp\left(-\xi^{dl}(b_t^f - b^f) - \xi^{fp}\left(\frac{e_{t+1}^{Gel/D}}{e_{t-11}^{Gel/D}} - 1\right)\right) - e_t^{Gel/D} B_t^f \tag{I.1.118}$$

$$\pi r_t^c = P_t^c C_t - P_t^d C_t^d - P_t^{mG} C_t^m \tag{I.1.119}$$

$$\pi r_t^i = P_t^i I_t - P_t^d I_t^d - P_t^{mG} I_t^m \tag{I.1.120}$$

$$\pi r_t^g = P_t^g G_t - P_t^d G_t^d - P_t^{mG} G_t^m \tag{I.1.121}$$

Total profit

$$\pi r_t^T = \pi r_t^d + \pi r_t^x + \pi r_t^e + \pi r_t^{fx} + \pi r_t^c + \pi r_t^i + \pi r_t^g \tag{I.1.122}$$

Aggregate nominal demand on domestic intermediate goods

$$P_t^d Y_t^d = P_t^d C_t^d + P_t^d I_t^d + P_t^d G_t^d + P_t^d X_t^d + \gamma(u_t) \overline{K}_t P_t^i \tag{I.1.123}$$

Aggregate demand on imported goods

$$M_t = Y_t^m + C_t^m + I_t^m + G_t^m + X_t^m \tag{I.1.124}$$

Definition of nominal gross domestic product

$$GDP_t = P_t^c C_t + P_t^g G_t + P_t^i I_t + (P_t^{xG} X_t - P_t^{mG} M_t) \tag{I.1.125}$$

GDP deflater.

$$P_t^Y = P_t^{cs_c} P_t^{gs_g} P_t^{is_I} \left(e_t^{Gel/D} P_t^x\right)^{s_x} \left(e_t^{Gel/D} P_t^{mf}\right)^{-s_m} \tag{I.1.126}$$

Real GDP:

$$GDP_t^r = \frac{GDP_t}{P_t^Y} \tag{I.1.127}$$

Definition of nominal absorption

$$ABS_t = P_t^c C_t + P_t^g G_t + P_t^i I_t \tag{I.1.128}$$

I.2 Stationary Equilibrium Conditions

To make the equilibrium conditions stationary, we apply the results of solving trends of model variables. The objective is to substitute model variables with their stationary components. As given in the trend cycle decomposition of the model variable, we denote the cyclical (stationary) component of any V_t variable as $\widetilde{V_t}$ while the trend of the variable is defined as $\overline{V_t}$, i.e. $V_t = \widetilde{V_t}\overline{V_t}$. The trend components of each model variable are derived in the appendix on solving trends of model variables;[31] After extracting those trend components we get equilibrium conditions into the stationary form.

[31]The online appendix on trend process is available upon your request

Households. Let's recall that the trend component of C_t^{uc} coincides z_t then the stationary form of the Euler equation is derived by applying the following steps:

$$R_t = \frac{E_t\psi_t\left(z_{t+1}\widetilde{C_{t+1}^{uc}} - hz_t\widetilde{C_t^{uc}}\right)\Pi_{t+1}^c}{E_t\beta\psi_{t+1}\left(z_t\widetilde{C_t^{uc}} - hz_{t-1}\widetilde{C_{t-1}^{uc}}\right)} = \frac{E_t\psi_t\left((1+\gamma_{t+1}^z)\widetilde{C_{t+1}^{uc}} - h\widetilde{C_t^{uc}}\right)\Pi_{t+1}^c}{E_t\beta\psi_{t+1}\left(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}}\right)} \tag{I.2.1}$$

Aggregate consumption

$$z_t\widetilde{C}_t = (1-\lambda)z_t\widetilde{C_t^{uc}} + \lambda z_t\widetilde{C_t^c}$$

implies

$$\widetilde{C}_t = (1-\lambda)\widetilde{C_t^{uc}} + \lambda\widetilde{C_t^c} \tag{I.2.2}$$

All variables in the optimal wage equation are already stationary:

$$\left(\frac{W_t^*}{W_t}\right)^{-(1+\eta_t^l\zeta)} = \frac{(\eta_t^l-1)(1-\tau^w)}{\eta_t^l(1+\tau^c)}\frac{C_{1t}}{C_{2t}} \tag{I.2.3}$$

The recursive form of C_{1t} and C_{2t} can be transformed as:

$$C_{1t} = \frac{\psi_t}{z_t\widetilde{C_t^{uc}} - hz_{t-1}\widetilde{C_{t-1}^{uc}}}\Pi_t^{w\eta_t^l}z_t\widetilde{W_t^r}L_t + \beta\theta_w E_t\frac{\Pi_t^w}{\Pi_{t+1}^w}C_{1t+1}$$

then

$$C_{1t} = \frac{\psi_t}{\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}}}\Pi_t^{w\eta_t^l}\widetilde{W_t^r}L_t + \beta\theta_w E_t\frac{\Pi_t^w}{\Pi_{t+1}^w}C_{1t+1} \tag{I.2.4}$$

Also,

$$C_{2t} = \chi\theta_t\left(\Pi_t^{w\eta_t^l}L_t\right)^{1+\zeta} + \beta\theta_w E_t C_{2t+1} \tag{I.2.5}$$

All variables included in the equation have already been stationary and it does not need any transformation.

Budget constraint of constrained HHs.

$$(1+\tau^c)C_t^c = (1-\tau^w)W_t^r L_t + T_t^{cr}$$

$$(1+\tau^c)z_t\widetilde{C_t^c} = (1-\tau^w)z_t\widetilde{W_t^r}L_t + z_t\widetilde{T_t^{cr}}$$

$$(1+\tau^c)\widetilde{C_t^c} = (1-\tau^w)\widetilde{W_t^r}L_t + \widetilde{T_t^{cr}} \tag{I.2.6}$$

The stationary component of the real wage could be written recursively as:

$$\widetilde{W_t^r} = \frac{W_t}{P_t^c z_t} = \frac{W_t}{P_t^c z_t}\frac{P_{t-1}^c z_{t-1}}{W_{t-1}}\frac{W_{t-1}}{P_{t-1}^c z_{t-1}} = \frac{\Pi_t^w}{\Pi_t^c(1+\gamma_t^z)}\widetilde{W_{t-1}^r} \tag{I.2.7}$$

Aggregate wage dynamic

$$\Pi_t^w = \Pi_{t-1}^w + \frac{1-\theta_w}{\theta_w}\Pi^w\left(\frac{W_t^*}{W_t} - 1\right) \tag{I.2.8}$$

Preference and labor supply shocks

$$\psi_t = (1-\rho_\psi)\psi + \rho_\psi\psi_{t-1} + \varepsilon_t^\psi \tag{I.2.9}$$

$$\theta_t = (1+\rho_\theta)\theta + \rho_\theta\theta_{t-1} + \varepsilon_t^\theta \tag{I.2.10}$$

Elasticity of substitution of labor inputs

$$\eta_t^l = (1-\rho^{\eta^l})\eta^l + \rho^{\eta^l}\eta_t^l + \varepsilon_t^{\eta^l} \tag{I.2.11}$$

Entrepreneurs The equation of real rental rate have already been stationary where P_t^i is the relative price of investment goods:

$$r_t^k = \gamma'(u_t)p_t^i \tag{I.2.12}$$

where,

$$\gamma'\left(u_t\right) = \sigma_a\sigma_b u_t + \sigma_b\left(1-\sigma_a\right) \tag{I.2.13}$$

$$\begin{aligned} p_t^i = {} & \tilde{\lambda}_t^e\left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)(1+\gamma_t^z)\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right) + \\ & + E_t\left[\frac{\beta\psi_{t+1}(z_t\widetilde{C_t^{uc}} - hz_{t-1}\widetilde{C_{t-1}^{uc}})}{\psi_t(z_{t+1}\widetilde{C_{t+1}^{uc}} - hz_t\widetilde{C_t^{uc}})}\tilde{\lambda}_{t+1}^e\tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)(1+\gamma_{t+1}^z)^2\frac{\widetilde{I_{t+1}}^2}{\widetilde{I}_t^2}\right] \end{aligned}$$

$$p_t^i = \tilde{\lambda}_t^e \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)(1+\gamma_t^z)\frac{\widetilde{I}_t}{\widetilde{I_{t-1}}}\right) +$$

$$+ E_t\left[\frac{\beta\psi_{t+1}(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}})}{\psi_t((1+\gamma_{t+1}^z)\widetilde{C_{t+1}^{uc}} - h\widetilde{C_t^{uc}})}\tilde{\lambda}_{t+1}^e \tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)(1+\gamma_{t+1}^z)^2 \frac{\widetilde{I_{t+1}}^2}{\widetilde{I}_t^2}\right] \tag{I.2.14}$$

$$\tilde{\lambda}_t^e = E_t\left[\frac{\beta\psi_{t+1}(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}})}{\psi_t((1+\gamma_{t+1}^z)\widetilde{C_{t+1}^{uc}} - h\widetilde{C_t^{uc}})}\left(r_{t+1}^k u_{t+1} - \gamma(u_{t+1})p_{t+1}^i\right)\right] +$$

$$+ (1-\delta)E_t\left[\frac{\beta\psi_{t+1}(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}})}{\psi_t((1+\gamma_{t+1}^z)\widetilde{C_{t+1}^{uc}} - h\widetilde{C_t^{uc}})}\tilde{\lambda}_{t+1}^e\right] \tag{I.2.15}$$

Where,

$$\gamma(u_t) = 0.5\sigma_a\sigma_b u_t^2 + \sigma_b(1-\sigma_a)u_t + \sigma_b\left(\frac{\sigma_a}{2} - 1\right) \tag{I.2.16}$$

The capital accumulation equation could be written in the following stationary form:

$$z_{t+1}\widetilde{\overline{K}_{t+1}} = (1-\delta)z_t\widetilde{\overline{K}}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)z_t\widetilde{I}_t$$

$$(1+\gamma_{t+1}^z)\widetilde{\overline{K}_{t+1}} = (1-\delta)\widetilde{\overline{K}}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)\widetilde{I}_t \tag{I.2.17}$$

Domestic intermediate input producers. *Optimal price of domestic intermediate input*

$$\frac{P_t^{d*}}{P_t^d} = \frac{\eta_t^d}{\eta_t^d - 1}\frac{D_{1t}}{D_{2t}} \tag{I.2.18}$$

The recursive form of D_{1t} and D_{2t} can be written as:

$$D_{1t} = \frac{\psi_t}{\left(z_t\widetilde{C_t^{uc}} - hz_{t-1}\widetilde{C_{t-1}^{uc}}\right)}\frac{P_t^d}{P_t^c}z_t\widetilde{Y_t^d}MC_t^{r^d} + \theta_d\beta E_t\left(\frac{\Pi_{t+1}^d}{\Pi_t^d}\right)^{\eta_t^d} D_{1t+1}$$

We can write the relative price of domestic intermediate inputs recursively:

$$p_t^d = \frac{P_t^d}{P_t^c} = \frac{P_t^d}{P_{t-1}^d}\frac{P_{t-1}^c}{P_t^c}\frac{P_{t-1}^d}{P_{t-1}^c} = \frac{\Pi_t^d}{\Pi_t^c}p_{t-1}^d \tag{I.2.19}$$

then

$$D_{1t} = \frac{\psi_t}{\left(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}}\right)} p_t^d \widetilde{Y_t^d} MC_t^{r^d} + \theta_d \beta \left(\frac{\Pi_{t+1}^d}{\Pi_t^d}\right)^{\eta_t^d} E_t D_{1t+1} \tag{I.2.20}$$

Also,

$$D_{2t} = \frac{\psi_t}{\left(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}}\right)} p_t^d \widetilde{Y_t^d} + \theta_d \beta E_t \left(\frac{\Pi_{t+1}^d}{\Pi_t^d}\right)^{\eta_t^d - 1} D_{2t+1} \tag{I.2.21}$$

Aggregate price index

$$\Pi_t^d = \Pi_{t-1}^d + \frac{1-\theta_d}{\theta_d} \Pi^d \left(\frac{P_t^{*d}}{P_t^d} - 1\right) \tag{I.2.22}$$

Real marginal cost in domestic intermediate input production

$$\begin{aligned} \frac{MC_t^d}{P_t^d} = & \frac{1}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{P_t^c}{P_t^d} z_t \widetilde{W_t^r}\right)^{\alpha_1} \left(\frac{P_t^c}{P_t^d} r_t^k\right)^{\alpha_2} \left(a_t^x \frac{P_t^c}{P_t^d} \frac{P_t^{mG}}{P_t^c}\right)^{1-\alpha_1-\alpha_2} \times \\ & \times \left(\frac{1}{\alpha_1^{\alpha_1} \alpha_2^{\alpha_2} (1-\alpha_1^{\alpha_1} - \alpha_2^{\alpha_2})^{1-\alpha_1^{\alpha_1} - \alpha_2^{\alpha_2}}}\right) \end{aligned}$$

The relative price of import can be written as:

$$p_t^{mG} = \frac{P_t^{mG} a_t^x}{P_t^c} = \frac{P_t^{mG} a_t^x}{P_t^c} \frac{P_{t-1}^C}{P_{t-1}^{mG} a_{t-1}^x} \frac{P_{t-1}^{mG} a_{t-1}^x}{P_{t-1}^c} = \frac{\Pi_t^{mG} (1+\gamma_t^{a^x})}{\Pi_t^c} p_{t-1}^{mG} \tag{I.2.23}$$

then

$$\widetilde{MC_t^{d^r}} = \left(\frac{1}{\alpha_1^{\alpha_1} \alpha_2^{\alpha_2} (1-\alpha_1^{\alpha_1} - \alpha_2^{\alpha_2})^{1-\alpha_1^{\alpha_1} - \alpha_2^{\alpha_2}}}\right) \frac{1}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{1}{p_t^d} z_t \widetilde{W_t^r}\right)^{\alpha_1} \left(\frac{1}{p_t^d} r_t^k\right)^{\alpha_2} \left(\frac{p_t^{mG}}{p_t^d}\right)^{1-\alpha_1-\alpha_2}$$

finally,

$$\widetilde{MC_t^{d^r}} = \left(\frac{1}{\alpha_1^{\alpha_1} \alpha_2^{\alpha_2} (1-\alpha_1^{\alpha_1} - \alpha_2^{\alpha_2})^{1-\alpha_1^{\alpha_1} - \alpha_2^{\alpha_2}}}\right) \frac{1}{\gamma_t} \left(\widetilde{W_t^r}\right)^{\alpha_1} \left(r_t^k\right)^{\alpha_2} \left(p_t^{mG}\right)^{1-\alpha_1-\alpha_2} \frac{1}{p_t^d} \tag{I.2.24}$$

Demand on labor input

$$L_t = \frac{1}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{{\alpha_1}^{1-\alpha_1}}{{\alpha_2}^{\alpha_2} (1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}}\right) \left(\frac{\frac{R_t^k}{P_t^c}^{\alpha_2} \left(\frac{a_t^x P_t^{mG}}{P_t^c}\right)^{1-\alpha_1-\alpha_2}}{\left(\frac{W_t}{P_t^c}\right)^{1-\alpha_1}}\right) z_t \left(\widetilde{Y}_t + \widetilde{F_t^d}\right)$$

$$L_t = \frac{1}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{{\alpha_1}^{1-\alpha_1}}{{\alpha_2}^{\alpha_2}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}} \right) \left(\frac{{r_t^k}^{\alpha_2} \left(p_t^{mG}\right)^{1-\alpha_1-\alpha_2}}{\left(z_t \widetilde{W_t^r}\right)^{1-\alpha_1}} \right) z_t \left(\widetilde{Y_t} + \widetilde{F_t^d} \right)$$

Finally, after substituting marginal cost into the previous equation we get:

$$L_t = \alpha_1 \frac{\widetilde{MC_t^{r^d} p_t^d}}{\widetilde{W_t^r}} \left(\widetilde{Y_t} + \widetilde{F_t^d} \right) \tag{I.2.25}$$

Also, the *Demand on capital input*

$$z_t \widetilde{K_t} = \frac{1}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{{\alpha_2}^{1-\alpha_2}}{{\alpha_1}^{\alpha_1}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}} \right) \left(\frac{\left(z_t \widetilde{W_t^r}\right)^{\alpha_1} \left(p_t^{mG}\right)^{1-\alpha_1-\alpha_2}}{{r_t^k}^{1-\alpha_2}} \right) z_t \left(\widetilde{Y_t} + \widetilde{F_t^d} \right)$$

$$\widetilde{K_t} = \alpha_2 \frac{\widetilde{MC_t^{r^d} p_t^d}}{r_t^k} \left(\widetilde{Y_t} + \widetilde{F_t^d} \right) \tag{I.2.26}$$

And, *Demand on imported intermediate input*

$$a_t^x z_t \widetilde{Y_t^m} = \frac{a_t^x}{\gamma_t {z_t}^{\alpha_1}} \left(\frac{(1-\alpha_1-\alpha_2)^{\alpha_1+\alpha_2}}{{\alpha_1}^{\alpha_1}{\alpha_2}^{\alpha_2}} \right) \left(\frac{\left(z_t \widetilde{W_t^r}\right)^{\alpha_1} {r_t^k}^{\alpha_2}}{(p_t^{mG})^{\alpha_1+\alpha_2}} \right) z_t \left(\widetilde{Y_t} + \widetilde{F_t^d} \right)$$

Finally,

$$\widetilde{Y_t^m} = (1-\alpha_1-\alpha_2) \frac{\widetilde{MC_t^{r^d} p_t^d}}{p_t^{mG}} \left(\widetilde{Y_t} + \widetilde{F_t^d} \right) \tag{I.2.27}$$

Exogenous TFP process

$$\gamma_t = (1-\rho^\gamma)\gamma + \rho^\gamma \gamma_{t-1} + \varepsilon_t^\gamma \tag{I.2.28}$$

Labor augmented productivity process (growth rate)

$$\gamma_t^z = (1-\rho_{\gamma^z})\gamma^z + \rho_{\gamma^z} \gamma_{t-1}^z + \varepsilon_t^z \tag{I.2.29}$$

Elasticity of substitution

$$\eta_t^d = (1-\rho^{\eta^d})\eta^d + \rho^{\eta^d} \eta_{t-1}^d + \varepsilon_t^{\eta^d} \tag{I.2.30}$$

Final goods producers. *The demand on domestic intermediate input in final consumption goods production*

$$z_t\widetilde{C_t^d} = (1-\omega_c)p_t^{d^{-\eta_c}} z_t\widetilde{C}_t$$

$$\widetilde{C_t^d} = (1-\omega_c)p_t^{d^{-\eta_c}} \widetilde{C}_t \tag{I.2.31}$$

Demand on imported input

$$\frac{a_t^x z_t\widetilde{C_t^m}}{a_t^x} = \omega_c\left(\frac{P_t^{mG}a_t^x}{P_t^c}\right)^{-\eta_c} z_t\widetilde{C}_t$$

$$\widetilde{C_t^m} = \omega_c\left(p_t^{mG}\right)^{-\eta_c}\widetilde{C}_t \tag{I.2.32}$$

Price index of final consumption goods

$$\frac{P_t^c}{P_{t-1}^c} = \left[(1-\omega_c)\left(\frac{P_t^d}{P_{t-1}^d}\frac{P_{t-1}^d}{P_{t-1}^c}\right)^{1-\eta_c} + \omega_c\left(\frac{P_t^{mG}a_t^x}{P_{t-1}^{mG}a_{t-1}^x}\frac{P_{t-1}^{mG}a_{t-1}^x}{P_{t-1}^c}\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}$$

$$\Pi_t^c = \left[(1-\omega_c)\left(\Pi_t^d p_{t-1}^d\right)^{1-\eta_c} + \omega_c\left(\Pi_t^{mG}(1+\gamma_t^{a^x})p_{t-1}^{mG}\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}} \tag{I.2.33}$$

Inefficiency technology in imported input usage

$$\gamma_t^{a^x} = (1-\rho_{\gamma^{a^x}})\gamma^{a^x} + \rho_{\gamma^{a^x}}\gamma_{t-1}^{a^x} + \varepsilon_t^{\gamma^{a^x}} \tag{I.2.34}$$

Demand on domestic input in final investment goods production

$$z_t\widetilde{I}_t^d = (1-\omega_i)\left(\frac{P_t^d}{P_t^c}\frac{P_t^c}{P_t^i}\right)^{-\eta_i} z_t\widetilde{I}_t$$

$$\widetilde{I}_t^d = (1-\omega_i)\left(\frac{p_t^d}{p_t^i}\right)^{-\eta_i}\widetilde{I}_t \tag{I.2.35}$$

Demand on imported input

$$\frac{a_t^x z_t\widetilde{I_t^m}}{a_t^x} = \omega_i\left(\frac{P_t^{mG}a_t^x}{P_t^c}\frac{P_t^c}{P_t^i}\right)^{-\eta_i} z_t\widetilde{I}_t$$

$$\widetilde{I_t^m} = \omega_i \left(\frac{p_t^{mG}}{p_t^i}\right)^{-\eta_i} \widetilde{I}_t \tag{I.2.36}$$

Price index of final investment goods

$$\frac{P_t^i}{P_{t-1}^i} = \left[(1-\omega_i)\left(\frac{P_t^d}{P_{t-1}^d}\frac{P_{t-1}^d}{P_{t-1}^c}\frac{P_{t-1}^c}{P_{t-1}^i}\right)^{1-\eta_i} + \omega_i\left(\frac{P_t^{mG}a_t^x}{P_{t-1}^{mG}a_{t-1}^x}\frac{P_{t-1}^{mG}a_{t-1}^x}{P_{t-1}^c}\frac{P_{t-1}^c}{P_{t-1}^i}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}$$

$$\Pi_t^i = \left[(1-\omega_i)\left(\Pi_t^d\frac{p_{t-1}^d}{p_{t-1}^i}\right)^{1-\eta_i} + \omega_i\left(\Pi_t^{mG}(1+\gamma_t^{a^x})\frac{p_{t-1}^{mG}}{p_{t-1}^i}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} \tag{I.2.37}$$

The relative price of final investment goods

$$p_t^i = \frac{P_t^i}{P_t^c} = \frac{P_t^i}{P_{t-1}^i}\frac{P_t^c}{P_{t-1}^c}\frac{P_{t-1}^i}{P_{t-1}^c} = \frac{\Pi_t^i}{\Pi_t^c}p_{t-1}^i \tag{I.2.38}$$

Import sector *Optimal price*

$$\frac{P_t^{*mf}}{P_t^{mf}} = \frac{\varepsilon_t^m}{\varepsilon_t^m - 1}\frac{A_{1t}/P_t^{mf}}{A_{2t}/P_t^{mf}}$$

Let's define:

$$a_{1t} = \frac{A_{1t}}{P_t^{mf}}$$

$$z_t a_t^x \widetilde{a_{1t}} = z_t a_t^x \widetilde{M_t} MC_t^{m^r} + \theta_m E_t Q_{t,t+1}^f \left(\frac{\Pi_{t+1}^{mf}}{\Pi_t^{mf}}\right)^{\varepsilon_t^m} \frac{A_{1t+1}}{P_{t+1}^{mf}}\frac{P_{t+1}^{mf}}{P_t^{mf}}$$

$$z_t a_t^x \widetilde{a_{1t}} = z_t a_t^x \widetilde{M_t} MC_t^{m^r} + \theta_m E_t Q_{t,t+1}^f z_{t+1} a_{t+1}^x \left(\frac{\Pi_{t+1}^{mf}}{\Pi_t^{mf}}\right)^{\varepsilon_t^m} \widetilde{a_{1t+1}} \Pi_{t+1}^{mf}$$

We assume that the foreign discount factor equals to the inverse of the foreign risk-free rate:

$$Q_{t,t+1}^f = \frac{1}{R_{t+1}^f}$$

Then

$$\widetilde{a_{1t}} = \widetilde{M_t} MC_t^{m^r} + \theta_m E_t \frac{1}{R_{t+1}^f}(1+\gamma_{t+1}^z)(1+\gamma_{t+1}^{a^x})\Pi_{t+1}^{mf}\left(\frac{\Pi_{t+1}^{mf}}{\Pi_t^{mf}}\right)^{\varepsilon_t^m} \widetilde{a_{1t+1}} \tag{I.2.39}$$

Also,

$$\widetilde{a_{2t}} = \widetilde{M_t} + \theta_m E_t \frac{1}{R^f_{t+1}} (1+\gamma^z_{t+1})(1+\gamma^{a^x}_{t+1}) \Pi^{mf}_t \left(\frac{\Pi^{mf}_{t+1}}{\Pi^{mf}_t} \right)^{\varepsilon^m_t - 1} \widetilde{a_{2t+1}} \tag{I.2.40}$$

The optimal price with stationary form:

$$\frac{P^{*mf}_t}{P^{mf}_t} = \frac{\varepsilon^m_t}{\varepsilon^m_t - 1} \frac{\widetilde{a_{1t}}}{\widetilde{a_{2t}}} \tag{I.2.41}$$

Real marginal cost

$$MC^{m^r}_t = \frac{P^c_t}{P^{mG}_t} \frac{1}{a^x_t} \widetilde{REER}_t$$

$$MC^{m^r}_t = \frac{1}{p^{mG}_t} \widetilde{REER}_t \tag{I.2.42}$$

Inflation (in USD) of imported goods

$$\Pi^{mf}_t = \Pi^{mf}_{t-1} + \frac{1-\theta_m}{\theta_m} \Pi^{mf} \Big(\frac{P^{*mf}_t}{P^{mf}_t} - 1 \Big) \tag{I.2.43}$$

Taking into account the trend component of the *real exchange rate* then its stationary component could be written as:

$$a^x_t REER_t = \frac{e^{Gel/R}_t P^R_t}{P^c_t} \frac{P^c_{t-1}}{e^{Gel/R}_{t-1} P^R_{t-1}} \frac{e^{Gel/R}_{t-1} P^R_{t-1}}{P^c_{t-1}} \frac{a^x_{t-1} a^x_t}{a^x_{t-1}}$$

$$\widetilde{REER}_t = \frac{(1+\gamma^{Gel/R}_t)(1+\gamma^{a^x}_t)\Pi^R_t}{\Pi^c_t} \widetilde{REER}_{t-1} \tag{I.2.44}$$

Nominal effective exchange rate

$$\frac{e^{Gel/R}_t}{e^{Gel/R}_{t-1}} = \frac{e^{Gel/D}_t}{e^{Gel/D}_{t-1}} \frac{e^{D/R}_t}{e^{D/R}_{t-1}}$$

$$(1+\gamma^{e^{Gel/R}}_t) = (1+\gamma^{e^{Gel/D}}_t)(1+\gamma^{e^{D/R}}_t) \tag{I.2.45}$$

Import price inflation in Lari

$$\frac{P^{mG}_t}{P^{mG}_{t-1}} = \frac{e^{Gel/D}_t}{e^{Gel/D}_{t-1}} \frac{P^{mf}_t}{P^{mf}_{t-1}}$$

$$\Pi_t^{mG} = (1+\gamma_t^{e^{Gel/D}})\Pi_t^{mf} \tag{I.2.46}$$

Elasticity of substitution in import sector

$$\varepsilon_t^m = (1-\rho^{\varepsilon^m})\varepsilon^m + \rho^{\varepsilon^m}\varepsilon_{t-1}^m + \varepsilon_t^{\varepsilon^m} \tag{I.2.47}$$

Export sector *optimal price*

$$\frac{P_t^{*xf}}{P_t^{xf}} = \frac{\varepsilon_t^x}{\varepsilon_t^x - 1}\frac{B_{1t}}{B_{2t}} \tag{I.2.48}$$

Recursive forms of B_{1t} and B_{2t}

$$B_{1t} = \frac{\psi_t}{(\widetilde{C_t^{uc}} - \frac{z_{t-1}}{z_t}h\widetilde{C_{t-1}^{uc}})z_t}\frac{P_t^{xG}}{P_t^c}\frac{a_t^x}{(a_t^r)^{\frac{2}{1-\eta_x}}}\frac{(a_t^r)^{\frac{2}{1-\eta_x}}}{a_t^x}a_t^{r\,\frac{-2\varepsilon^x}{1-\eta_x}}z_t^*\widetilde{X}_t MC_t^{x^r} + \theta_x\beta E_t\left(\frac{\Pi_{t+1}^{xf}}{\Pi_t^{xf}}\right)^{\varepsilon_t^x} B_{1t+1}$$

Let's denote

$$\begin{aligned} p_t^{xG} &= \frac{P_t^{xG}}{P_t^c}\frac{a_t^x}{(a_t^r)^{\frac{2}{1-\eta_x}}} = \frac{P_t^{xG}}{P_t^c}\frac{a_t^x}{(a_t^r)^{\frac{2}{1-\eta_x}}}\frac{P_{t-1}^c}{P_{t-1}^{xG}}\frac{(a_{t-1}^r)^{\frac{2}{1-\eta_x}}}{a_{t-1}^x}\frac{P_{t-1}^{xG}}{P_{t-1}^c}\frac{a_{t-1}^x}{(a_{t-1}^r)^{\frac{2}{1-\eta_x}}} = \\ &= \frac{\Pi_t^{xG}}{\Pi_t^c}\frac{1+\gamma_t^{a^x}}{(1+\gamma_t^{a^r})^{\frac{2}{1-\eta_x}}}p_{t-1}^{xG} \end{aligned} \tag{I.2.49}$$

Then

$$B_{1t} = \frac{\psi_t}{(\widetilde{C_t^{uc}} - \frac{z_{t-1}}{z_t}h\widetilde{C_{t-1}^{uc}})z_t}p_t^{xG}\frac{(a_t^r)^{\frac{2(1-\varepsilon_t^x)}{1-\eta_x}}}{a_t^x}z_t^*\widetilde{X}_t MC_t^{x^r} + \theta_x\beta E_t\left(\frac{\Pi_{t+1}^{xf}}{\Pi_t^{xf}}\right)^{\varepsilon_t^x} B_{1t+1}$$

We define stationary relative technology as

$$\begin{aligned} \widetilde{a_t} &\equiv \frac{a_t^{r\,\frac{2(1-\varepsilon_t^x)}{1-\eta_x}}z_t^*}{a_t^x z_t} = \frac{a_t^{r\,\frac{2(1-\varepsilon_t^x)}{1-\eta_x}}z_t^*}{a_t^x z_t}\frac{a_{t-1}^x z_{t-1}}{a_{t-1}^{r\,\frac{2(1-\varepsilon_t^x)}{1-\eta_x}}z_{t-1}^*}\frac{a_{t-1}^{r\,\frac{2(1-\varepsilon_t^x)}{1-\eta_x}}z_{t-1}^*}{a_{t-1}^x z_{t-1}} = \\ &= \frac{(1+\gamma_t^{a^r})^{\frac{2(1-\varepsilon_t^x)}{1-\eta_x}}(1+\gamma_t^{z^*})}{(1+\gamma_t^{a^x})(1+\gamma_t^z)}\widetilde{a_{t-1}} \end{aligned} \tag{I.2.50}$$

Taking into account restrictions on trends (implied by stationarity of current account balance), i.e. $\frac{(1+\gamma^{a^r})^{\frac{2(1-\varepsilon^x)}{1-\eta_x}}(1+\gamma^{z^*})}{(1+\gamma^{a^x})} = (1+\gamma^z)$. The $\widetilde{a_t}$ is stationary auxiliary variable,

then

$$B_{1t} = \frac{\psi_t}{(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}})} p_t^{xG} \widetilde{a}_t \widetilde{X}_t MC_t^{x^r} + \theta_x \beta E_t \left(\frac{\Pi_{t+1}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon_t^x} B_{1t+1} \tag{I.2.51}$$

By applying same steps to B_{2t}, we get:

$$B_{2t} = \frac{\psi_t}{(\widetilde{C_t^{uc}} - \frac{h}{1+\gamma_t^z}\widetilde{C_{t-1}^{uc}})} p_t^{xG} \widetilde{a}_t \widetilde{X}_t + \theta_x \beta E_t \left(\frac{\Pi_{t+1}^{xf}}{\Pi_t^{xf}} \right)^{\varepsilon_t^x - 1} B_{2t+1} \tag{I.2.52}$$

Marginal cost in exported goods sector

$$\frac{MC_t^x}{P_t^{xG}} = a_t^{r - \frac{2}{\eta_x - 1}} \left[(1-\omega_x) \left(\frac{P_t^d}{P_t^c} \frac{P_t^c}{a_t^x P_t^{xG}} a_t^{r \frac{2}{1-\eta_x}} a_t^{r \frac{-2}{1-\eta_x}} \right)^{1-\eta_x} + \omega_x \left(\frac{P_t^{mG} a_t^x}{P_t^c} \frac{P_t^c}{a_t^x P_t^{xG}} a_t^{r \frac{2}{1-\eta_x}} a_t^{r \frac{-2}{1-\eta_x}} \right)^{1-\eta_x} \right]^{\frac{1}{1-\eta_x}}$$

$$MC_t^{x^r} = \left[(1-\omega_x) \left(\frac{p_t^d}{p_t^{xG}} \right)^{1-\eta_x} + \omega_x \left(\frac{p_t^{mG}}{p_t^{xG}} \right)^{1-\eta_x} \right]^{\frac{1}{1-\eta_x}} \tag{I.2.53}$$

Exported goods inflation in USD

$$\Pi_t^{xf} = \Pi_{t-1}^{xf} + \frac{1-\theta_x}{\theta_x} \Pi^{xf} \Big(\frac{P_t^{*xf}}{P_t^{xf}} - 1 \Big) \tag{I.2.54}$$

Exported goods inflation in Gel

$$\Pi_t^{xG} = (1 + \gamma_t^{e^{Gel/D}}) \Pi_t^{xf} \tag{I.2.55}$$

Demand on exported goods

$$a_t^{r \frac{-2\varepsilon_t^x}{1-\eta_x}} z_t^* \widetilde{X}_t = \omega_w \alpha_t \left(\frac{P_t^{xf}}{P_t^*} a_t^{r \frac{-2}{1-\eta_x}} a_t^{r \frac{2}{1-\eta_x}} \right)^{-\varepsilon_t^x} z_t^* \widetilde{Y}_t^*$$

We define the relative price of exported goods (in USD) recursively:

$$p_t^{xf} = \frac{P_t^{xf}}{P_t^*} a_t^{r \frac{-2}{1-\eta_x}} = \frac{P_t^{xf}}{P_t^*} a_t^{r \frac{-2}{1-\eta_x}} \frac{P_{t-1}^*}{P_{t-1}^{xf}} a_{t-1}^{r \frac{2}{1-\eta_x}} \frac{P_{t-1}^{xf}}{P_{t-1}^*} a_{t-1}^{r \frac{-2}{1-\eta_x}} = \frac{\Pi_t^{xf}}{\Pi_t^*} p_{t-1}^{xf} \tag{I.2.56}$$

By taking into account the relative price of exported goods, the demand function could be written as:

$$\widetilde{X}_t = \omega_x \alpha_t \left(p_t^{xf} \right)^{-\varepsilon_t^x} \widetilde{Y}_t^* \tag{I.2.57}$$

Demand on domestic inputs in exported goods sector

$$z_t^* a_t^{r\frac{2(1-\varepsilon_t^x)}{1-\eta_x}} \widetilde{X_t^d} = (1-\omega_x) a_t^{r2} \left(\frac{P_t^d}{P_t^c} \frac{P_t^c}{a_t^x P_t^{xG}} \frac{P_t^{xG}}{MC_t^x} \right)^{-\eta_x} a_t^{r\frac{-2\varepsilon_t^x}{1-\eta_x}} z_t^* \left(\widetilde{X_t} + \widetilde{F_t^x} \right)$$

$$z_t^* a_t^{r\frac{2(1-\varepsilon_t^x)}{1-\eta_x}} \widetilde{X_t^d} = (1-\omega_x) a_t^{r2} \left(p_t^d \frac{P_t^c}{a_t^x P_t^{xG}} a_t^{r\frac{2}{1-\eta_x}} a_t^{r\frac{-2}{1-\eta_x}} \frac{1}{MC_t^{x^r}} \right)^{-\eta_x} a_t^{r\frac{-2\varepsilon^x}{1-\eta_x}} z_t^* \left(\widetilde{X_t} + \widetilde{F_t^x} \right)$$

$$z_t^* a_t^{r\frac{2(1-\varepsilon_t^x)}{1-\eta_x}} \widetilde{X_t^d} = (1-\omega_x) \left(\frac{p_t^d}{p_t^{xG} MC_t^{x^r}} \right)^{-\eta_x} a_t^{r\frac{2}{1-\eta_x}} a_t^{r\frac{-2\varepsilon_t^x}{1-\eta_x}} z_t^* \left(\widetilde{X_t} + \widetilde{F_t^x} \right)$$

$$\widetilde{X_t^d} = (1-\omega_x) \left(\frac{p_t^d}{p_t^{xG} MC_t^{x^r}} \right)^{-\eta_x} \left(\widetilde{X_t} + \widetilde{F_t^x} \right) \tag{I.2.58}$$

Demand on imported input in exported goods production

$$z_t^* a_t^{r\frac{2(1-\varepsilon_t^x)}{1-\eta_x}} \widetilde{X_t^m} = \omega_x a_t^{r2} \left(\frac{P_t^{mG} a_t^x}{P_t^c} \frac{P_t^c}{P_t^{xG} a_t^x} a_t^{r\frac{2}{1-\eta_x}} a_t^{r\frac{-2}{1-\eta_x}} \frac{P_t^{xG}}{MC_t^x} \right)^{-\eta_x} a_t^{r\frac{-2\varepsilon_t^x}{1-\eta_x}} z_t^* \left(\widetilde{X_t} + \widetilde{F_t^x} \right)$$

$$z_t^* a_t^{r\frac{2(1-\varepsilon_t^x)}{1-\eta_x}} \widetilde{X_t^m} = \omega_x a_t^{r2} \left(\frac{p_t^{mG}}{p_t^{xG}} \frac{1}{MC_t^{x^r}} \right)^{-\eta_x} a_t^{\frac{2\eta_x}{1-\eta_x}} a_t^{r\frac{-2\varepsilon_t^x}{1-\eta_x}} z_t^* \left(\widetilde{X_t} + \widetilde{F_t^x} \right)$$

$$\widetilde{X_t^m} = \omega_x \left(\frac{p_t^{mG}}{p_t^{xG}} \frac{1}{MC_t^{x^r}} \right)^{-\eta_x} \left(\widetilde{X_t} + \widetilde{F_t^x} \right) \tag{I.2.59}$$

General technology in export production

$$\gamma_t^{a^r} = (1-\rho_{\gamma^{a^r}}) \gamma^{a^r} + \rho_{\gamma^{a^r}} \gamma_{t-1}^{a^r} + \varepsilon_t^{\gamma^{a^r}} \tag{I.2.60}$$

Elasticity of substitution of differentiated exported inputs

$$\varepsilon_t^x = (1+\rho^{\varepsilon^x}) \varepsilon^x + \rho^{\varepsilon^x} \varepsilon_{t-1}^x + \varepsilon_t^{\varepsilon^x} \tag{I.2.61}$$

Foreign preference shock

$$\alpha_t = \rho^\alpha \alpha_{t-1} + (1-\rho^\alpha)\alpha + \varepsilon_t^\alpha \tag{I.2.62}$$

Monetary policy *The monetary policy rule*

$$i_t = \delta_1 i_{t-1} + (1-\delta_1) \left[i_t^N + \delta_2 E_t \left(\pi_{t+4} - \pi_{t+4}^{tar} \right) \right] + \epsilon_t^i \tag{I.2.63}$$

Gross interest rate

$$R_t = \frac{1}{1+i_t} \tag{I.2.64}$$

Real neutral interest rate

$$1 + r_t^{nut} = \rho^r(1 + r_{t-1}^{nut}) + (1-\rho^r)\frac{1}{1+\gamma_t^{ax}}(1 + r_t^{fnut})R_t^{\rho^{nut}} + \varepsilon_t^{rnut} \tag{I.2.65}$$

Nominal neutral interest rate

$$1 + i_t^N = E_t(1 + \pi_t^{exp})(1 + r_t^{nut}) \tag{I.2.66}$$

The expected inflation

$$\begin{aligned}\pi_t^{exp} = &\rho^{exp1}\pi_{t-1}^{exp} + (1-\rho^{exp1})(\omega^\pi \pi_{t-1}^c + (1-\omega^\pi)(\rho^{exp2}\pi_{t+1}^c + \\ &+ (1-\rho^{exp2})\pi_t^{tar})) + \varepsilon_t^{exp}\end{aligned} \tag{I.2.67}$$

Trend component of sovereign risk premium

$$R_t^{\rho^{nut}} = \rho^{\rho nut}R_{t-1}^{\rho^{nut}} + (1-\rho^{\rho nut})R^{\rho^{nut}} + \varepsilon_t^{\rho nut} \tag{I.2.68}$$

Total sovereign risk premium

$$R_t^{\rho} = R_t^{\rho^{nut}}\widehat{R_t^{\rho}} \tag{I.2.69}$$

Monetary policy shock

$$\epsilon_t^i = \rho_i \epsilon_{t-1}^i + \varepsilon_t^i \tag{I.2.70}$$

Inflation target

$$\pi_t^{tar} = \pi_{t-1}^{tar} + \epsilon_t^{tar} \tag{I.2.71}$$

Fiscal Sector

$$gb_t = \rho_b gb_{t-1} + \phi\left(d_t - d\right) + u_t^g \tag{I.2.72}$$

All variables included in the budget balance rule are already stationary.

Government spending shock

$$u_t^g = \rho_u u_{t-1}^g + \varepsilon_t^{u^g} \tag{I.2.73}$$

Law of motion of public debt (budget constraint of the government)

$$d_t = (1 + i_{t-1}) \frac{1}{\Pi_t^d (1 + \gamma_t^y)} d_{t-1} - gb_t \tag{I.2.74}$$

Growth rate of output

$$1 + \gamma_t^y = (1 + \gamma_t^z) \frac{\widetilde{Y}_t}{\widetilde{Y_{t-1}}} \tag{I.2.75}$$

Definition of budget balance

$$gb_t = \frac{P_t^c}{P_t^d Y_t^d} \left(\frac{T_t}{P_t^c} - \frac{G_t}{P_t^c} - \frac{TR_t}{P_t^c} \right)$$

$$gb_t = \frac{1}{z_t p_t^d \widetilde{Y}_t} \left(z_t \widetilde{T_t^r} - z_t \widetilde{G}_t - z_t \widetilde{TR}_t \right)$$

$$gb_t = \frac{1}{p_t^d \widetilde{Y}_t} \left(\widetilde{T_t^r} - \widetilde{G}_t - \widetilde{TR}_t \right) \tag{I.2.76}$$

Tax revenue

$$\frac{T_t}{P_t^c} = \tau^c C_t + \tau^w \frac{W_t}{P_t^c} L_t + \tau^{\pi r} \frac{\pi r_t^T}{P_t^c}$$

$$z_t \widetilde{T_t^r} = \tau^c z_t \widetilde{C}_t + \tau^w z_t \widetilde{W_t^r} L_t + \tau^{\pi r} z_t \widetilde{\pi r_t^{T^r}}$$

$$\widetilde{T_t^r} = \tau^c \widetilde{C}_t + \tau^w \widetilde{W_t^r} L_t + \tau^{\pi r} \widetilde{\pi r_t^{T^r}} \tag{I.2.77}$$

where T_t^r and $\pi r_t^{T^r}$ are real total tax and real profit tax revenue.

Market clears on public goods market

$$P_t^g Y_t^g = G_t$$

$$\frac{P_t^g Y_t^g}{P_t^c} = \frac{G_t}{P_t^c}$$

Let's denote:

$$p_t^g = \frac{P_t^g}{P_t^c} \frac{P_{t-1}^c}{P_{t-1}^g} \frac{P_{t-1}^g}{P_{t-1}^c}$$

$$p_t^g = \frac{\Pi_t^g}{\Pi_t^c} p_{t-1}^g \tag{I.2.78}$$

$$p_t^g z_t \widetilde{Y_t^g} = z_t \widetilde{G_t}$$

Finally,

$$\widetilde{Y_t^g} = \frac{1}{p_t^g} \widetilde{G_t} \tag{I.2.79}$$

Demand on domestic input in public goods production

$$z_t \widetilde{G_t^d} = (1-\omega_g) \left(\frac{P_t^d}{P_t^c} \frac{P_t^c}{P_t^g} \right)^{-\eta_g} z_t \widetilde{Y_t^g}$$

$$\widetilde{G_t^d} = (1-\omega_g) \left(\frac{p_t^d}{p_t^g} \right)^{-\eta_g} \widetilde{Y_t^g} \tag{I.2.80}$$

Demand on imported input in public goods production

$$\frac{a_t^x z_t \widetilde{G_t^m}}{a_t^x} = \omega_g \left(\frac{P_t^{mG} a_t^x}{P_t^c} \frac{P_t^c}{P_t^g} \right)^{-\eta_g} z_t \widetilde{Y_t^g}$$

$$\widetilde{G_t^m} = \omega_g \left(\frac{p_t^{mG}}{p_t^g} \right)^{-\eta_g} \widetilde{Y_t^g} \tag{I.2.81}$$

Price index of public goods

$$\frac{P_t^g}{P_{t-1}^g} = \left((1-\omega_g) \left(\frac{P_t^d}{P_{t-1}^d} \frac{P_{t-1}^d}{P_{t-1}^c} \frac{P_{t-1}^c}{P_{t-1}^g} \right)^{1-\eta_g} + \omega_g \left(\frac{P_t^{mG} a_t^x}{P_{t-1}^{mG} a_{t-1}^x} \frac{P_{t-1}^{mG}}{P_{t-1}^c} \frac{P_{t-1}^c}{P_{t-1}^g} a_{t-1}^x \right)^{1-\eta_g} \right)^{\frac{1}{1-\eta_g}}$$

$$\Pi_t^g = \left((1-\omega_g) \left(\Pi_t^d \frac{p_{t-1}^d}{p_{t-1}^g} \right)^{1-\eta_g} + \omega_g \left(\Pi_t^{mG} (1+\gamma_t^{a^x}) \frac{p_{t-1}^{mG}}{p_{t-1}^g} \right)^{1-\eta_g} \right)^{\frac{1}{1-\eta_g}} \tag{I.2.82}$$

Transfers to output ratio

$$t_t^{cr} = (1-\rho_{tr}^{cr}) t^{cr} + \rho_{tr} t_{t-1}^{cr} + \epsilon_t^{t^{cr}} \tag{I.2.83}$$

$$t_t^{ucr} = (1-\rho_{tr}^{ucr}) t^{ucr} + \rho_{tr} t_{t-1}^{ucr} + \epsilon_t^{t^{ucr}} \tag{I.2.84}$$

Transfers to HHs.

$$\widetilde{TR_t^{cr}} = t_t^{cr} p_t^d \widetilde{Y}_t \tag{I.2.85}$$

$$\widetilde{TR_t^{ucr}} = t_t^{ucr} p_t^d \widetilde{Y}_t \tag{I.2.86}$$

Total transfers

$$\widetilde{TR}_t = \widetilde{TR_t^{ucr}} + \widetilde{TR_t^{cr}} \tag{I.2.87}$$

Balance of payment Firstly, we define foreign bonds to GDP and current account to GDP ratios that are stationary variables.

$$b_t^f \equiv \frac{e_t^{Gel/D} B_t^f}{P_t^d Y_t}$$

The foreign bonds to GDP is stationary variable taking into account trends of the variables:

$$b_t^f = \frac{e_t^{Gel/D} e_t^{D/R} P_t^R a_t^{\frac{2(1-\varepsilon_t^x)}{1-\eta x}} z_t^* \widetilde{B_t^f}}{z_t P_t^c (P_t^d / P_t^c) \widetilde{Y}_t} = \widetilde{a}_t \widetilde{REER}_t \frac{\widetilde{B_t^f}}{p_t^d \widetilde{Y}_t}$$

Note that we have used the definitions of real exchange rate and the stationary relative technology auxiliary variable (defined in the export sector). Similar, to the foreign bonds to output ratio, the current account to output ratio is also stationary.

$$ca_t = \frac{CA_t}{P_t^d Y_t} = \frac{e_t^{Gel/D} e_t^{D/R} P_t^R a_t^{\frac{2(1-\varepsilon^x)}{1-\eta x}} z_t^* \widetilde{CA_t}}{z_t P_t^c (P_t^d / P_t^c) \widetilde{Y}_t} = \widetilde{a}_t \widetilde{REER}_t \frac{\widetilde{CA_t}}{p_t^d \widetilde{Y}_t}$$

Therefore, the law of motion of foreign debt could be written in terms of stationary variables:

$$\frac{e_t^{Gel/D} B_t^f}{P_t^d Y_t} = \frac{e_t^{Gel/D} CA_t}{P_t^d Y_t} + \frac{e_{t-1}^{Gel/D} B_{t-1}^f}{P_{t-1}^d Y_{t-1}} \frac{e_t^{Gel/D} P_{t-1}^d Y_{t-1}}{e_{t-1}^{Gel/D} P_t^d Y_t} R_t^f R_t^\rho \left(-\xi^{dl}(b_t^f - b^f) - \xi^{fp} \left(\frac{e_{t+1}^{Gel/D}}{e_{t-1}^{Gel/D}} - 1 \right) \right)$$

Or,

$$b_t^f = ca_t + R_t^f R_t^\rho \exp\left(-\xi^{dl}(b_t^f - b^f) - \xi^{fp} \left(\left(1 + \gamma_{t+1}^{e^{Gel/D}}\right) \left(1 + \gamma_t^{e^{Gel/D}}\right) - 1 \right) \right) \frac{1 + \gamma_t^{e^{Gel/D}}}{\Pi_t^d (1 + \gamma_t^y)} b_{t-1}^f \tag{I.2.88}$$

The growth rate of nominal GDP

$$1 + \gamma_t^{GDP} = \frac{GDP_t}{GDP_{t-1}} = \frac{GDP_t}{GDP_{t-1}} \frac{P_{t-1}^c z_{t-1}}{P_t^c z_t} \frac{P_t^c z_t}{P_{t-1}^c z_{t-1}} = (1 + \gamma_t^z) \Pi_t^c \frac{\widetilde{GDP_t}}{\widetilde{GDP_{t-1}}} \tag{I.2.89}$$

At the same time, the stationary version of the definition of the current account balance is given by:

$$\frac{e_t^{Gel/D} CA_t}{P_t^d Y_t} = \frac{P_t^{xG} X_t}{P_t^d Y_t} - \frac{P_t^{mG} M_t}{P_t^d Y_t}$$

Taking into account the trends of export and import and the corresponding price levels, we can write:

$$ca_t = \frac{a_t^x P_t^{xG}}{P_t^c} {a_t^r}^{\frac{-2}{1-\eta x}} {a_t^r}^{\frac{2}{1-\eta x}} \frac{1}{a_t^x} {a_t^r}^{\frac{-2\varepsilon_t^x}{1-\eta x}} z_t^* \frac{\widetilde{X}_t}{(P_t^d/P_t^c) z_t \widetilde{Y}_t} - \frac{P_t^{mG} z_t a_t^x \widetilde{M}_t}{P_t^c (P_t^d/P_t^c) z_t \widetilde{Y}_t}$$

Finally,

$$ca_t = p_t^{xG} \widetilde{a}_t \frac{\widetilde{X}_t}{p_t^d \widetilde{Y}_t} - p_t^{mG} \frac{\widetilde{M}_t}{p_t^d \widetilde{Y}_t} \tag{I.2.90}$$

UIP To save space, we substitute foreign bonds gross growth rate with Γ_t^{Bf} in the UIP condition.

$$R_t = E_0(1 + \gamma_{t+1}^{eGel/D}) R_t^f R_t^\rho exp\left(-\xi^{dl}(b_t^f - b^f) - \xi^{fp}\left(\left(1 + \gamma_{t+1}^{eGel/D}\right)\left(1 + \gamma_t^{eGel/D} - 1\right)\right)\right) \tag{I.2.91}$$

The country risk premium

$$R_t^\rho = (1 - \rho_{prem}) R^\rho + \rho_{prem} R_{t-1}^\rho + \eta_t \tag{I.2.92}$$

Foreign block *Foreign inflation (in trade partners' currency)*

$$\Pi_t^R = (1 - \rho_{\Pi^R})\,\Pi^R + \rho_{\Pi^R} \Pi_{t-1}^R + \varepsilon_t^{\Pi^R} \tag{I.2.93}$$

Foreign inflation (in USD)

$$\Pi_t^f = \rho_{\Pi^f} \Pi_{t-1}^f + (1 - \rho_{\Pi^f})((1 + \gamma_t^{eD/R}) \Pi_t^R) + \varepsilon_t^{\Pi^f} \tag{I.2.94}$$

Foreign interest rate (USD)

$$R_t^f = \frac{1}{1 + i_t^f} \tag{I.2.95}$$

$$i_t^f = (1 - \rho_{i^f}) i^f + \rho_{i^f} i_{t-1}^f + \varepsilon_t^{i^f} \tag{I.2.96}$$

Foreign real neutral rate

$$r_t^{fnut} = \rho^{fnut} r_{t-1}^{fnut} + (1-\rho^{fnut}) r^{fnut} + \varepsilon_t^{fnut} \tag{I.2.97}$$

foreign real rate

$$r_t^f = i_t^f - E_t \pi_{t+1}^f \tag{I.2.98}$$

The foreign real interest rate gap

$$r_t^f = r_t^{fnut} + \widehat{r_t^f} \tag{I.2.99}$$

Foreign interest rate (ROW)

$$i_t^{rw} = \rho_{irw} i_{t-1}^{rw} + (1-\rho_{irw}) i^{rw} + \varepsilon_t^{irw} \tag{I.2.100}$$

UIP condition assets in USD vs ROW

$$\frac{(1+i_t^{rw})(1+E_t\gamma_{t+1}^{e^{R/D}})}{1+i_t^f} = \exp\left(\rho_{rwuip}((1+\gamma_t^{e^{R/D}})(1+E_t\gamma_{t+1}^{e^{R/D}})-1)\right) \tag{I.2.101}$$

Definition of foreign economic growth

$$(1+\gamma_t^{Y^*}) = \frac{Y_t^*}{Y_{t-1}^*} = \frac{z_t^*\widetilde{Y_t^*}}{z_{t-1}^*\widetilde{Y_{t-1}^*}} = (1+\gamma_t^{z^*})\frac{\widetilde{Y_t^*}}{\widetilde{Y_{t-1}^*}} \tag{I.2.102}$$

We assume that the dynamic of *growth rate of productivity abroad* follows the AR(1) process:

$$\gamma_t^{z^*} = (1-\rho_{\gamma^{z^*}})\gamma^{z^*} + \rho_{\gamma^{z^*}}\gamma_{t-1}^{z^*} + \varepsilon_t^{\gamma^{z^*}} \tag{I.2.103}$$

Also, the *Growth rate of foreign GDP* is given with the following AR(1) process:

$$\gamma_t^{Y^*} = (1-\rho_{\gamma^{Y^*}})\gamma^{Y^*} + \rho_{\gamma^{Y^*}}\gamma_{t-1}^{Y^*} + \varepsilon_t^{\gamma^{Y^*}} \tag{I.2.104}$$

Market clearing *Market clears on domestic intermediate input market*

$$z_t\widetilde{Y}_t = d_t^d z_t\widetilde{Y_t^d}$$

$$\widetilde{Y}_t = d_t^d\widetilde{Y_t^d} \tag{I.2.105}$$

Law of motion of domestic intermediate input price dispersion

$$d_t^d = (1-\theta_d)\left(\frac{P_t^{*d}}{P_t^d}\right)^{-\eta_t^d} + \theta_d {\Pi_{t-1}^d}^{-\eta_t^d} {\Pi_t^d}^{\eta_t^d} d_{t-1}^d \tag{I.2.106}$$

Law of motion of wage dispersion

$$d_t^w = (1-\theta_w)\left(\frac{W_t^*}{W_t}\right)^{-\eta_t^l} + \theta_w {\Pi_{t-1}^w}^{-\eta_t^l} {\Pi_t^w}^{\eta_l} d_{t-1}^w \tag{I.2.107}$$

Labor market clears

$$L_t^s = d_t^w L_t \tag{I.2.108}$$

Real effective wage

$$\frac{w_t}{P_t^c} z_t = \frac{W_t}{P_t^c}$$

$$\widetilde{w_t^r} = \widetilde{W_t^r} \tag{I.2.109}$$

Market clears on capital market

$$z_t\widetilde{K}_t = u_t z_t\widetilde{\overline{K_t}}$$

$$\widetilde{K}_t = u_t\widetilde{\overline{K_t}} \tag{I.2.110}$$

Real profit in domestic intermediate input production

$$\frac{\pi r_t^d}{P_t^c} = \frac{P_t^d}{P_t^c} Y_t^d - \frac{R_t^k}{P_t^c} K_t - \frac{w_t}{P_t^c}(z_t L_t) - \frac{P_t^{mG}}{P_t^c} Y_t^m$$

$$z_t\widetilde{\pi r_t^{dr}} = p_t^d z_t\widetilde{Y_t^d} - r_t^k z_t\widetilde{K_t} - \widetilde{w_t^r}(z_t L_t) - \frac{P_t^{mG} a_t^x}{P_t^c} z_t\widetilde{Y_t^m}$$

$$\widetilde{\pi r_t^{dr}} = p_t^d\widetilde{Y_t^d} - r_t^k\widetilde{K_t} - \widetilde{w_t^r} L_t - p_t^{mG}\widetilde{Y_t^m} \tag{I.2.111}$$

Profit in differentiated exported goods production

$$\frac{\pi r_t^x}{P_t^c} = \frac{P_t^{xG}}{P_t^c} X_t - \frac{P_t^d}{P_t^c} X_t^d - \frac{P_t^{mG}}{P_t^c} X_t^m$$

$$z_t \widetilde{\pi r_t^{xr}} = \frac{P_t^{xG} a_t^x}{P_t^c} a_t^{r\,\frac{-2}{1-\eta_x}} a_t^{r\,\frac{2}{1-\eta_x}} a_t^{r\,\frac{-2\varepsilon_t^x}{1-\eta_x}} z_t^* \frac{1}{a_t^x} \widetilde{X}_t - \frac{P_t^d}{P_t^c} z_t \widetilde{X_t^d} - \frac{P_t^{mG} a_t^x}{P_t^c} z_t \widetilde{X_t^m}$$

$$\widetilde{\pi r_t^{xr}} = p_t^{xG} a_t^{r\,\frac{2(1-\varepsilon_t^x)}{1-\eta_x}} \frac{z_t^*}{z_t} \frac{1}{a_t^x} \widetilde{X}_t - p_t^d \widetilde{X_t^d} - p_t^{mG} \widetilde{X_t^m}$$

$$\widetilde{\pi r_t^{xr}} = p_t^{xG} \widetilde{a}_t \widetilde{X}_t - p_t^d \widetilde{X_t^d} - p_t^{mG} \widetilde{X_t^m} \tag{I.2.112}$$

Entrepreneurs' profit

$$\frac{\pi r_t^e}{P_t^c} = \frac{R_t^k}{P_t^c} \overline{K}_t u_t - \gamma(u_t) \frac{P_t^i}{P_t^c} \overline{K}_t - \frac{P_t^i}{P_t^c} I_t$$

$$z_t \widetilde{\pi r_t^e} = r_t^k z_t \widetilde{\overline{K}}_t u_t - \gamma(u_t) P_t^i z_t \widetilde{\overline{K}}_t - P_t^i z_t \widetilde{I}_t$$

$$\widetilde{\pi r_t^e} = r_t^k \widetilde{\overline{K}}_t u_t - \gamma(u_t) P_t^i \widetilde{\overline{K}}_t - P_t^i \widetilde{I}_t \tag{I.2.113}$$

Profit of forex dealer To make the forex dealer‘s profit stationary, we normalize the profit by dividing with a nominal output

$$\begin{aligned}\frac{\pi r_t^{fx}}{P_t^d Y_t} = & \frac{e_t^{Gel/D} B_{t-1}^f}{P_t^d Y_t} \frac{e_{t-1}^{Gel/D} P_{t-1}^d Y_{t-1}}{P_{t-1}^d Y_{t-1}} R_t^f R_t^\rho \times \\ & \times \exp\left(-\xi^{lb}(b_t^f - b^f) - \xi^{fp}\left(\left(1+\gamma_{t+1}^{e^{Gel/D}}\right)\left(1+\gamma_t^{e^{Gel/D}}\right) - 1\right)\right) - \frac{e_t^{Gel/D} B_t^f}{P_t^d Y_t}\end{aligned}$$

By taking into account the definition of the foreign bonds to output and trends of nominal output and profit, we can rewrite the forex dealer's profit function as:

$$\begin{aligned}\widetilde{\pi r_t^{fx}} = & \frac{1+\gamma_t^{e^{Gel/D}}}{\Pi_t^d (1+\gamma_t^y)} R_t^f R_t^\rho \times \\ & \times \exp\left(-\xi^{lb}(b_t^f - b^f) - \xi^{fp}\left(\left(1+\gamma_{t+1}^{e^{Gel/D}}\right)\left(1+\gamma_t^{e^{Gel/D}}\right) - 1\right)\right) b_{t-1}^f - b_t^f\end{aligned} \tag{I.2.114}$$

Profit in final consumption goods production

$$\frac{\widetilde{\pi r_t^c}}{P_t^c} = C_t - \frac{P_t^d}{P_t^c} C_t^d - \frac{P_t^{mG}}{P_t^c} C_t^m$$

$$z_t\widetilde{\pi r_t^{cr}} = z_t\widetilde{C}_t - p_t^d z_t\widetilde{C_t^d} - \frac{P_t^{mG}a_t^x}{P_t^c}z_t\widetilde{C_t^m}$$

$$\widetilde{\pi r_t^{cr}} = \widetilde{C}_t - p_t^d\widetilde{C_t^d} - p_t^{mG}\widetilde{C_t^m} \tag{I.2.115}$$

Also, profit generated in the final investment and public goods production can be written as:

$$\widetilde{\pi r_t^{i^r}} = P_t^i\widetilde{I}_t - p_t^d\widetilde{I_t^d} - p_t^{mG}\widetilde{I_t^m} \tag{I.2.116}$$

$$\widetilde{\pi r_t^{gr}} = \widetilde{G}_t - p_t^d\widetilde{G_t^d} - p_t^{mG}\widetilde{G_t^m} \tag{I.2.117}$$

Total profit

$$\frac{\pi r_t^T}{P_t^c} = \frac{\pi r_t^d}{P_t^c} + \frac{\pi r_t^x}{P_t^c} + \frac{\pi r_t^e}{P_t^c} + \frac{\pi r_t^{fx}}{P_t^c} + \frac{\pi r_t^c}{P_t^c} + \frac{\pi r_t^i}{P_t^c} + \frac{\pi r_t^g}{P_t^c}$$

$$z_t\widetilde{\pi r_t^{T^r}} = z_t\widetilde{\pi r_t^{d^r}} + z_t\widetilde{\pi r_t^{xr}} + z_t\widetilde{\pi r_t^{er}} + z_t\widetilde{\pi r_t^{fx^r}} + z_t\widetilde{\pi r_t^{cr}} + z_t\widetilde{\pi r_t^{i^r}} + z_t\widetilde{\pi r_t^{gr}}$$

$$\widetilde{\pi r_t^{T^r}} = \widetilde{\pi r_t^{d^r}} + \widetilde{\pi r_t^{xr}} + \widetilde{\pi r_t^{er}} + \widetilde{\pi r_t^{fx^r}} + \widetilde{\pi r_t^{cr}} + \widetilde{\pi r_t^{i^r}} + \widetilde{\pi r_t^{gr}} \tag{I.2.118}$$

Aggregate demand on imported goods

$$a_t z_t\widetilde{M}_t = a_t z_t\widetilde{Y_t^m} + a_t z_t\widetilde{C_t^m} + a_t z_t\widetilde{I_t^m} + a_t z_t\widetilde{G_t^m} + a_t z_t\widetilde{X_t^m}$$

$$\widetilde{M}_t = \widetilde{Y_t^m} + \widetilde{C_t^m} + \widetilde{I_t^m} + \widetilde{G_t^m} + \widetilde{X_t^m} \tag{I.2.119}$$

Stationary component of the nominal GDP.

$$P_t^c z_t\widetilde{GDP_t} = P_t^c z_t\widetilde{C}_t + P_t^g z_t\widetilde{Y_t^g} + P_t^i z_t\widetilde{I}_t + (e_t^{Gel/D}P_t^{xf}X_t - e_t^{Gel/D}P_t^{mf}M_t)$$

$$\widetilde{GDP_t} = \widetilde{C}_t + \widetilde{G}_t + \frac{P_t^i}{P_t^c}\widetilde{I}_t + \left(\frac{P_t^{xG}}{P_t^c}\frac{a_t^x}{{a_t^r}^{\frac{2}{1-\eta_x}}}\frac{{a_t^r}^{\frac{2}{1-\eta_x}}}{a_t^x}\frac{z_t^*}{z_t}{a_t^r}^{\frac{-2\varepsilon_t^x}{1-\eta_x}}\widetilde{X}_t - \frac{P_t^{mG}}{P_t^c}\frac{a_t^x z_t}{z_t}\widetilde{M}_t\right)$$

$$\widetilde{GDP_t} = \widetilde{C}_t + \widetilde{G}_t + P_t^i\widetilde{I}_t + \left(p_t^{xG}\frac{{a_t^r}^{\frac{2}{1-\eta_x}}}{a_t^x}\frac{z_t^*}{z_t}{a_t^r}^{\frac{-2\varepsilon_t^x}{1-\eta_x}}\widetilde{X}_t - p_t^{mG}\widetilde{M}_t\right)$$

Now, recall the definition of $\widetilde{a}_t$, then

$$\widetilde{GDP_t} = \widetilde{C}_t + \widetilde{G}_t + P_t^i \widetilde{I}_t + \left(p_t^{xG} \widetilde{a}_t \widetilde{X}_t - p_t^{mG} \widetilde{M}_t \right) \tag{I.2.120}$$

The relative price index of GDP deflator. We define the relative price index of the GDP deflator as:

$$p_t^Y \equiv \frac{P_t^Y}{P_t^c} \left(\frac{a_t^x}{a_t^{r\,\frac{2}{1-\eta_x}}} \right)^{s_x} (a_t^x)^{-s_m}$$

and it could be written recursively:

$$p_t^Y = \frac{P_t^Y}{P_t^c} \left(\frac{a_t^x}{a_t^{r\,\frac{2}{1-\eta_x}}} \right)^{s_x} (a_t^x)^{-s_m} \frac{P_{t-1}^c}{P_{t-1}^c} \frac{P_{t-1}^Y}{P_{t-1}^Y} \left(\frac{a_{t-1}^{r\,\frac{2}{1-\eta_x}}}{a_{t-1}^x} \right)^{s_x} \left(\frac{1}{a_{t-1}^x} \right)^{-s_m} \left(\frac{a_{t-1}^x}{a_{t-1}^{r\,\frac{2}{1-\eta_x}}} \right)^{s_x} \left(a_{t-1}^x \right)^{-s_m}$$

Finally;

$$p_t^Y = \frac{\Pi_t^Y}{\Pi_t^c} \left(\frac{\left(1 + \gamma_t^{a^r}\right)^{\frac{2}{1-\eta_x}}}{1 + \gamma_t^{a^x}} \right)^{s_x} \left(\frac{1}{1 + \gamma_t^{a^x}} \right)^{-s_m} p_{t-1}^Y \tag{I.2.121}$$

Inflation of GDP deflator

$$\Pi_t^Y = (\Pi_t^c)^{s_c} \left(\Pi_t^I\right)^{s_i} \left(\Pi_t^G\right)^{s_g} \left(\Pi_t^{xG}\right)^{s_x} \left(\Pi_t^{mG}\right)^{-s_m} \tag{I.2.122}$$

Demand on aggregate domestic intermediate input

$$P_t^d Y_t^d = P_t^d C_t^d + P_t^d I_t^d + P_t^d G_t^d + P_t^d X_t^d + P_t^i \gamma(u_t) \overline{K}_t$$

Implies that,

$$Y_t^d = C_t^d + I_t^d + G_t^d + X_t^d + \frac{P_t^i}{P_t^c} \frac{P_t^c}{P_t^d} \gamma(u_t) \overline{K}_t$$

$$z_t \widetilde{Y_t^d} = z_t \widetilde{C_t^d} + z_t \widetilde{I_t^d} + z_t \widetilde{G_t^d} + z_t \widetilde{X_t^d} + \frac{P_t^i}{p_t^d} z_t \gamma(u_t) \widetilde{\overline{K}}_t$$

Finally,

$$\widetilde{Y_t^d} = \widetilde{C_t^d} + \widetilde{I_t^d} + \widetilde{G_t^d} + \widetilde{X_t^d} + \frac{p_t^i}{p_t^d} \gamma(u_t) \widetilde{\overline{K}}_t \tag{I.2.123}$$

Real GDP. By taking into account the trend process around which real GDP is stationary, we could extract the stationary part of it by applying some steps of transformations; firstly, we recall the stationary component of the nominal GDP, then the real

GDP could be written as:

$$GDP_t^r = \frac{GDP_t}{P_t^Y} = \frac{z_t P_t^c \widetilde{GDP_t}}{P_t^Y}$$

Let's multiply both sides of the equation by $\frac{1}{z_t}\left(\frac{{a_t^r}^{\frac{2}{1-\eta x}}}{a_t^x}\right)^{s_x}\left(\frac{1}{a_t^x}\right)^{-s_m}$; note, this is the inverse of the stochastic trend of real GDP, i.e. symmetrically $\left(\frac{{a_t^r}^{\frac{2}{1-\eta x}}}{a_t^x}\right)^{s_x}\left(\frac{1}{a_t^x}\right)^{-s_m}$ is the trend of the relative price index of GDP deflator $\frac{P_t^Y}{P_t^c}$, then,

$$\frac{1}{z_t}\left(\frac{{a_t^r}^{\frac{2}{1-\eta x}}}{a_t^x}\right)^{s_x}\left(\frac{1}{a_t^x}\right)^{-s_m} GDP_t^r = \widetilde{GDP}_t \frac{P_t^c}{P_t^Y}\left(\frac{{a_t^r}^{\frac{2}{1-\eta x}}}{a_t^x}\right)^{s_x}\left(\frac{1}{a_t^x}\right)^{-s_m}$$

Finally,

$$\widetilde{GDP_t^r} = \frac{\widetilde{GDP_t}}{p_t^y} \tag{I.2.124}$$

Domestic absorption

$$\frac{ABS_t}{P_t^c} = C_t + \frac{G_t}{P_t^c} + \frac{P_t^i}{P_t^c} I_t$$

$$z_t \widetilde{ABS_t^r} = z_t \widetilde{C}_t + z_t \widetilde{G_t^r} + P_t^i z_t \widetilde{I}_t$$

$$\widetilde{ABS_t^r} = \widetilde{C}_t + \widetilde{G}_t + P_t^i \widetilde{I}_t \tag{I.2.125}$$

The stationary equations are ready enough to implement in dynare. Here, we have introduced 129 variables, and 125 equations, however, we note that the functional forms of two more variables $S\left(\frac{I_t}{I_{t-1}}\right)$ and $S'\left(\frac{I_t}{I_{t-1}}\right)$ are given in the Appendix B.1. As for the stationary component of the remaining two variables (fixed costs in domestic intermediate input and differentiated exported goods production) $\widetilde{F_t^d}$ and $\widetilde{F_t^x}$, based on the definitions of those variables, they are constants and we treat them as parameters in the model. Finally, we have 130 variables and the same number of equations. The steady-state values of those variables are solved in the remaining part of the text.

I.3 Steady State of the Stationary Model

As the model equilibrium conditions have already been written in a stationary form, the next step is to derive the steady state conditions. It works twofold. On the one hand, we can solve the steady state of the model analytically around which the model is simulated, and on the other hand, the equilibrium conditions in the steady-state set restrictions for calibrating some of the model parameters, which is crucial on calibration stage.

We express variables in a steady state without a time index and assume that exogenous shocks are muted and are not in place. Also, we calibrate steady-state values of the following exogenous stationary processes ψ_t, θ_t, α_t, γ_t as one. Additional assumptions on steady-state values of some part of variables will be discussed in the next section.

In the first stage, we rewrite stationary equations in steady-state to prepare the ground for solving the steady-state values of model variables.

Household *Euler equation*

$$R = \frac{\psi\left((1+\gamma^z)\widetilde{C^{uc}} - h\widetilde{C^{uc}}\right)\Pi^C}{\beta\psi\left(\frac{(1+\gamma^z)\widetilde{C^{uc}} - h\widetilde{C^{uc}}}{1+\gamma^z}\right)} = \frac{(1+\gamma^z)\Pi^C}{\beta} \tag{I.3.1}$$

i.e. the long-run value of the nominal interest rate is shaped with the steady-state value of inflation (determined with targeted inflation), the exogenous growth rate of productivity, and subjective discount rate.

Aggregate consumption

$$\widetilde{C} = (1-\lambda)\widetilde{C^{uc}} + \lambda\widetilde{C^{c}} \tag{I.3.2}$$

The *Aggregate wage dynamic* in steady state implies that

$$\frac{W^*}{W} = 1 \tag{I.3.3}$$

The steady state values of auxiliary variables C_{1t} and C_{2t} are derived as:

$$C_1 = \frac{\psi}{\widetilde{C^{uc}} - \frac{h}{1+\gamma^z}\widetilde{C^{uc}}}\widetilde{W^r}L + \beta\theta_w C_1 \tag{I.3.4}$$

By using the assumption that $\psi = 1$, then

$$C_1 = \frac{1}{1-\beta\theta_w}\frac{1+\gamma^z}{1+\gamma^z - h}\frac{\widetilde{W^r}L}{\widetilde{C^{uc}}} \tag{I.3.5}$$

While

$$C_2 = \frac{1}{1-\beta\theta_w}\theta\chi L^{1+\zeta} \tag{I.3.6}$$

Then by taking into account the nonlinear wage Philips curve:

$$1 = \frac{(\eta_l - 1)(1-\tau^w)}{\eta_l(1+\tau^c)} \frac{\frac{1+\gamma^z}{1+\gamma^z-h}\frac{\widetilde{W^r}L}{C^{uc}}}{\psi\chi L^{1+\zeta}} \tag{I.3.7}$$

It implies:

$$\widetilde{C^{uc}} = \frac{(\eta_l - 1)(1-\tau^w)}{\eta_l(1+\tau^c)} \frac{1+\gamma^z}{1+\gamma^z-h} \frac{\widetilde{W^r}L^{-\zeta}}{\theta\chi} \tag{I.3.8}$$

While the consumption of constrained HHs is:

$$\widetilde{C^c} = \frac{(1-\tau^w)}{(1+\tau^c)}\widetilde{W^r}L + \frac{1}{(1+\tau^c)}\widetilde{T^{cr}} \tag{I.3.9}$$

Also, the recursive definition of real wages implies that in the steady-state change in nominal wage is determined by the inflation target and growth rate of productivity.

$$\Pi^w = \Pi^c(1+\gamma^z) \tag{I.3.10}$$

Entrepreneurs The *real rental rate of capital*

$$r^k = \gamma'(u)p^i \tag{I.3.11}$$

Using the assumption on the steady state level of capital utilization i.e. u=1, then

$$\gamma'(u) = \sigma_b \tag{I.3.12}$$

The equation I.2.14 in steady state implies that:

$$p^i = \widetilde{\lambda}^e \tag{I.3.13}$$

Also,

$$\tilde{\lambda}^e = \left[\frac{\beta(\widetilde{C^{uc}} - \frac{h}{1+\gamma^z}\widetilde{C^{uc}})}{((1+\gamma^z)\widetilde{C^{uc}} - h\widetilde{C^{uc}})}(r^k u - \gamma(u)p^i) \right] + (1-\delta) \left[\frac{\beta(\widetilde{C^{uc}} - \frac{h}{1+\gamma^z}\widetilde{C^{uc}})}{((1+\gamma^z)\widetilde{C^{uc}} - h\widetilde{C^{uc}})}\tilde{\lambda}^e \right] \tag{I.3.14}$$

Note, here we use the fact that $\gamma(u) = 0$ in SS, then:

$$\tilde{\lambda}^e = \frac{\beta}{1+\gamma^z} r^k + (1-\delta)\frac{\beta}{1+\gamma^z}\tilde{\lambda}^e \tag{I.3.15}$$

It follows that:

$$\tilde{\lambda}^e = \frac{\beta}{1+\gamma^z - (1-\delta)\beta} r^k \tag{I.3.16}$$

The law of motion of capital implies the following relationship between capital and investment in SS:

$$\widetilde{I} = (\gamma^z + \delta)\widetilde{\overline{K}} \tag{I.3.17}$$

Domestic intermediate input producers. The equation of aggregate price index implies that:

$$\frac{P^{*d}}{P^d} = 1 \tag{I.3.18}$$

Now let's solve for auxiliary variables, D_1, and D_2.

$$D_1 = \frac{\psi}{\frac{((1+\gamma^z)-h)\widetilde{C^{uc}}}{1+\gamma^z}} p^d \widetilde{Y^d} MC^{r^d} + \theta_d \beta D_1 \tag{I.3.19}$$

It follows that

$$D_1 = \frac{1}{1-\theta_d\beta}\frac{\psi(1+\gamma^z)}{1+\gamma^z-h}\frac{p^d\widetilde{Y^d}MC^{r^d}}{\widetilde{C^{uc}}} \tag{I.3.20}$$

Also,

$$D_2 = \frac{1}{1-\theta_d\beta}\frac{\psi(1+\gamma^z)}{1+\gamma^z-h}\frac{p^d\widetilde{Y^d}}{\widetilde{C^{uc}}} \tag{I.3.21}$$

From the non-linear Philips curve:

$$1 = \frac{\eta_d}{\eta_d - 1} MC^{r^d} \tag{I.3.22}$$

The recursive definition of the price index of domestic intermediate input implies that:

$$\Pi^d = \Pi^c \tag{I.3.23}$$

The real marginal cost

$$\widetilde{MC^{dr}} = \frac{1}{\alpha_1{}^{\alpha_1}\alpha_2{}^{\alpha_2}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}}\frac{1}{\gamma}\frac{1}{p^d}\left(\widetilde{W^r}\right)^{\alpha_1}\left(r^k\right)^{\alpha_2}\left(p_t^{mG}\right)^{1-\alpha_1-\alpha_2} \tag{I.3.24}$$

Here, we used the assumption that the stationary productivity shock $\gamma = 1$ in SS. The demand on production factors is given by:

$$L = \alpha_1 \frac{MC^{r^d}p^d}{\widetilde{W^r}}\left(\widetilde{Y} + \widetilde{F^d}\right) \tag{I.3.25}$$

$$\widetilde{K} = \alpha_2 \frac{MC^{r^d}p^d}{r^k}\left(\widetilde{Y} + \widetilde{F^d}\right) \tag{I.3.26}$$

$$\widetilde{Y^m} = (1-\alpha_1-\alpha_2)\frac{MC^{r^d}p^d}{p^{mG}}\left(\widetilde{Y} + \widetilde{F^d}\right) \tag{I.3.27}$$

From the recursive definition of relative import price:

$$\Pi^{mG} = \frac{\Pi^c}{1+\gamma^{a^x}} \tag{I.3.28}$$

Final goods producers. The demand for domestic and imported inputs used in final consumption goods production is given by:

$$\widetilde{C^d} = (1-\omega_c){p^d}^{-\eta_c}\widetilde{C} \tag{I.3.29}$$

$$\widetilde{C^m} = \omega_c {p^{mG}}^{-\eta_c}\widetilde{C} \tag{I.3.30}$$

The CPI inflation is the CES aggregate of domestic intermediate input price inflation and imported inflation.

$$\Pi^c = \left[(1-\omega_c)\left(\Pi^d p^d\right)^{1-\eta_c} + \omega_c\left(\Pi^{mG}(1+\gamma^{a^x})p^{mG}\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}} \tag{I.3.31}$$

Note, that the CPI inflation is anchored by the central bank and its steady state value equals targeted inflation; by taking into account that in SS $\Pi^c = \Pi^d = \Pi^{mG}(1+\gamma^{a^x})$, from the CPI inflation equation, we can derive the relative price index of domestic intermediate input price in terms of the relative imported price index:

$$1 = \left[(1-\omega_c)\, p^{d^{1-\eta_c}} + \omega_c p^{mG^{1-\eta_c}}\right]^{\frac{1}{1-\eta_c}} \tag{I.3.32}$$

$$p^d = \left(\frac{1}{1-\omega_c}\left(1-\omega_c p^{mG^{1-\eta_c}}\right)\right)^{\frac{1}{1-\eta_c}} \tag{I.3.33}$$

Demand on domestic and imported inputs in final investment goods production

$$\widetilde{I}^d = (1-\omega_i)\left(\frac{p^d}{p^i}\right)^{-\eta_i}\widetilde{I} \tag{I.3.34}$$

$$\widetilde{I^m} = \omega_i\left(\frac{p^{mG}}{p^i}\right)^{-\eta_i}\widetilde{I} \tag{I.3.35}$$

$$\Pi^I = \left[(1-\omega_i)\left(\Pi^d\frac{p^d}{p^i}\right)^{1-\eta_i} + \omega_i\left(\Pi^{mG}(1+\gamma^{a^x})\frac{p^{mG}}{p^i}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} \tag{I.3.36}$$

We further note, that $\Pi^c = \Pi^i$ in SS, then the relative price index of final investment goods is given by:

$$p^i = \left[(1-\omega_i)\, p^{d^{1-\eta_i}} + \omega_i p^{mG^{1-\eta_i}}\right]^{\frac{1}{1-\eta_i}} \tag{I.3.37}$$

Import Sector The optimal price in the import sector:

$$\frac{P^{*mf}}{P^{mf}} = \frac{\varepsilon^m}{\varepsilon^m - 1}\frac{\widetilde{a}_1}{\widetilde{a}_2} \tag{I.3.38}$$

While the inflation equation of imported goods implies that:

$$\frac{P^{*mf}}{P^{mf}} = 1 \tag{I.3.39}$$

The auxiliary variables $\widetilde{a_{1t}}$ and $\widetilde{a_{2t}}$ in steady state:

$$\widetilde{a_1} = \widetilde{M}MC^{r^m} + \theta_m \frac{1}{R^f}(1+\gamma^z)(1+\gamma^{a^x})\Pi^{mf}\widetilde{a_1} \tag{I.3.40}$$

$$\widetilde{a_1} = \frac{1}{1 - \frac{\theta_m}{R^f}(1+\gamma^z)(1+\gamma^{a^x})\Pi^{mf}} \widetilde{M}MC^{m^r} \tag{I.3.41}$$

While $\widetilde{a_2}$ is:

$$\widetilde{a_2} = \widetilde{M} + \theta_m \frac{1}{R^f}\Pi^{mf}(1+\gamma^z)(1+\gamma^{a^x})\widetilde{a_2} \tag{I.3.42}$$

$$\widetilde{a_2} = \frac{1}{1 - \frac{\theta_m}{R^f}(1+\gamma^z)(1+\gamma^{a^x})\Pi^{mf}} \widetilde{M} \tag{I.3.43}$$

The real marginal cost in the import sector is related to the real exchange rate in the following way:

$$MC^{m^r} = \frac{1}{p^{mG}}\widetilde{REER} \tag{I.3.44}$$

The recursive form of real exchange rate implies that:

$$1 + \gamma^{Gel/R} = \frac{\Pi^c}{(1+\gamma^{a^x})\Pi^R} \tag{I.3.45}$$

Inflation in the import sector in GEL:

$$\Pi^{mG} = (1 + \gamma^{e^{Gel/D}})\Pi^{mf} \tag{I.3.46}$$

Export sector The optimal price in the export sector:

$$\frac{P^{*xf}}{P^{xf}} = \frac{\varepsilon^x}{\varepsilon^x - 1}\frac{B_1}{B_2} \tag{I.3.47}$$

Also, the inflation equation in the export sector implies that:

$$\frac{P^{*xf}}{P^{xf}} = 1 \tag{I.3.48}$$

The auxiliary variables in the export sector:

$$B_1 = \frac{\psi}{(\widetilde{C^{uc}} - \frac{h}{1+\gamma^z}\widetilde{C^{uc}})} p^{xG}\widetilde{a}\widetilde{X}MC^{x^r} + \theta_x \beta B_1 \tag{I.3.49}$$

$$B_1 = \frac{1}{1-\beta\theta_x}\frac{\psi(1+\gamma^z)}{(1+\gamma^z-h)\widetilde{C^{uc}}}p^{xG}\widetilde{a}\widetilde{X}MC^{x^r} \tag{I.3.50}$$

Also,

$$B_2 = \frac{1}{1-\beta\theta_x}\frac{\psi(1+\gamma^z)}{(1+\gamma^z-h)\widetilde{C^{uc}}}p^{xG}\widetilde{a}\widetilde{X} \tag{I.3.51}$$

The real marginal cost in the export sector:

$$MC^{x^r} = \left[(1-\omega_x)\left(\frac{p^d}{p^{xG}}\right)^{1-\eta_x} + \omega_x\left(\frac{p^{mG}}{p^{xG}}\right)^{1-\eta_x}\right]^{\frac{1}{1-\eta_x}} \tag{I.3.52}$$

The recursive definition of the relative price of exported goods in USD implies that:

$$\Pi^{xf} = (1+\gamma^{e^{D/R}})(1+\gamma^{a^r})^{\frac{2}{1-\gamma_x}}\Pi^R \tag{I.3.53}$$

While

$$\Pi^{xG} = (1+\gamma^{e^{Gel/D}})\Pi^{xf} \tag{I.3.54}$$

The demand equations on aggregate exported goods and on inputs used in exported goods production are given by:

$$\widetilde{X} = \omega_w\left(p^{xf}\right)^{-\epsilon_x}\widetilde{Y^*} \tag{I.3.55}$$

$$\widetilde{X^d} = (1-\omega_x)\left(\frac{p^d}{p^{xG}MC^{x^r}}\right)^{-\eta_x}\left(\widetilde{X}+\widetilde{F^x}\right) \tag{I.3.56}$$

$$\widetilde{X^m} = \omega_x\left(\frac{p^{mG}}{p^{xG}MC^{x^r}}\right)^{-\eta_x}\left(\widetilde{X}+\widetilde{F^x}\right) \tag{I.3.57}$$

Monetary policy

$$\Pi_4 = \Pi^{tar} \tag{I.3.58}$$

$$\Pi^c = \Pi^{tar} \tag{I.3.59}$$

$$i^N = (1+\pi^{exp})(1+r^{nut}) - 1 \tag{I.3.60}$$

$$\pi^{exp} = \Pi^{tar} - 1 \tag{I.3.61}$$

$$i = i^N \tag{I.3.62}$$

$$R = \frac{1}{1+i} \tag{I.3.63}$$

Finally, the real neutral interest rate is given by:

$$1 + r^{nut} = \frac{(1 + r^{fnut})R^{\rho^{nut}}}{1 + \gamma^{a^x}} \tag{I.3.64}$$

Fiscal sector We assume that the Government keeps debt to output level at the target level d in SS, therefore, $d_t = d$. Then from the law of motion of government debt:

$$gb = \left(\frac{1+i}{\Pi^d(1+\gamma^y)} - 1 \right) d \tag{I.3.65}$$

Total real tax revenue in steady state:

$$\widetilde{T^r} = \tau^c \widetilde{C} + \tau^w \widetilde{W^r} L + \tau^{\pi r} \widetilde{\pi r^{T^r}} \tag{I.3.66}$$

From the definition of primary balance we can derive the steady state level of government spending:

$$gb = \frac{1}{p^d Y}(\widetilde{T^r} - \widetilde{G} - \widetilde{TR}) \tag{I.3.67}$$

The recursive definition of the relative price index of public goods implies, that

$$\Pi^g = \Pi^c \tag{I.3.68}$$

The supply and demand of public goods:

$$\widetilde{Y^g} = \frac{1}{p^g} \widetilde{G} \tag{I.3.69}$$

The demand for domestic and imported inputs in the production of the public good

$$\widetilde{G^d} = (1 - \omega_g) \left(\frac{p^d}{p^g} \right)^{-\eta_g} \widetilde{Y^g} \tag{I.3.70}$$

$$\widetilde{G^m} = \omega_g \left(\frac{p^{mG}}{p^g}\right)^{-\eta_g} \widetilde{Y^g} \tag{I.3.71}$$

The relative price index of public goods in SS can be derived from the public goods inflation equation.

$$p^g = \left((1-\omega_g){p^d}^{1-\eta_g} + \omega_g {p^{mG}}^{1-\eta_g}\right)^{\frac{1}{1-\eta_g}} \tag{I.3.72}$$

Balance of payment The law of motion of the external debt to output

$$b^f = ca + R^f R^\rho \frac{1+\gamma^{e^{Gel/D}}}{\Pi^d(1+\gamma^y)} b^f \tag{I.3.73}$$

The SS value of debt-to-output ratio is calibrated, therefore, we can derive the SS current account-to-output ratio consistent with the debt-to-output ratio as:

$$ca = \left(1 - R^f R^\rho \frac{1+\gamma^{e^{Gel/D}}}{\Pi^d(1+\gamma^y)}\right) b^f \tag{I.3.74}$$

The definition of the current account in steady state:

$$ca = p^{xG}\widetilde{a}\frac{\widetilde{X}}{p^d\widetilde{Y}} - p^{mG}\frac{\widetilde{M}}{p^d\widetilde{Y}} \tag{I.3.75}$$

Growth rate of nominal GDP

$$1+\gamma^{GDP} = (1+\gamma^z)\,\Pi^c \tag{I.3.76}$$

UIP At the steady state the UIP condition implies:

$$R = R^f R^\rho (1+\gamma^{e^{Gel/D}}) \tag{I.3.77}$$

Foreign sector

$$\Pi^f = (1+\gamma^{e^{D/R}})\Pi^R \tag{I.3.78}$$

$$R^f = \frac{1}{1+i^f} \tag{I.3.79}$$

$$1+r^f = \frac{R^f}{\Pi^f} \tag{I.3.80}$$

Market clearing condition *Law of motion of domestic intermediate input price dispersion* in steady-state:

$$d^d = (1-\theta_d)\left(\frac{P^{*d}}{P^d}\right)^{-\eta_d} + \theta_d {\Pi^d}^{-\eta_d} {\Pi^d}^{\eta_d} d^d \tag{I.3.81}$$

Then,

$$d^d = \left(\frac{P^{*d}}{P^d}\right)^{-\eta_d} \tag{I.3.82}$$

As it was mentioned above in SS $\frac{P^{*d}}{P^d} = 1$, therefore:

$$d^d = 1 \tag{I.3.83}$$

Then the market clearing condition on the domestic intermediate input market is written as:

$$\widetilde{Y} = \widetilde{Y^d} \tag{I.3.84}$$

The law of motion of wage dispersion together with optimal wage equation in steady state implies that,

$$d^w = 1 \tag{I.3.85}$$

Then *Labor market clears*

$$L^s = L \tag{I.3.86}$$

The real effective wage equals to a stationary component of real wage:

$$\widetilde{w^r} = \widetilde{W^r} \tag{I.3.87}$$

Market clearing condition on capital

$$\widetilde{K} = \widetilde{\widetilde{K}} \tag{I.3.88}$$

The profits in steady state in different sectors is given by

$$\widetilde{\pi r^{d^r}} = p^d \widetilde{Y^d} - r^k \widetilde{K} - \widetilde{W^r} L - p^{mG} \widetilde{Y^m} \tag{I.3.89}$$

$$\widetilde{\pi r^{xr}} = p^{xG}\widetilde{a}\widetilde{X} - p^d\widetilde{X^d} - p^{mG}\widetilde{X^m} \tag{I.3.90}$$

$$\widetilde{\pi r^e} = r^k\widetilde{\widetilde{K}} - p^I\widetilde{I} \tag{I.3.91}$$

$$\widetilde{\pi r^{fx}} = \left(R^f R^\rho \frac{1+\gamma^{e^{Gel/D}}}{\Pi^d(1+\gamma^y)} - 1\right) b^f p^d \widetilde{y} \tag{I.3.92}$$

$$\widetilde{\pi r^{cr}} = \widetilde{C} - p^d\widetilde{C^d} - p^{mG}\widetilde{C^m} \tag{I.3.93}$$

$$\widetilde{\pi r^{i^r}} = p^i\widetilde{I} - p^d\widetilde{I^d} - p^{mG}\widetilde{I^m} \tag{I.3.94}$$

$$\widetilde{\pi r^{gr}} = P^g\widetilde{G} - p^d\widetilde{G^d} - p^{mG}\widetilde{G^m} \tag{I.3.95}$$

$$\widetilde{\pi r^{T^r}} = \widetilde{\pi r^{d^r}} + \widetilde{\pi r^{xr}} + \widetilde{\pi r^{er}} + \widetilde{\pi r^{fx^r}} + \widetilde{\pi r^{cr}} + \widetilde{\pi r^{i^r}} + \widetilde{\pi r^{gr}} \tag{I.3.96}$$

Aggregate demand on imported goods

$$\widetilde{M} = \widetilde{Y^m} + \widetilde{C^m} + \widetilde{I^m} + \widetilde{G^m} + \widetilde{X^m} \tag{I.3.97}$$

Stationary component of the nominal GDP.

$$\widetilde{GDP} = \widetilde{C} + \widetilde{G} + p^i\widetilde{I} + \left(p^{xG}\widetilde{a}\widetilde{X} - p^{mG}\widetilde{M}\right) \tag{I.3.98}$$

The Real GDP

$$\widetilde{GDP^r} = \frac{\widetilde{GDP}}{p^y} \tag{I.3.99}$$

GDP deflator inflation

$$\Pi^Y = (\Pi^c)^{s_c}\left(\Pi^i\right)^{s_i}(\Pi^g)^{s_g}\left(\Pi^{xG}\right)^{s_x}\left(\Pi^{mG}\right)^{-s_m} \tag{I.3.100}$$

To derive the steady state relative price index of the GDP deflator, we divide the GDP deflator by the CPI index:

$$\frac{P_t^Y}{P_t^c} = \left(\frac{P_t^g}{P_t^c}\right)^{s_g} \left(\frac{P_t^i}{P_t^c}\right)^{s_i} \left(\frac{P_t^{xG}}{P_t^c}\right)^{s_x} \left(\frac{P_t^{mG}}{P_t^c}\right)^{-s_m} \tag{I.3.101}$$

In steady state:

$$p^Y = P^{g s_g} p^{i s_i} p^{xG s_x} p^{mG - s_m} \tag{I.3.102}$$

Demand on aggregate domestic intermediate input

$$\widetilde{Y^d} = \widetilde{C^d} + \widetilde{I^d} + \widetilde{G^d} + \widetilde{X^d} \tag{I.3.103}$$

The domestic absorption in steady state

$$\widetilde{ABS^r} = \widetilde{C} + \widetilde{G} + p^i \widetilde{I} \tag{I.3.104}$$

I.4 Solving the Steady State Conditions

The equilibrium conditions in steady state are solved recursively, and values of the variables in SS are derived in terms of parameters and with already known variables. But also, steady-state values of some variables are determined outside of the model, for example, the SS value of the following *exogenous shock process* is set to one: $\psi, \theta, \gamma, \alpha$, while the values of monetary policy and government spending shocks ε^i, u^g are set to zero. And following *growth rates* will be calibrated exogenously: $\gamma^z, \gamma^{a^x}, \gamma^{D/R}, \gamma^{Y^*}, \gamma^{z^*}$. The monetary authority determines the targeted inflation π^{tar}, as well as, the foreign inflation rate Π^R is calibrated consistent to the long-run inflation trend in trade partners' economies. Also, the foreign nominal interest rate i^f is calibrated based on the data on foreign interest rates. We further assume that values of the following variables: $\widetilde{REER}, \widetilde{a}, u, p^{xf}$ equal to one while the variables: S, S' are assumed as zero in SS (S'' is calibrated based on literature and IRFs analysis). Moreover, we assume that workers spend 1/3 of their time on working places i.e. $L = 1/3$. Finally, government debt to output and external debt to output ratios d, b^f are calibrated consistent to the debt sustainability conditions in the country. The transfers to output ratios are calibrated, then the SS value of transfers is given as $\widetilde{T^{cr}} = \widetilde{t^{cr}} p^d \widetilde{Y}$. Also, SS values of elasticity of substitution $\eta^d, \eta^l, \varepsilon^m, \varepsilon^x$ are calibrated using firms' data. The foreign interest rate gap (net) is assumed to be $\widehat{r^f} = 0$, while the risk premium gap $\widehat{R^\rho} = 1$.

Given that there are 129 variables in the stationary version of the model, and as said we exogenously determine SS values of 31 variables, then we are left with 98 variables of which SS values must be determined endogenously by solving the equilibrium conditions of the model in SS. The analytical solution of the model in SS is outlined in the rest of the text.

Firstly, we start with defining variables that are directly linked to the exogenously determined variables or are derived without interaction with other variables.

Consumer price inflation

$$\Pi^c = \Pi^{tar} \tag{I.4.1}$$

Expected inflation

$$\pi^{exp} = \Pi^{tar} - 1 \tag{I.4.2}$$

Inflation of domestic intermediate inputs price is determined from trend relations.

$$\Pi^d = \Pi^c \tag{I.4.3}$$

Inflation of final public goods prices

$$\Pi^g = \Pi^c \tag{I.4.4}$$

Imported inflation I.3.28

$$\Pi^{mG} = \frac{\Pi^c}{1 + \gamma^{a^x}} \tag{I.4.5}$$

Growth rate of output

$$\gamma^y = \gamma^z \tag{I.4.6}$$

Rate of appreciation of nominal exchange rate is derived from I.3.45

$$1 + \gamma^{Gel/R} = \frac{\Pi^c}{(1 + \gamma^{a^x})\Pi^R} \tag{I.4.7}$$

From the definition of nominal effective rate and exogenous trend rate of appreciation of effective exchange rate of USD w.r.t. trade partners currencies, we can derive the rate of appreciation of GEL/USD:

$$1+\gamma^{e^{Gel/D}} = \frac{1+\gamma^{e^{Gel/R}}}{1+\gamma^{e^{D/R}}} \tag{I.4.8}$$

Inflation of exported goods in USD

$$\Pi^{xf} = (1+\gamma^{e^{D/R}})(1+\gamma^{a^r})^{\frac{2}{1-\gamma_x}}\Pi^R \tag{I.4.9}$$

Inflation of exported goods in GEL

$$\Pi^{xG} = (1+\gamma^{e^{Gel/D}})\Pi^{xf} \tag{I.4.10}$$

Foreign inflation in USD

$$\Pi^f = (1+\gamma^{e^{D/R}})\Pi^R \tag{I.4.11}$$

Imported inflation in USD

$$\Pi^{mf} = \frac{\Pi^{mG}}{1+\gamma^{e^{Gel/D}}} \tag{I.4.12}$$

Export specific productivity growth

$$1+\gamma^{a^r} = \left(\frac{(1+\gamma^{a^x})(1+\gamma^z)}{(1+\gamma^{z^*})}\right)^{\frac{1-\eta_x}{2(1-\varepsilon^x)}} \tag{I.4.13}$$

The relative optimal prices of domestic intermediate input, differentiated imported and exported goods, as well as relative optimal wage in SS are given by equations I.3.3, I.3.18, I.3.39 and I.3.48:

$$\frac{W^*}{W} = 1 \tag{I.4.14}$$

$$\frac{P^{*d}}{P^d} = 1 \tag{I.4.15}$$

$$\frac{P^{*mf}}{P^{mf}} = 1 \tag{I.4.16}$$

$$\frac{P^{*xf}}{P^{xf}} = 1 \tag{I.4.17}$$

Foreign gross interest rate, USD

$$R^f = 1 + i^f \tag{I.4.18}$$

Foreign gross interest rate, ROW

$$1 + i^{rw} = 1 + i^f \tag{I.4.19}$$

Foreign real interest rate

$$r^f = \frac{1 + i^f}{\Pi^f} - 1 \tag{I.4.20}$$

Foreign real neutral rate

$$r^{fnut} = r^f - \widehat{r^f} \tag{I.4.21}$$

From market clearing condition the price and wage dispersion in SS is given by I.3.85 and I.3.87

$$d^d = 1 \tag{I.4.22}$$

$$d^w = 1 \tag{I.4.23}$$

Labor supply I.3.86

$$L^s = L \tag{I.4.24}$$

From trend relations:

$$1 + \gamma^{GDP} = (1 + \gamma^z)\Pi^c \tag{I.4.25}$$

The equations I.3.38 , I.3.41 and I.3.43 together with I.3.39 implies that

$$MC^{mr} = \frac{\varepsilon^m - 1}{\varepsilon^m} \tag{I.4.26}$$

Taking into account our assumption that $\widetilde{REER} = 1$ and by using the equation I.3.44, we get that:

$$p^{mG} = \frac{1}{MC^{mr}} \widetilde{REER} = \frac{\varepsilon^m}{\varepsilon^m - 1} \tag{I.4.27}$$

The equation determines relative price of imported goods, given that the equation

I.3.33 pins down the relative price of domestic intermediate inputs

$$p^d = \left(\frac{1}{1-\omega_c} \left(1 - \omega_c {p^{mG}}^{1-\eta_c} \right)^{\frac{1}{1-\eta_c}} \right) \tag{I.4.28}$$

From the equations I.3.37 and I.3.72, the relative prices of final investment and public goods are given by:

$$p^i = \left((1-\omega_i)\, {p^d}^{1-\eta_i} + \omega_i {p^{mG}}^{1-\eta_i} \right)^{\frac{1}{1-\eta_i}} \tag{I.4.29}$$

$$p^g = \left((1-\omega_g) {p^d}^{1-\eta_g} + \omega_g {p^{mG}}^{1-\eta_g} \right)^{\frac{1}{1-\eta_g}} \tag{I.4.30}$$

The equations I.3.47, I.3.50 and I.3.51 implies:

$$\frac{P^{*xf}}{P^{xf}} = \frac{\varepsilon^x}{\varepsilon^x - 1} MC^{x^r}$$

Then using the equation I.3.48

$$MC^{x^r} = \frac{\varepsilon^x - 1}{\varepsilon^x} \tag{I.4.31}$$

From equation I.3.52 the relative price of exported goods is given by:

$$p^{xG} = \frac{\varepsilon^x}{\varepsilon^x - 1} \left[(1-\omega_x) {p^d}^{1-\eta_x} + \omega_x {p^{mG}}^{1-\eta_x} \right]^{\frac{1}{1-\eta_x}} \tag{I.4.32}$$

As long as the relative price indexes of all GDP components have already been defined in SS, then the relative price index of GDP deflator is determined with the equation I.3.102

$$p^Y = {p^g}^{s_g} {p^I}^{s_I} {p^{xG}}^{s_x} {p^{mG}}^{-s_m} \tag{I.4.33}$$

Inflation of final investment goods I.3.36

$$\Pi^I = \left[(1-\omega_i) \left(\Pi^d \frac{p^d}{p^I} \right)^{1-\eta_i} + \omega_i \left(\Pi^{mG} (1+\gamma^{a^x}) \frac{p^{mG}}{p^I} \right)^{1-\eta_i} \right]^{\frac{1}{1-\eta_i}} \tag{I.4.34}$$

Inflation of GDP deflator I.3.100

$$\Pi^Y = (\Pi^c)^{s_c} \left(\Pi^I\right)^{s_i} (\Pi^g)^{s_g} \left(\Pi^{xG}\right)^{s_x} \left(\Pi^{mG}\right)^{-s_m} \tag{I.4.35}$$

By taking into account the equation I.3.13, then the equation I.3.16 could be written as:

$$r^k = \left(\frac{1+\gamma^z-(1-\delta)\beta}{\beta}\right)p^I \tag{I.4.36}$$

Also,

$$\widetilde{\lambda^e} = p^I \tag{I.4.37}$$

From I.3.12

$$\gamma'(u) = \sigma_b \tag{I.4.38}$$

While the equation I.3.11

$$r^k = \sigma_b p^i$$

Taking into account our assumption that $u = 1$, then

$$\gamma(u) = 0 \tag{I.4.39}$$

The above two conditions of real rental rate implies following restriction to calibrate following the parameters:

$$\sigma_b = \left(\frac{1+\gamma^z-(1-\delta)\beta}{\beta}\right)$$

From the equation I.3.22 the real marginal cost of domestic intermediate input producer is:

$$MC^{d^r} = \frac{\eta_d - 1}{\eta_d} \tag{I.4.40}$$

Then from the equation I.3.24 the steady state level of real wage is determined with:

$$\widetilde{W^r} = \left(\frac{1}{\alpha_1{}^{\alpha_1}\alpha_2{}^{\alpha_2}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}}\frac{\eta_l}{\eta_l-1}\frac{1}{\gamma}\frac{1}{p^d}\left(r^k\right)^{\alpha_2}\left(p^{mG}\right)^{1-\alpha_1-\alpha_2}\right)^{-\frac{1}{\alpha_1}} \tag{I.4.41}$$

Using I.3.87

$$\widetilde{w^r} = \widetilde{W^r} \tag{I.4.42}$$

The equation I.3.25 could be rearranged as:

$$L = \left(\frac{{\alpha_1}^{1-\alpha_1}}{{\alpha_2}^{\alpha_2}(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}} \right) \left(\frac{\widetilde{W^r}^{\alpha_1} r^{k\alpha_2} \left(p^{mG}\right)^{1-\alpha_1-\alpha_2}}{\widetilde{W^r}} \right) \widetilde{Y}$$

After substituting the term $\widetilde{W^r}^{\alpha_1} r^{k\alpha_2} \left(p^{mG}\right)^{1-\alpha_1-\alpha_2}$ in the above equation with I.3.24, we get

$$L = \alpha_1 \frac{MC^{d^r}}{\widetilde{W^r}} p^d \widetilde{Y^d}$$

We assume that workers in the economy spend 1/3 of their time at their working places, i.e. $L = 1/3$, then the above equation determines steady-state level of domestic intermediate inputs, also, from the market clearing condition:

$$\widetilde{Y^d} = \widetilde{Y} \tag{I.4.43}$$

$$\widetilde{Y} = \frac{1}{\alpha_1} \frac{\widetilde{W^r}}{MC^{d^r} p^d} L \tag{I.4.44}$$

Applying the same transformation to the demand on capital and imported inputs in domestic intermediate input production, then

$$\widetilde{K} = \alpha_2 \frac{MC^{d^r}}{r^k} p^d \widetilde{Y} \tag{I.4.45}$$

And

$$\widetilde{Y^m} = (1-\alpha_1-\alpha_2) \frac{MC^{d^r}}{p^{mG}} p^d \widetilde{Y} \tag{I.4.46}$$

The fixed cost in SS is calibrated so as to equalize the profit of domestic intermediate input producers to zero

$$F^d = \frac{1 - MC^{r^d}}{MC^{r^d}} \widetilde{Y} \tag{I.4.47}$$

From the entrepreneurs' problem, also, using the market clearing condition:

$$\widetilde{\widetilde{K}} = \widetilde{K} \tag{I.4.48}$$

Then from the equation I.3.17:

$$\widetilde{I} = (\gamma^z + \delta) \widetilde{K} \tag{I.4.49}$$

Now the ground is ready to derive the steady state values of variables related to Households decision, taking into account that consumer price inflation is anchored with the inflation target, then

$$R = \frac{(1+\gamma^z)\Pi^c}{\beta} \tag{I.4.50}$$

Exogenous risk premium

$$R^\rho = \frac{R}{R^f(1+\gamma^{e^{Gel/D}})} \tag{I.4.51}$$

The domestic real neutral interest rate is given by:

$$1 + r^{nut} = \frac{R^f R^\rho}{1+\gamma^{a^x}} \tag{I.4.52}$$

Sovereign risk premium (neutral)

$$R^{\rho^{nut}} = \frac{R^\rho}{\widehat{R^\rho}} \tag{I.4.53}$$

With the gross nominal rate in hand:

$$i^N = R - 1 \tag{I.4.54}$$

Then

$$i = i^N \tag{I.4.55}$$

The wage inflation is given by the equation I.3.10

$$\Pi^w = \Pi^c(1+\gamma^z) \tag{I.4.56}$$

Now, let's derive profits in final goods production in the steady-state. After plugging the equations I.3.29 and I.3.30 into I.3.93:

$$\widetilde{\pi r^{cr}} = \widetilde{C} - (1-\omega_c){p^d}^{1-\eta_c}\widetilde{C} - \omega_c {p^{mG}}^{1-\eta_c}\widetilde{C}$$

By using the equation I.3.31

$$\widetilde{\pi r^{cr}} = \widetilde{C} - \widetilde{C} = 0 \tag{I.4.57}$$

Moreover, it implies that:

$$\widetilde{C} = p^d \widetilde{C^d} + p^{mG} \widetilde{C^m}$$

Also, by using equations I.3.34, I.3.35 and I.3.94, the real profit in final investment goods sector:

$$\widetilde{\pi r^{I^r}} = p^I \widetilde{I} - p^d \left(1 - \omega_i\right) \left(\frac{p^d}{p^I}\right)^{-\eta_i} \widetilde{I} - p^{mG} \omega_i \left(\frac{p^{mG}}{p^I}\right)^{-\eta_i} \widetilde{I}$$

Then

$$\widetilde{\pi r^{I^r}} = p^I \widetilde{I} - \left(1 - \omega_i\right) p^{I^{\eta_i}} \left(p^d\right)^{1-\eta_i} \widetilde{I} - \omega_i p^{I^{\eta_i}} \left(p^{mG}\right)^{1-\eta_i} \widetilde{I}$$

$$\widetilde{\pi r^{I^r}} = p^I \widetilde{I} - \left(\left(1 - \omega_i\right) \left(p^d\right)^{1-\eta_i} \widetilde{I} + \omega_i \left(p^{mG}\right)^{1-\eta_i}\right) p^{I^{\eta_i}} \widetilde{I}$$

By using the equation I.3.37

$$\widetilde{\pi r^{I^r}} = p^I \widetilde{I} - p^I \widetilde{I} = 0 \tag{I.4.58}$$

It implies:

$$p^I \widetilde{I} = p^d \widetilde{I^d} + p^{mG} \widetilde{I^m}$$

Taking into account the equations I.4.81 and I.4.82, the *Profit in public goods production* is rewritten as:

$$\widetilde{\pi r^{g^r}} = \widetilde{G} - \left(1 - \omega_g\right) p^{d^{1-\eta_g}} p^{g^{\eta_g}} \widetilde{Y^G} - \omega_g p^{mG^{1-\eta_g}} p^{g^{\eta_g}} \widetilde{Y^g}$$

$$\widetilde{\pi r^{g^r}} = \widetilde{G} - \left(\left(1 - \omega_g\right) p^{d^{1-\eta_g}} + \omega_g p^{mG^{1-\eta_g}}\right) p^{g^{\eta_g}} \widetilde{Y^g}$$

Using the equations I.3.69 and I.3.72 , we conclude that the profit in public goods production is zero.

$$\widetilde{\pi r^{g^r}} = p^g \widetilde{G} - p^{g^{1-\eta_g}} p^{g^{\eta_g}} \widetilde{Y^G} = \widetilde{G^r} - p^g \widetilde{Y^g} = 0 \tag{I.4.59}$$

Also, it implies that

$$\widetilde{G} = p^d \widetilde{G^d} + p^{mG} \widetilde{G^m}$$

After substituting the inputs used in export production with their demand functions I.3.56 and I.3.57,

$$\widetilde{\pi r^{xr}} = p^{xG}\widetilde{a}\widetilde{X} - (1-\omega_x){p^{d}}^{1-\eta_x}\left(p^{xG}MC^{x^r}\right)^{\eta_x}\left(\widetilde{X}+\widetilde{F^x}\right) - \omega_x {p^{mG}}^{1-\eta_x}\left(p^{xG}MC^{x^r}\right)^{\eta_x}\left(\widetilde{X}+\widetilde{F^x}\right)$$

After rearranging terms and using I.3.52 we get

$$\widetilde{\pi r^{xr}} = p^{xG}\widetilde{a}\widetilde{X} - \left((1-\omega_x){p^{d}}^{1-\eta_x} - \omega_x {p^{mG}}^{1-\eta_x}\right)\left(p^{xG}MC^{x^r}\right)^{\eta_x}\left(\widetilde{X}+\widetilde{F^x}\right)$$

Finally,

$$\widetilde{\pi r^{xr}} = p^{xG}\widetilde{a}\widetilde{X} - \left(p^{xG}MC^{x^r}\right)\left(\widetilde{X}+\widetilde{F^x}\right)$$

We assumed that the relative technology factor equals to one in SS, i.e. $\widetilde{a} = 1$, then

$$\widetilde{\pi r^{xr}} = \left(1 - MC^{x^r}\right)p^{xG}\widetilde{X} - p^{xG}MC^{x^r}\widetilde{F^x} \tag{I.4.60}$$

The fixed cost is calibrated by making the assumption that the firm's profit is zero after paying the cost, then:

$$\widetilde{F^x} = \frac{1 - MC^{x^r}}{MC^{x^r}}\widetilde{X} \tag{I.4.61}$$

Also, it follows that

$$MC^{x^r}p^{xG}\left(\widetilde{X}+\widetilde{F^x}\right) = p^d\widetilde{X^d} + p^{mG}\widetilde{X^m}$$

The real marginal cost in the export sector is given by:

$$MC^{x^r} = \frac{\varepsilon^x - 1}{\varepsilon^x}$$

Subsequently, positive profit is generated in the export sector in the steady-state.
Let's substitute the steady state values of inputs used in domestic intermediate input producer's profit function, we get:

$$\widetilde{\pi r^{d^r}} = p^d\widetilde{Y^d} - \alpha_2 MC^{d^r}p^d\left(\widetilde{Y}+\widetilde{F^d}\right) - \alpha_1 MC^{d^r}p^d\left(\widetilde{Y}+\widetilde{F^d}\right) - (1-\alpha_1-\alpha_2)MC^{d^r}p^d\left(\widetilde{Y}+\widetilde{F^d}\right)$$

It follows that

$$\widetilde{\pi r^{d^r}} = \left(1 - MC^{d^r}\right) p^d \widetilde{Y} - MC^{d^r} p^d \widetilde{F^d} \tag{I.4.62}$$

Given that the real marginal cost is less than one in SS, positive profit is generated in domestic intermediate input production in SS before paying the fixed cost. Here, we assume that firm's profit after paying the amortization cost is zero in SS.

Now to derive the steady state value of profit of the entrepreneur, we substitute the capital with investment in its profit function in SS, and we get:

$$\widetilde{\pi r^e} = \frac{r^k}{\gamma^z + \delta} \widetilde{I} - p^I \widetilde{I} = \left(\frac{r^k}{\gamma^z + \delta} - p^I\right) \widetilde{I}$$

Using the steady state relationship between rental rate and the relative price of investment:

$$\widetilde{\pi r^e} = \left(\frac{1 + \gamma^z - (1-\delta)\beta}{\beta} \frac{1}{\gamma^z + \delta} - 1\right) p^I \widetilde{I}$$

After rearranging some terms, we get:

$$\widetilde{\pi r^e} = \left(\frac{(1 + \gamma^z)(1 - \beta)}{\beta(\gamma^z + \delta)}\right) p^I \widetilde{I} \tag{I.4.63}$$

Given that the discount factor is less than one, the entrepreneur earns positive profit in SS.

The sustainable ratio of foreign assets to output, meaning the ratio in the steady-state consistent with the debt sustainability conditions, is calibrated outside of the model, given that the profit of forex dealer I.3.92 is rewritten as:

$$\widetilde{\pi r^{fx}} = \left(R^f R^\rho \frac{1 + \gamma^{e^{Gel/D}}}{\Pi^d (1 + \gamma^y)} - 1\right) b^f p^d \widetilde{Y}$$

Taking into account trend relations we can substitute $1 + \gamma^y = \Pi^c (1 + \gamma^z)$ from euler equation, then

$$\widetilde{\pi r^{fx}} = \left(R^f R^\rho \frac{1 + \gamma^{e^{Gel/D}}}{R\beta} - 1\right) b^f p^d \widetilde{Y}$$

Furthermore, if we substitute the steady state value of R using the UIP condition in

SS.

$$\widetilde{\pi r^{fx}} = \left(\frac{1-\beta}{\beta}\right) b^f p^d \widetilde{Y} \tag{I.4.64}$$

As long as $\beta < 1$ the forex dealer earns positive profit in the steady-state. As mentioned profits in final consumption, investment, and public goods production are zero in SS, as well as profits of domestic intermediate input producers and differentiated export goods producers are zero after paying fixed costs. By taking into account those facts the total profit generated in the economy could be written as:

$$\widetilde{\pi r^{T^r}} = \left(\frac{(1+\gamma^z)(1-\beta)}{\beta(\gamma^z+\delta)}\right) p^I \widetilde{I} + \left(\frac{1-\beta}{\beta}\right) b^f p^d \widetilde{Y}$$

After substituting the investment in the above equation:

$$\widetilde{\pi r^{T^r}} = \left(\frac{(1+\gamma^z)(1-\beta)}{\beta(\gamma^z+\delta)}\right) p^I (\gamma^z+\delta)\beta \frac{MC^{d^r}}{r^k} p^d \widetilde{Y} + \left(\frac{1-\beta}{\beta}\right) b^f p^d \widetilde{Y}$$

Let's collect the same terms:

$$\widetilde{\pi r^{T^r}} = \left(\left(\frac{(1+\gamma^z)(1-\beta)}{\beta(\gamma^z+\delta)}\right) p^I (\gamma^z+\delta)\beta \frac{MC^{d^r}}{r^k} + \left(\frac{1-\beta}{\beta}\right) b^f\right) p^d \widetilde{Y} \tag{I.4.65}$$

The government balance (to output) is given by:

$$gb = \left(1 - \frac{R}{\Pi^d(1+\gamma^y)}\right) d \tag{I.4.66}$$

CA balance (to output):

$$ca = \left(1 - R^f R^\rho \frac{1+\gamma^{e^{Gel/D}}}{\Pi^d(1+\gamma^y)}\right) b^f \tag{I.4.67}$$

In domestic intermediate inputs production market clearing requires that:

$$p^d \widetilde{Y^d} = p^d \widetilde{X^d} + p^d \widetilde{G^d} + p^d \widetilde{C^d} + p^d \widetilde{I^d}$$

Now our objective is to derive the SS value of $\widetilde{C}$ as a function of $\widetilde{Y}$ which helps us to solve values of other variables which are dependent on $\widetilde{C}$. Let's add and then subtract

$p^{mG}\widetilde{X^m}$ in the market clearing condition of domestic intermediate inputs

$$p^d\widetilde{Y} = p^d\widetilde{C^d} + p^d\widetilde{I^d} + p^d\widetilde{G^d} + p^d\widetilde{X^d} + p^{mG}\widetilde{X^m} - p^{mG}\widetilde{X^m}$$

Now recall that $p^{xG}\widetilde{X} = MC^{x^r}p^{xG}\left(\widetilde{X} + \widetilde{F^d}\right) = p^d\widetilde{X^d} + p^{mG}\widetilde{X^m}$ from the profit function of exported goods producer, then

$$p^d\widetilde{Y} = p^d\widetilde{C^d} + p^d\widetilde{I^d} + p^d\widetilde{G^d} + p^{xG}\widetilde{X} - p^{mG}\widetilde{X^m}$$

Now, let's substitute $p^{xG}\widetilde{X}$ from the CA balance:

$$p^d\widetilde{Y} = p^d\widetilde{C^d} + p^d\widetilde{I^d} + p^d\widetilde{G^d} + cap^d\widetilde{Y} + p^{mG}\widetilde{M} - p^{mG}\widetilde{X^m}$$

Using the market clearing condition on imported input we can write:

$$p^d\widetilde{Y} = p^d\widetilde{C^d} + p^d\widetilde{I^d} + p^d\widetilde{G^d} + cap^d\widetilde{Y} + p^{mG}\left(\widetilde{Y^m} + \widetilde{C^m} + \widetilde{I^m} + \widetilde{G^m} + \widetilde{X^m}\right) - p^{mG}\widetilde{X^m}$$

After rearranging the same terms:

$$p^d\widetilde{Y} = p^d\widetilde{C^d} + p^{mG}\widetilde{C^m} + p^d\widetilde{I^d} + p^{mG}\widetilde{I^m} + p^d\widetilde{G^d} + p^{mG}\widetilde{G^m} + p^{mG}\widetilde{Y^m} + cap^d\widetilde{Y}$$

Now, let's recall the results from the derivations of profits in final consumption, investment, and public goods production in SS, we can write:

$$p^d\widetilde{Y} = \widetilde{C} + p^I\widetilde{I} + \widetilde{G} + p^{mG}\widetilde{Y^m} + cap^d\widetilde{Y}$$

Also, it could be written that:

$$\widetilde{GDP} = p^d\widetilde{Y} - p^{mG}\widetilde{Y^m} \tag{I.4.68}$$

We have already solved all variables except $\widetilde{C}$ in real tax revenue [32]:

$$\widetilde{T^r} = \tau^c \widetilde{C} + \tau^w \widetilde{W^r} L + \tau^{\pi r} \widetilde{\pi^{T^r}} \tag{I.4.69}$$

The government spending could be written as:

$$\widetilde{G} = \widetilde{T^r} - \widetilde{T^{tr}} - gbp^d \widetilde{Y} \tag{I.4.70}$$

Then by substituting $\widetilde{G}$ from the government's budget balance, we get:

$$p^d \widetilde{Y} = \widetilde{C} + p^I \widetilde{I} + \widetilde{T^r} - \widetilde{T^{tr}} - gbp^d \widetilde{Y} + p^{mG} \widetilde{Y^m} + cap^d \widetilde{Y}$$

Also, let's substitute $\widetilde{Y^m}$ and real transfers to HHs.

$$p^d \widetilde{Y} = \widetilde{C} + p^I \widetilde{I} + (\tau^c \widetilde{C} + \tau^w \widetilde{W^r} L + \tau^{\pi r} \widetilde{\pi^{T^r}}) - t^{tr} p^d \widetilde{Y} - gbp^d \widetilde{Y} + (1 - \alpha_1 - \alpha_2) p^d \widetilde{Y^d} + cap^d \widetilde{Y}$$

We note that $\widetilde{I} = (\gamma^z + \delta) \widetilde{K}$ in SS, in addition, as long as the SS value of $\widetilde{K}$ has already been derived, we can write:

$$\begin{aligned} p^d \widetilde{Y} =& \widetilde{C} + \alpha_2 (\gamma^z + \delta) p^I \frac{p^d}{r^k} \widetilde{Y} + (\tau^c C + \tau^w \widetilde{W^r} L + \tau^{\pi r} \widetilde{\pi^{T^r}}) - t^{tr} p^d \widetilde{Y} - \\ & - gbp^d \widetilde{Y} + (1 - \alpha_1 - \alpha_2) p^d \widetilde{Y} + cap^d \widetilde{Y} \end{aligned}$$

After rearranging same terms we get:

$$\left(1 - \alpha_2 (\gamma^z + \delta) \frac{p^I}{r^k} + t^{tr} + gb - ca - (1 - \alpha_1 - \alpha_2)\right) p^d \widetilde{Y} = (1 + \tau^c) \widetilde{C} + \tau^w \widetilde{W^r} L + \tau^{\pi r} \widetilde{\pi^{T^r}}$$

Let's denote:

$$\Gamma_0 \equiv \left(1 - \alpha_2 (\gamma^z + \delta) \frac{p^I}{r^k} + t^{tr} + gb - ca - (1 - \alpha_1 - \alpha_2)\right)$$

[32] Note, that ss values of all variables within the expression are not known yet, but we keep it here to save the space and avoid rewriting it at the end, same is true about the $\widetilde{G}$ as well.

Then $\widetilde{C}$ as a function of $\widetilde{Y}$ could be given as:

$$\widetilde{C} = \frac{1}{1+\tau^c}\left(\Gamma_0 p^d \widetilde{Y} - \tau^w \widetilde{W^r} L - \tau^{\pi r} \widetilde{\pi^{T^r}}\right) \tag{I.4.71}$$

The SS value of $\widetilde{C^c}$ is:

$$\widetilde{C^c} = \frac{(1-\tau^w)}{(1+\tau^c)}\widetilde{W^r} L + \frac{1}{(1+\tau^c)}\widetilde{T^{cr}} \tag{I.4.72}$$

With $\widetilde{C}$ and $\widetilde{C^c}$ in hand the $\widetilde{C^{uc}}$ is given by:

$$\widetilde{C^{uc}} = \frac{1}{1-\lambda}\left(\widetilde{C} - \lambda \widetilde{C^c}\right) \tag{I.4.73}$$

The SS values set restrictions on the values of some parameters:

$$\chi = \left(\frac{\eta^l(1+\zeta)(1+\tau^c)}{(\eta^l-1)(1-\tau^w)}\frac{(1+\gamma^z-h)\widetilde{C^{uc}}}{\psi(1+\gamma^z)\widetilde{W^r}}\theta L\right)^{-1}$$

The auxiliary variables C_1 and C_2 are given by I.3.5 and I.3.6

$$C_1 = \frac{1}{1-\beta\theta_w}\frac{1+\gamma^z}{1+\gamma^z-h}\frac{\Pi^{w\eta_l}\widetilde{W^r}L}{\widetilde{C^{uc}}} \tag{I.4.74}$$

$$C_2 = \frac{1}{1-\beta\theta_w}\psi\chi L^{1+\zeta} \tag{I.4.75}$$

SS values of domestic I.3.29 and imported inputs I.3.30 in final consumption goods production:

$$\widetilde{C^d} = (1-\omega_c) p^{d^{-\eta_c}} \widetilde{C} \tag{I.4.76}$$

$$\widetilde{C^m} = \omega_c p^{mG^{-\eta_c}} \widetilde{C} \tag{I.4.77}$$

Also, I.3.34 and I.3.35 determine values of inputs in final investment goods production

$$\widetilde{I^d} = (1-\omega_i)\left(\frac{p^d}{p^i}\right)^{-\eta_i}\widetilde{I} \tag{I.4.78}$$

$$\widetilde{I^m} = \omega_i\left(\frac{p^{mG}}{p^i}\right)^{-\eta_i}\widetilde{I} \tag{I.4.79}$$

The market clearing condition on public goods I.3.69

$$p^g \widetilde{Y^g} = \widetilde{G} \tag{I.4.80}$$

The demand for domestic and imported inputs in public goods production:

$$\widetilde{G^d} = (1-\omega_g)\left(\frac{p^d}{p^g}\right)^{-\eta_g} \widetilde{Y^g} \tag{I.4.81}$$

$$\widetilde{G^m} = \omega_g\left(\frac{p^{mG}}{p^g}\right)^{-\eta_g} \widetilde{Y^g} \tag{I.4.82}$$

Let's solve SS values for other fiscal-related variables:

$$\widetilde{T^{ucr}} = t^{ucr} p^d \widetilde{Y} \tag{I.4.83}$$

$$\widetilde{T^{cr}} = t^{cr} p^d \widetilde{Y} \tag{I.4.84}$$

Total transfers

$$\widetilde{T^{tr}} = \widetilde{T^{ucr}} + \widetilde{T^{cr}} \tag{I.4.85}$$

While the auxiliary variables in domestic intermediate input production D_1 and D_2 are determined with I.3.20 and I.3.21

$$D_1 = \frac{1}{1-\theta_d\beta}\frac{\psi(1+\gamma^z)}{1+\gamma^z-h}\frac{p^d\widetilde{Y}MC^{r^d}}{\widetilde{C^{uc}}} \tag{I.4.86}$$

And,

$$D_2 = \frac{1}{1-\theta_d\beta}\frac{\psi(1+\gamma^z)}{1+\gamma^z-h}\frac{p^d\widetilde{Y}}{\widetilde{C^{uc}}} \tag{I.4.87}$$

To derive the SS value of export, we need to rewrite the ca balance

$$p^{xG}\widetilde{X} = cap^d\widetilde{Y} + p^{mG}\left(\widetilde{C^m} + \widetilde{I^m} + \widetilde{G^m} + \widetilde{Y^m} + \omega_x\left(\frac{p^{mG}}{p^{xG}MC^{r^x}}\right)^{-\eta^x}\frac{\widetilde{X}}{MC^{r^x}}\right)$$

Then

$$\left(p^{xG} - \omega_x \left(\frac{p^{mG}}{p^{xG} MC^{r^x}}\right)^{-\eta^x} \frac{1}{MC^{r^x}}\right) \widetilde{X} = cap^d \widetilde{Y} + p^{mG} \left(\widetilde{C^m} + \widetilde{I^m} + \widetilde{G^m} + \widetilde{Y^m}\right)$$

Let's introduce the following definition:

$$\Gamma_1 \equiv \left(p^{xG} - \omega_x \left(\frac{p^{mG}}{p^{xG} MC^{r^x}}\right)^{-\eta^x} \frac{1}{MC^{r^x}}\right)$$

Finally,

$$\widetilde{X} = \frac{1}{\Gamma_1} \left(cap^d \widetilde{Y} + p^{mG} \left(\widetilde{C^m} + \widetilde{I^m} + \widetilde{G^m} + \widetilde{Y^m}\right)\right) \tag{I.4.88}$$

After that, we can derive the SS value of foreign demand from I.3.55

$$\widetilde{Y^*} = \frac{1}{\omega_w} \widetilde{X} \tag{I.4.89}$$

The demand for inputs in exported goods production in SS:

$$\widetilde{X^d} = (1 - \omega_x) \left(\frac{p^d}{p^{xG} MC^{x^r}}\right)^{-\eta_x} \widetilde{X} \tag{I.4.90}$$

$$\widetilde{X^m} = \omega_x \left(\frac{p^{mG}}{p^{xG} MC^{x^r}}\right)^{-\eta_x} \widetilde{X} \tag{I.4.91}$$

The real GDP from I.3.99

$$\widetilde{GDP^r} = \frac{\widetilde{GDP}}{p^y} \tag{I.4.92}$$

The domestic absorption in steady state I.3.104

$$\widetilde{ABS^r} = \widetilde{C} + \widetilde{G^r} + p^I \widetilde{I} \tag{I.4.93}$$

The market clearing on imported inputs I.3.97

$$\widetilde{M} = \widetilde{Y^m} + \widetilde{C^m} + \widetilde{I^m} + \widetilde{G^m} + \widetilde{X^m} \tag{I.4.94}$$

The auxiliary variables related to Phillips curve in import sector I.3.41 and I.3.43

$$\widetilde{a_1} = \frac{1}{1 - \frac{\theta_m}{R^f}(1+\gamma^z)(1+\gamma^{a^x})\Pi^{mf}} \widetilde{M}MC^{m^r} \tag{I.4.95}$$

$$\widetilde{a_2} = \frac{1}{1 - \frac{\theta_m}{R^f}(1+\gamma^z)(1+\gamma^{a^x})\Pi^{mf}} \widetilde{M} \tag{I.4.96}$$

While the auxiliary variables in export sector is given by I.3.49 and I.3.50

$$B_1 = \frac{\psi}{(\widetilde{C^{uc}} - \frac{h}{1+\gamma^z}\widetilde{C^{uc}})} p^{xG} \widetilde{a}\widetilde{X}MC^{x^r} + \theta_x \beta E_t B_1$$

$$B_1 = \frac{1}{1-\beta\theta_x} \frac{\psi(1+\gamma^z)}{(1+\gamma^z - h)\widetilde{C^{uc}}} p^{xG} \widetilde{a}\widetilde{X}MC^{x^r} \tag{I.4.97}$$

also,

$$B_2 = \frac{1}{1-\beta\theta_x} \frac{\psi(1+\gamma^z)}{(1+\gamma^z - h)\widetilde{C^{uc}}} p^{xG} \widetilde{a}\widetilde{X} \tag{I.4.98}$$

I.5 Model properties

I.5.1 IRFs

Figure 2: Monetary policy shock

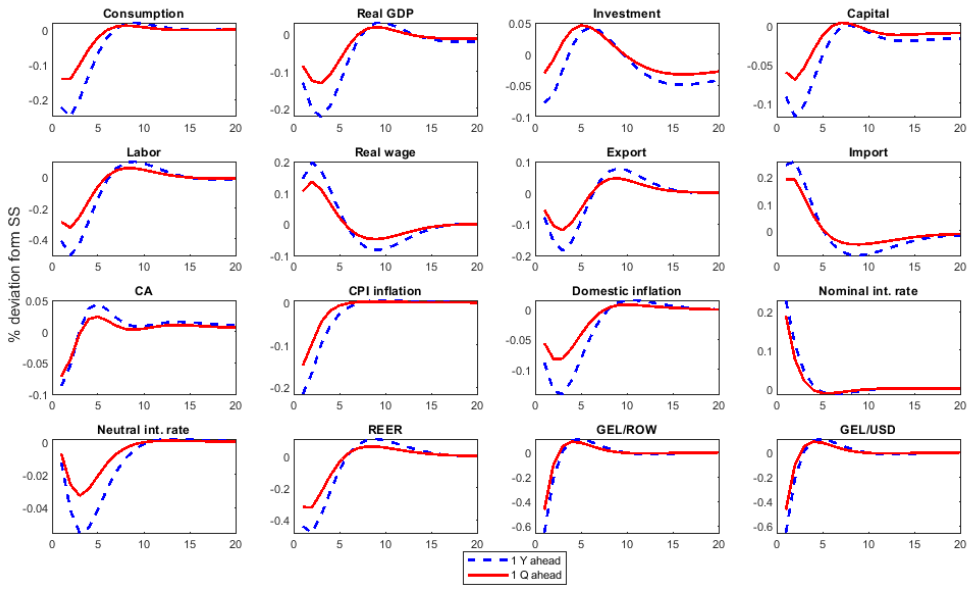


Figure 3: Shock to inflation target

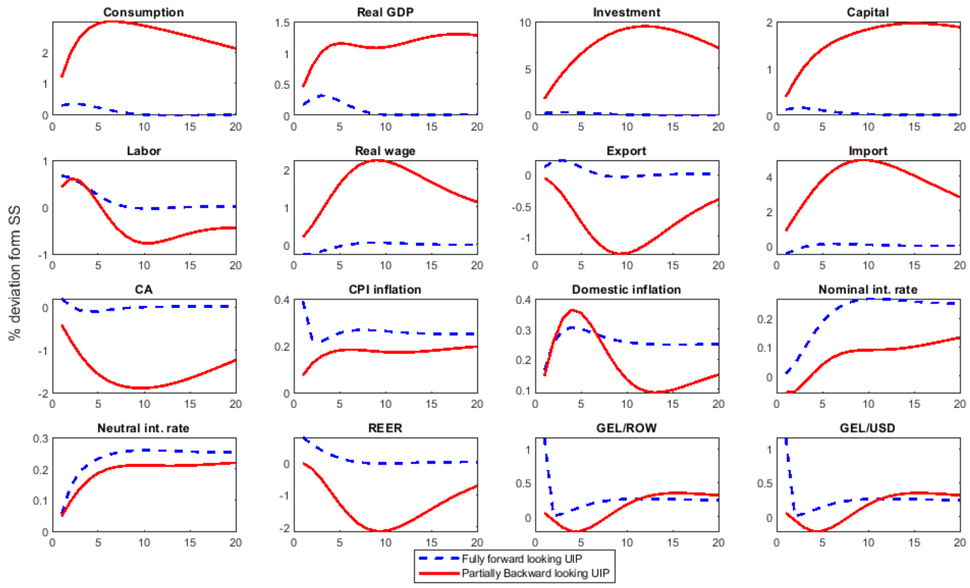


Figure 4: Fiscal consolidation shock

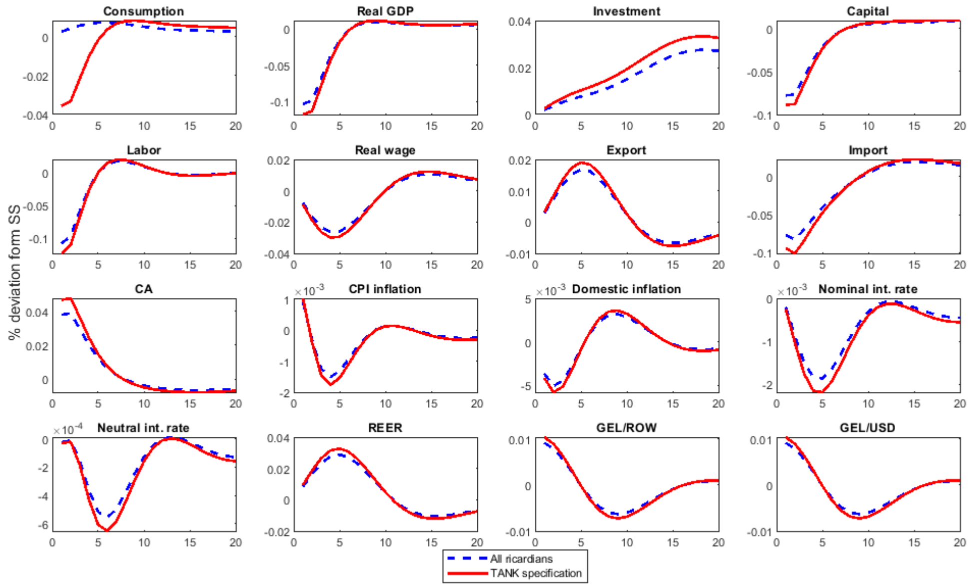


Figure 5: Trnasfers to HHs.

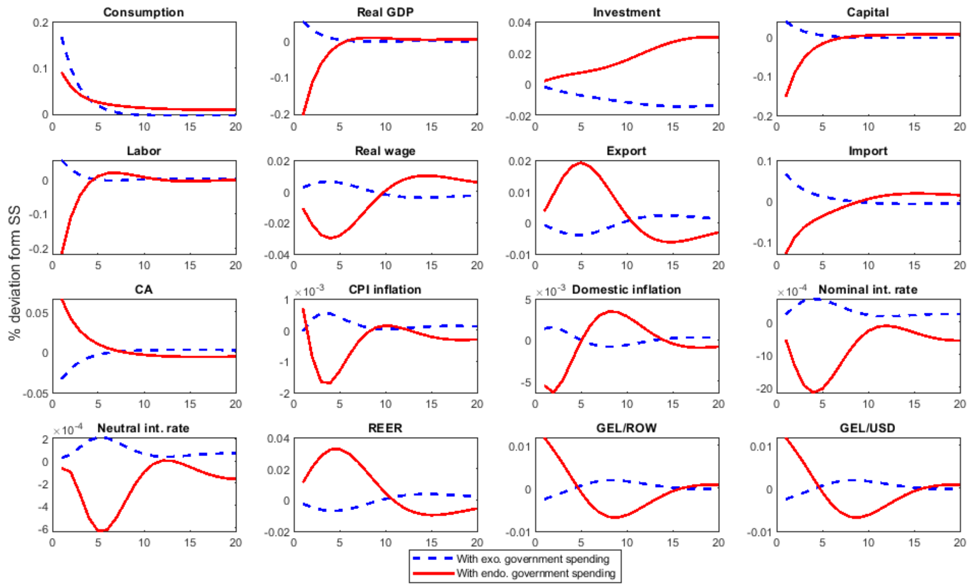


Figure 6: Preference shock

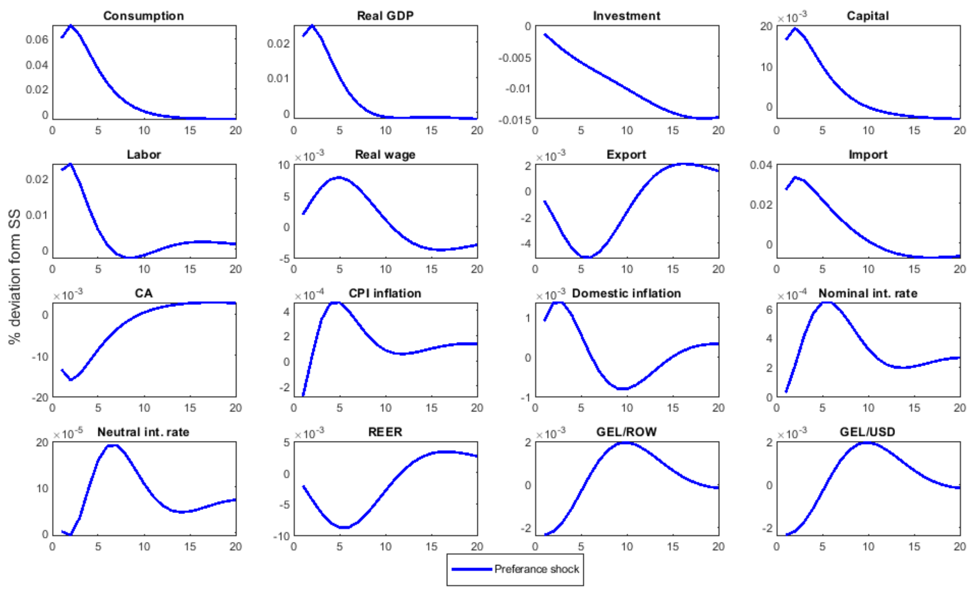


Figure 7: Real neutral interest rate shock

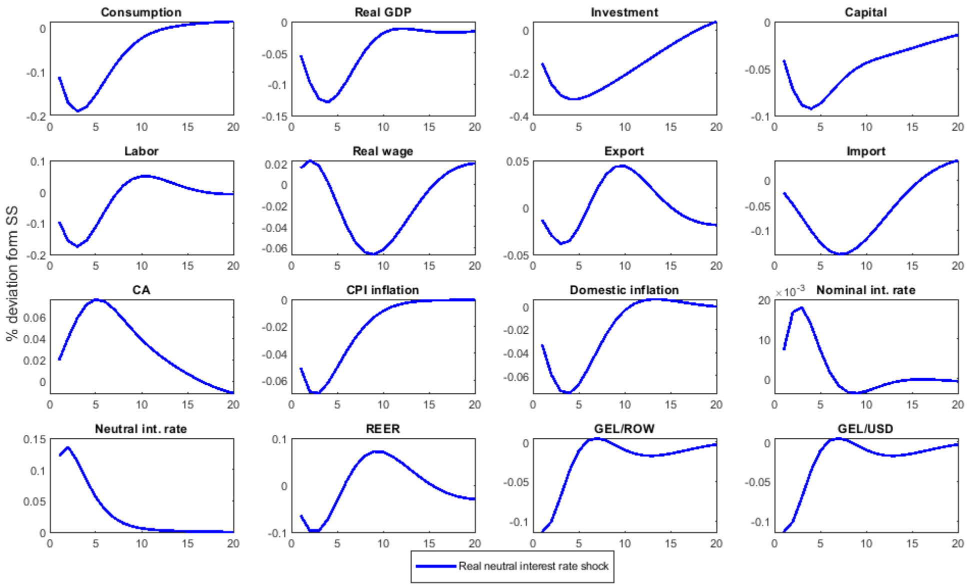


Figure 8: Labor supply shock

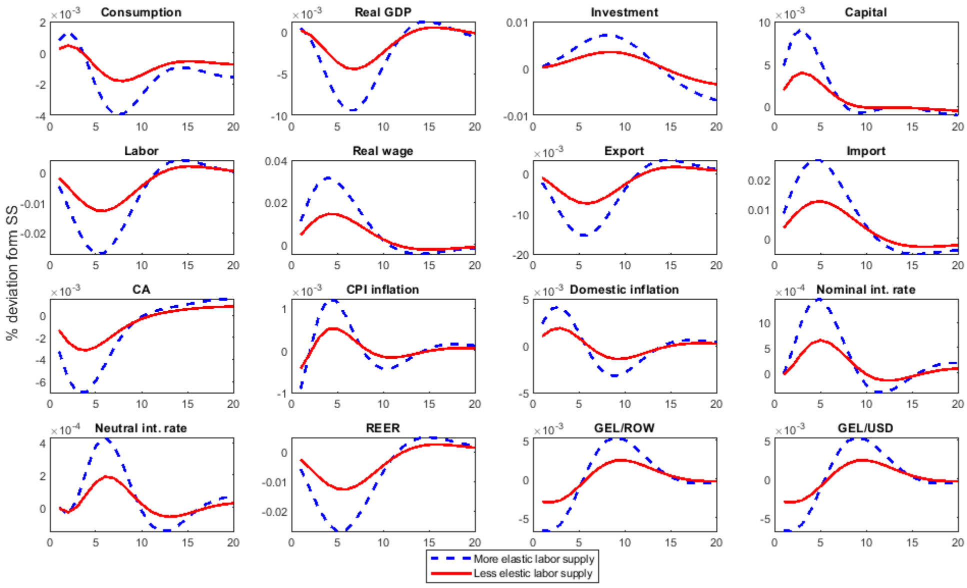


Figure 9: TFP shock

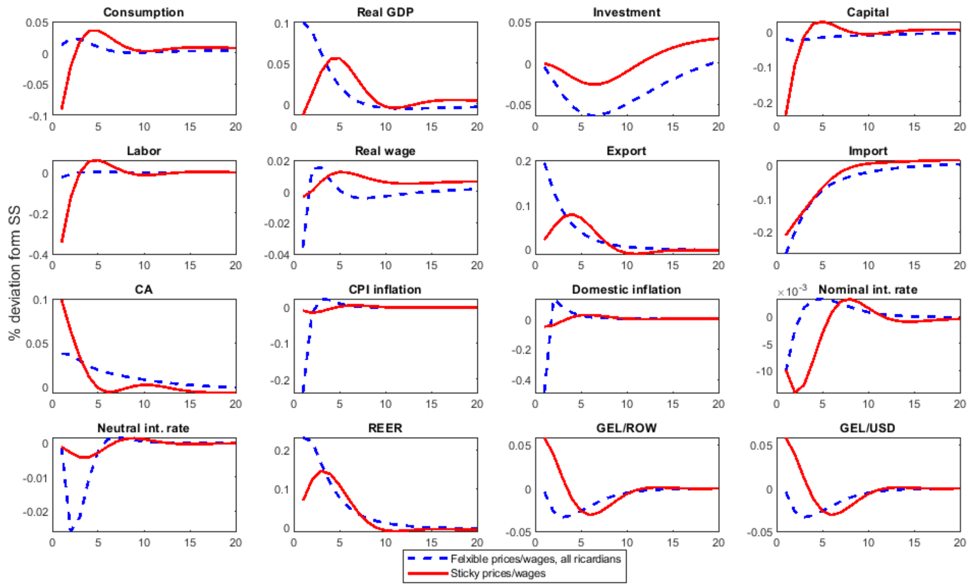


Figure 10: Labor augmented technology shock

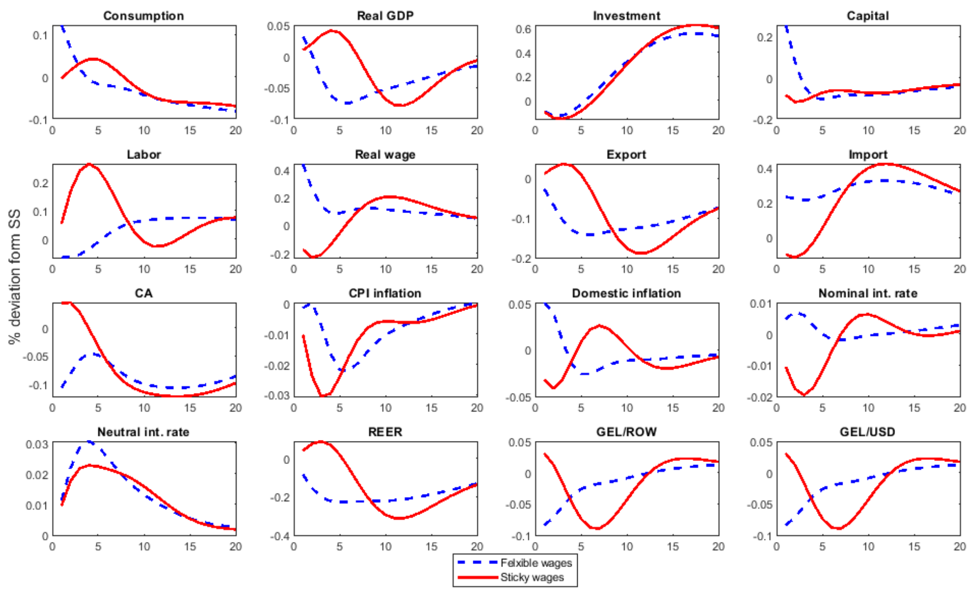


Figure 11: Inefficiency technology shock of imported goods

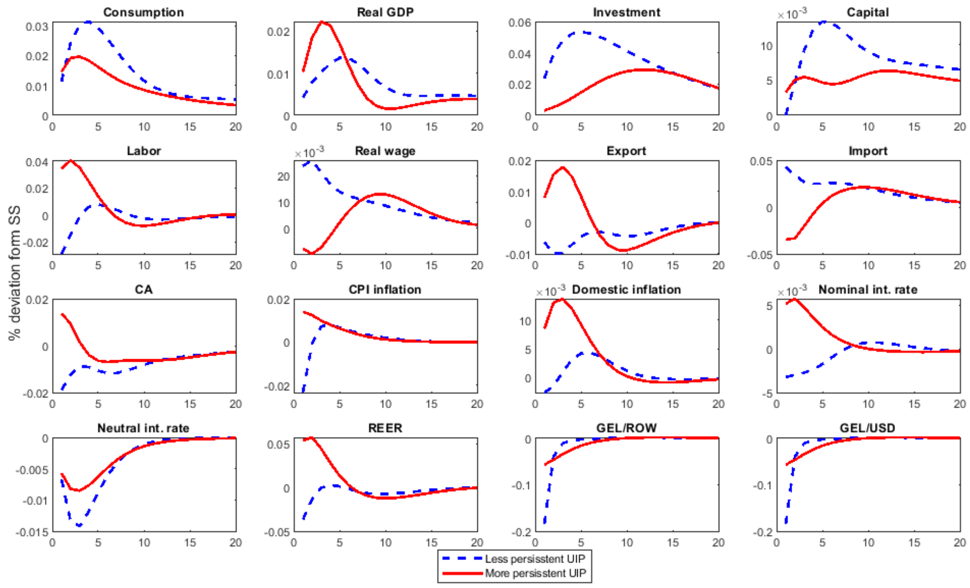


Figure 12: Comparison of mark up shocks

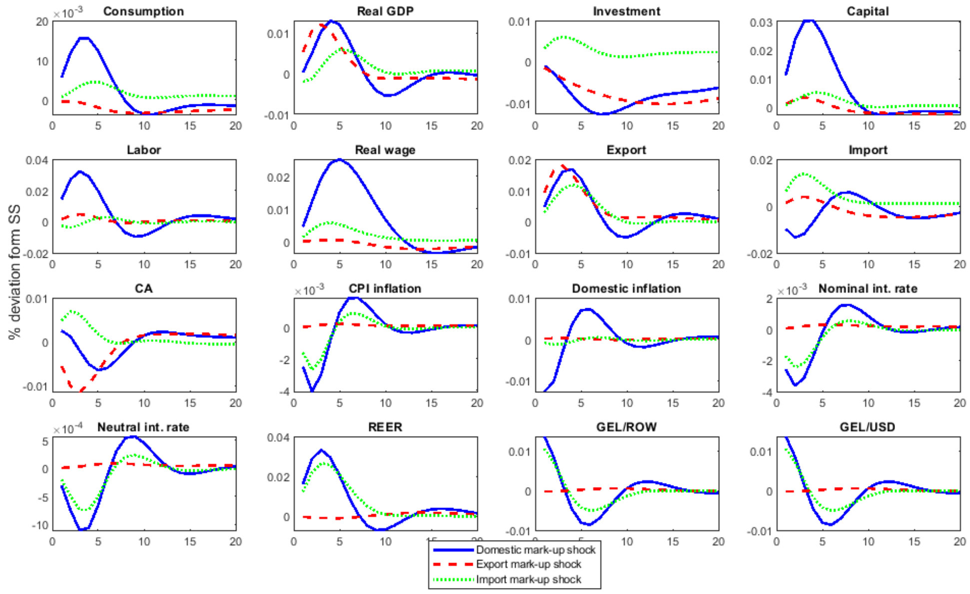


Figure 13: Wage mark up shocks

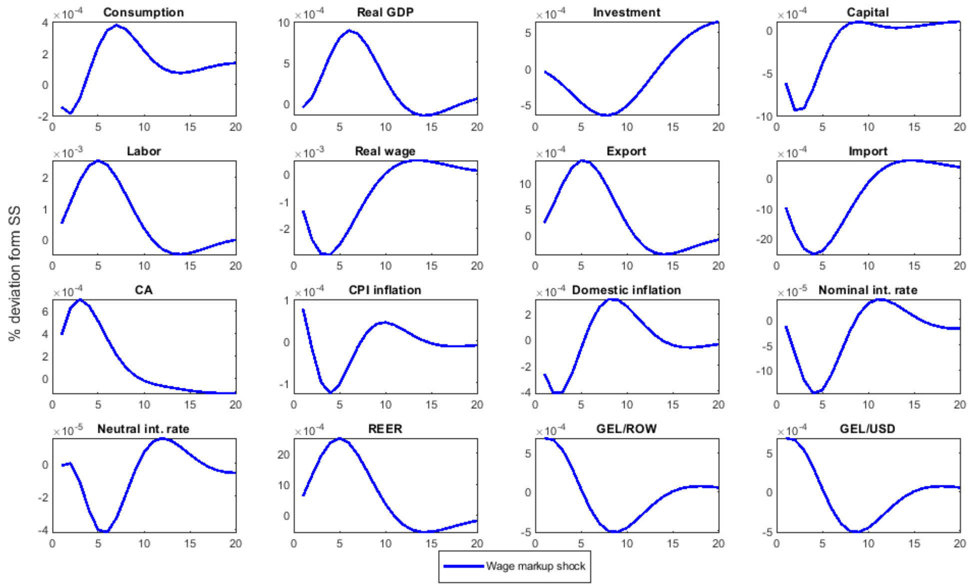


Figure 14: Foreign inflation rate shock

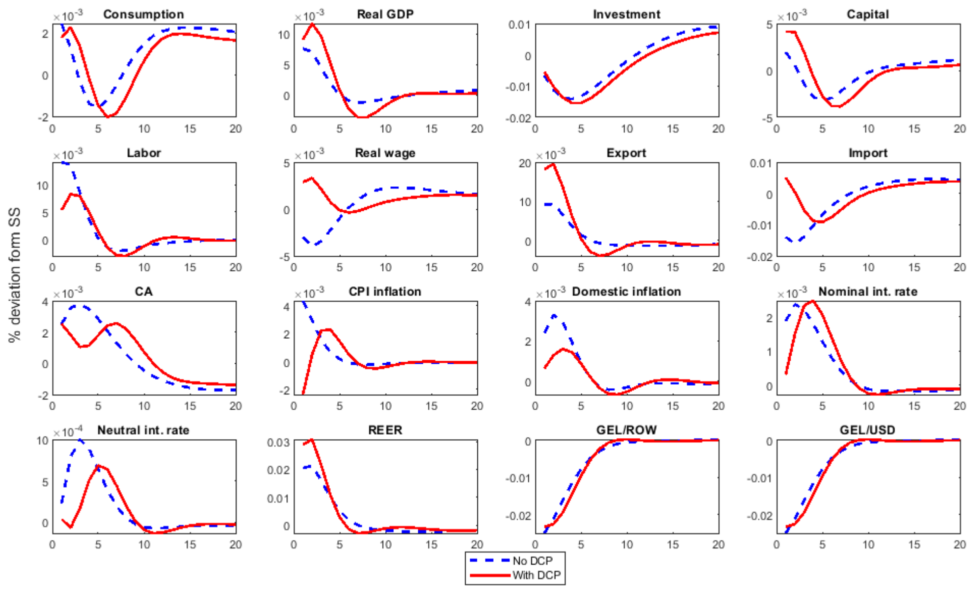


Figure 15: Foreign interest rate shock–UIP persistence

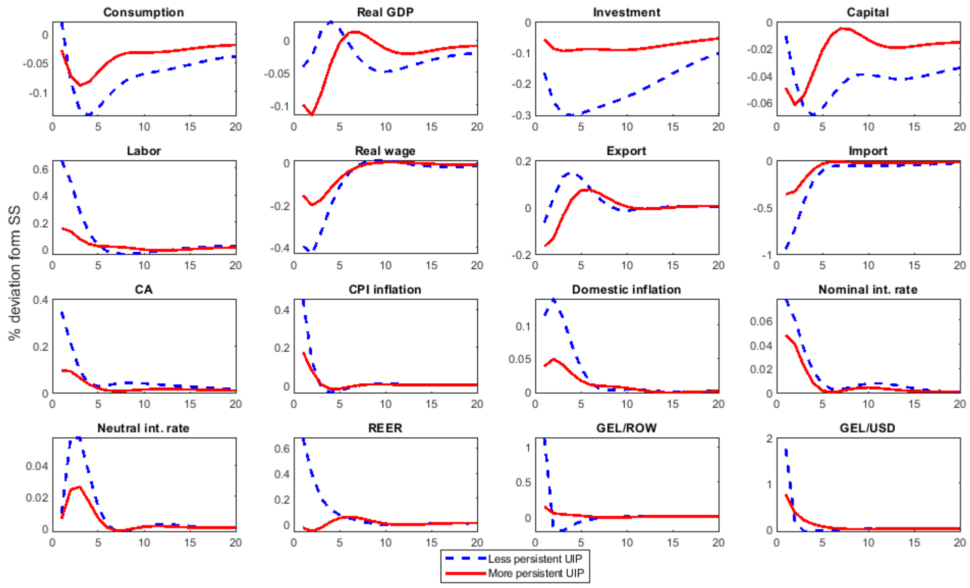


Figure 16: Foreign interest rate shock–Dollar pricing

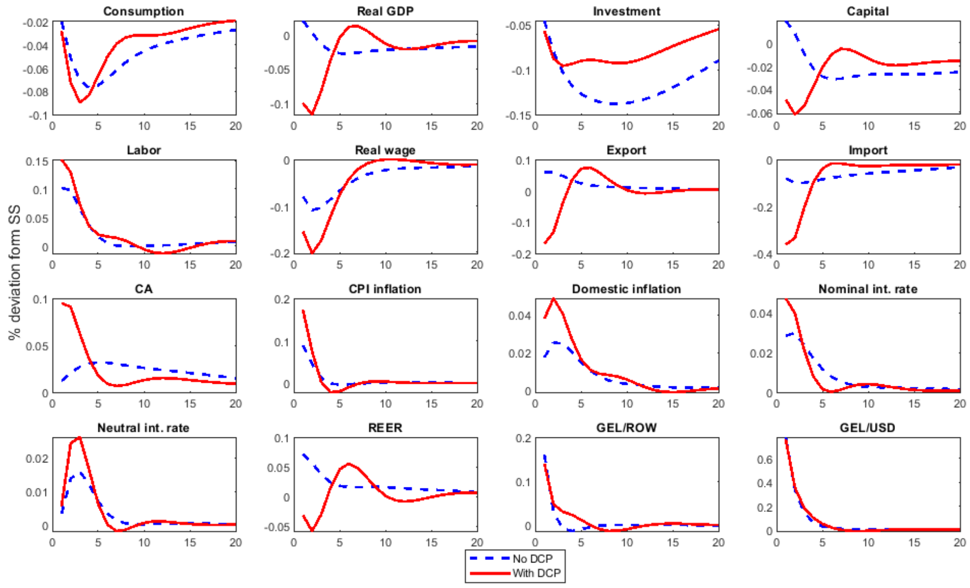


Figure 17: Foreign int. rate vs risk premium shocks

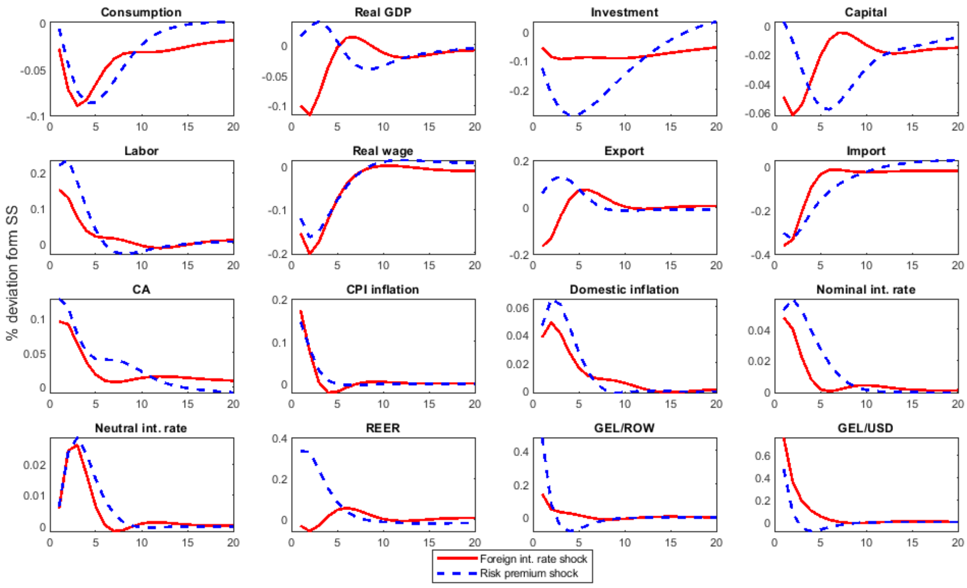


Figure 18: Foreign GDP growth shocks

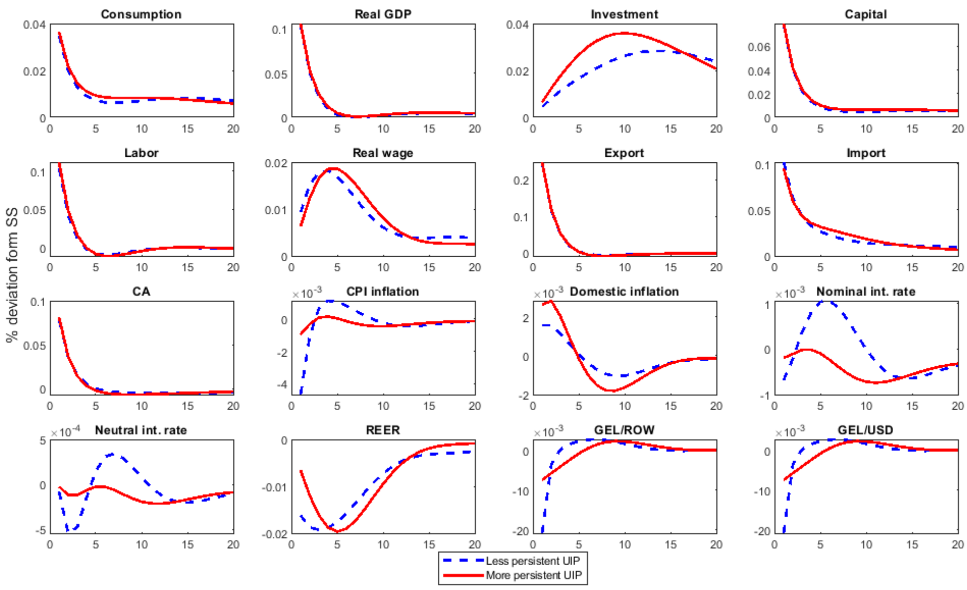
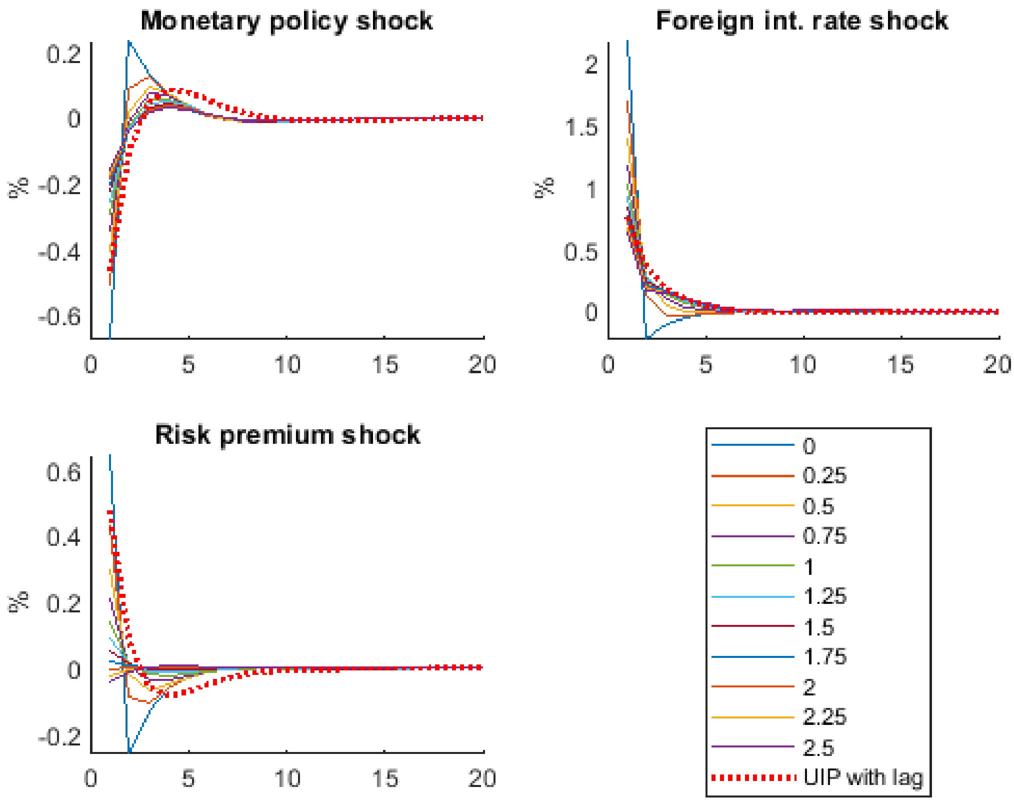


Figure 19: Exchange rate reaction: lagged UIP vs modified UIP



I.5.2 Filtration[33]

Figure 20: Trend cycle decomposition of real GDP

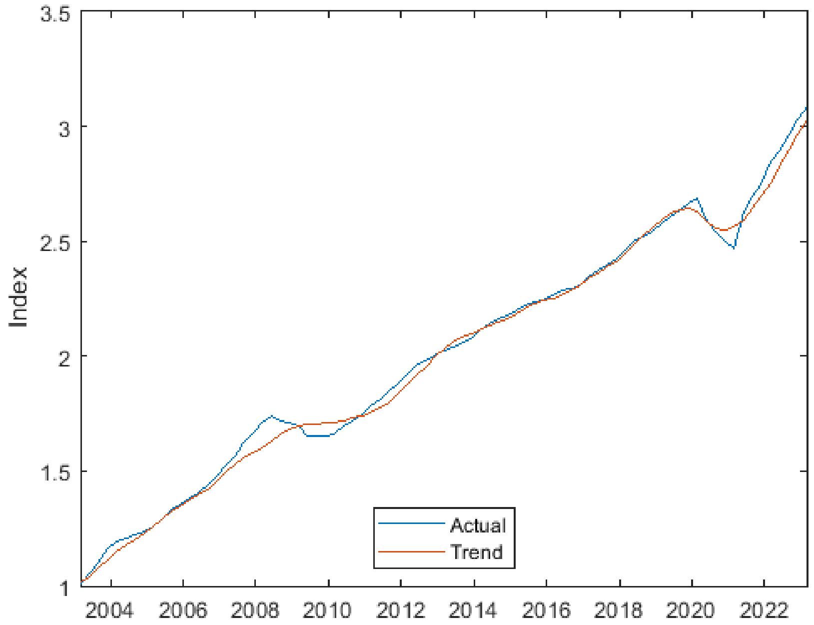
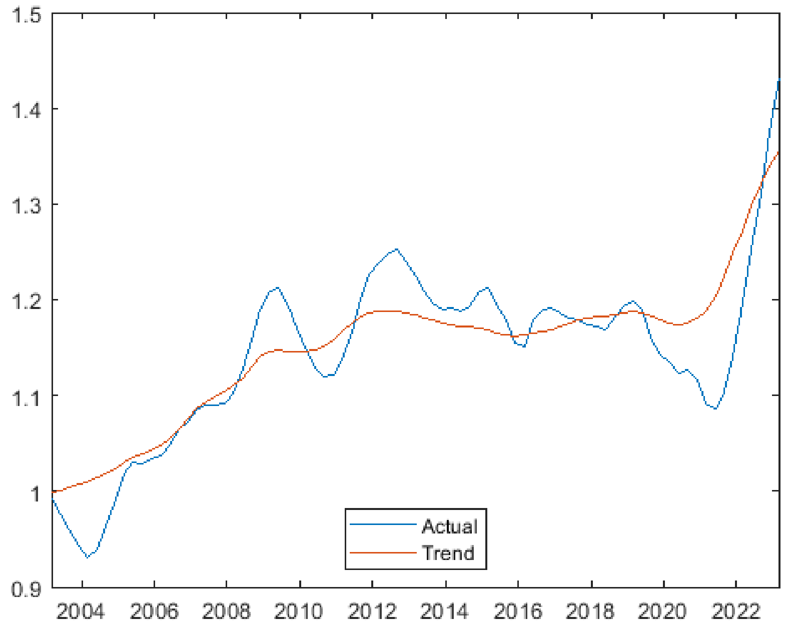


Figure 21: Real Exchange Rate (trend and actual values)



[33]The observable equations in the model include measurement errors, implying the actual time series plotted on Figures 20 and 21 are smoother than their corresponding values in data

Figure 22: Real neutral, nominal neutral and policy rates

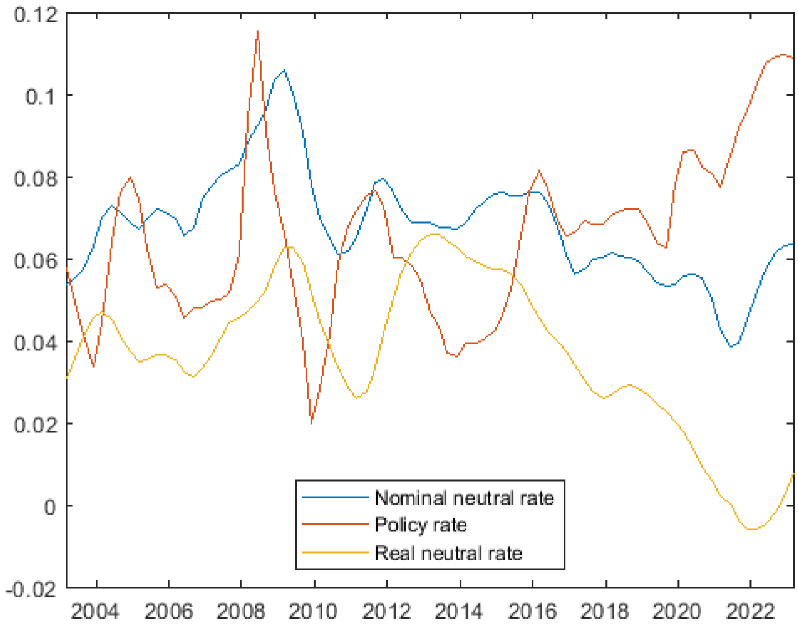


Figure 23: Headline inflation

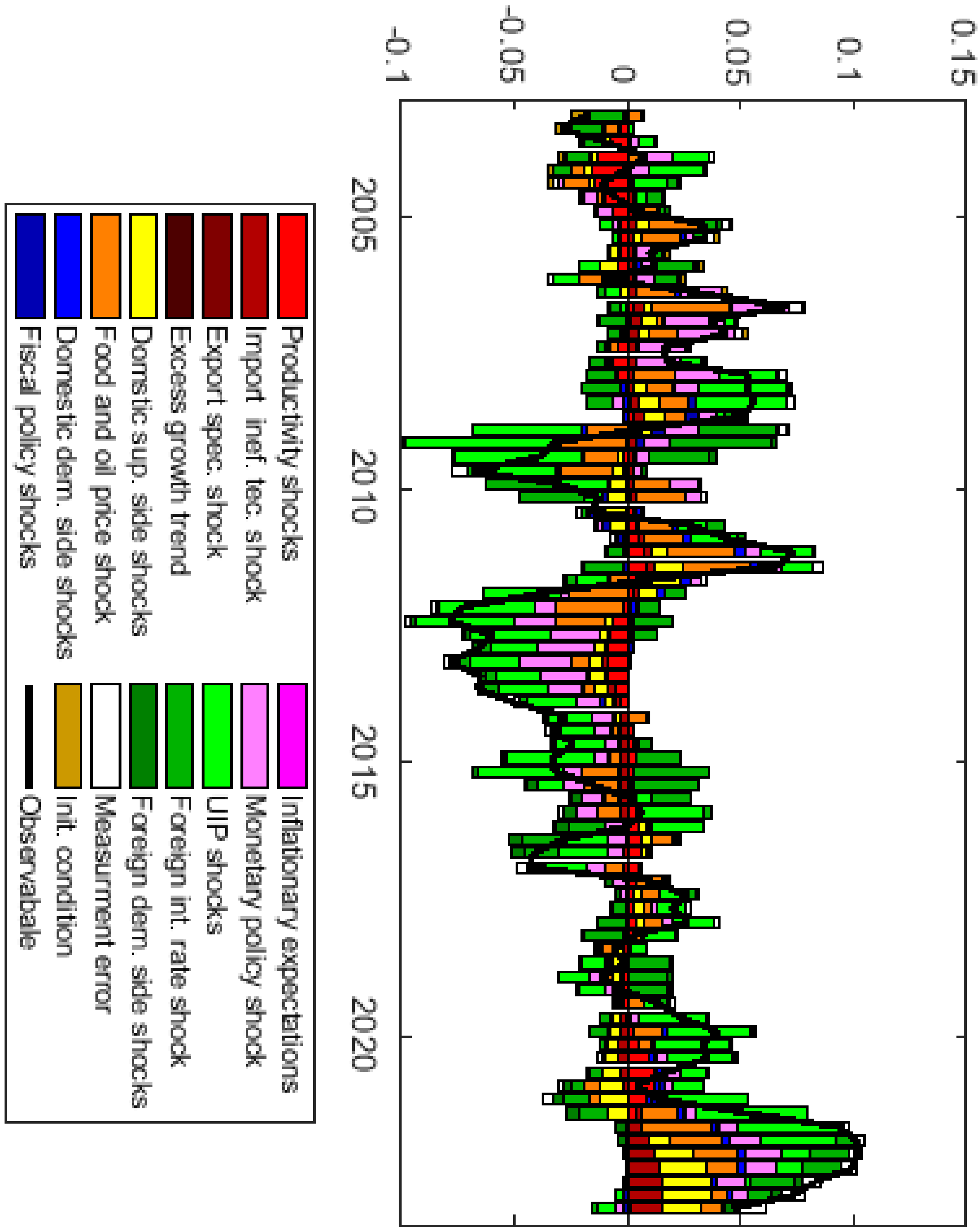


Figure 24: Real GDP growth

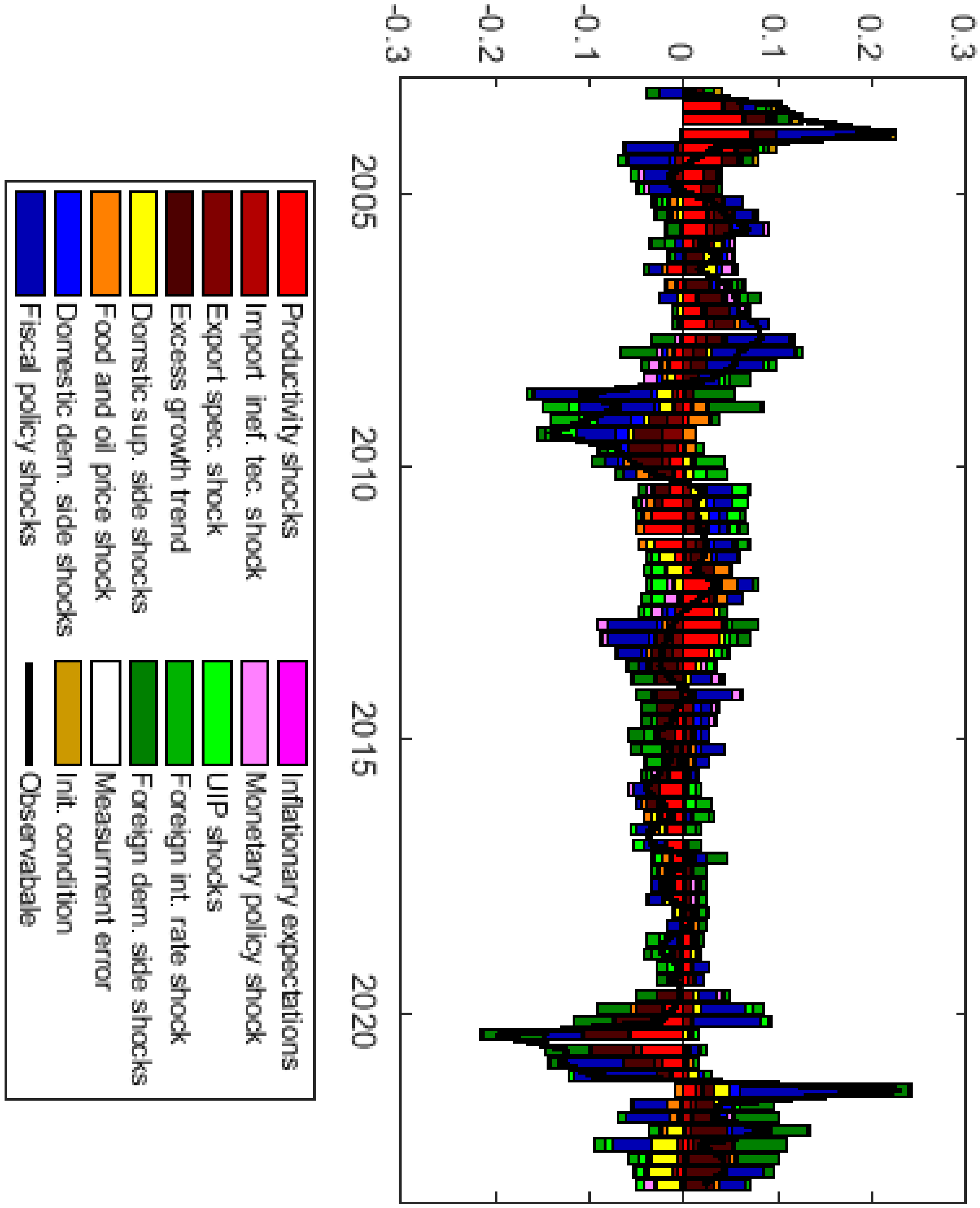


Figure 25: Monetary policy rate

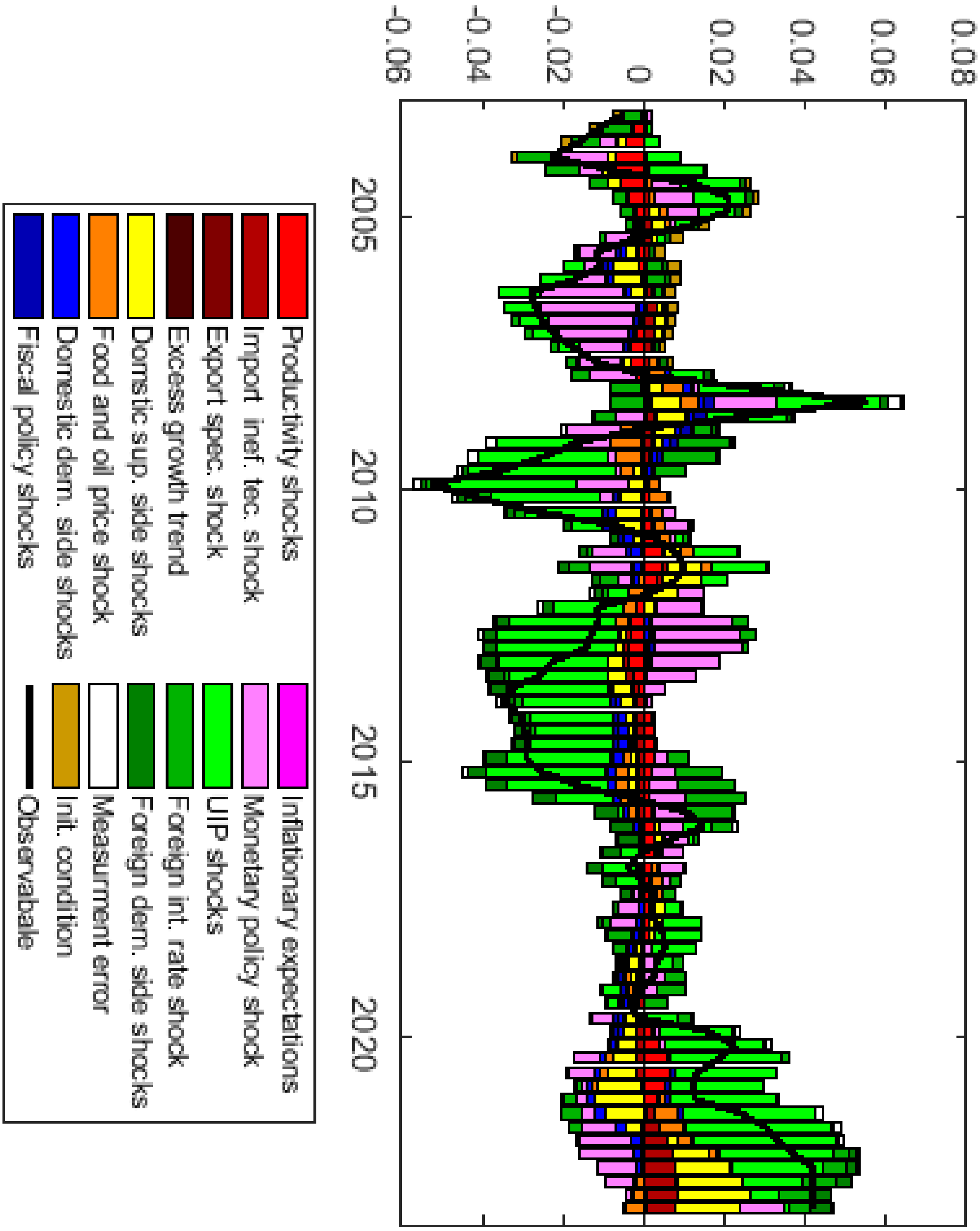


Figure 26: Real Exchange Rate

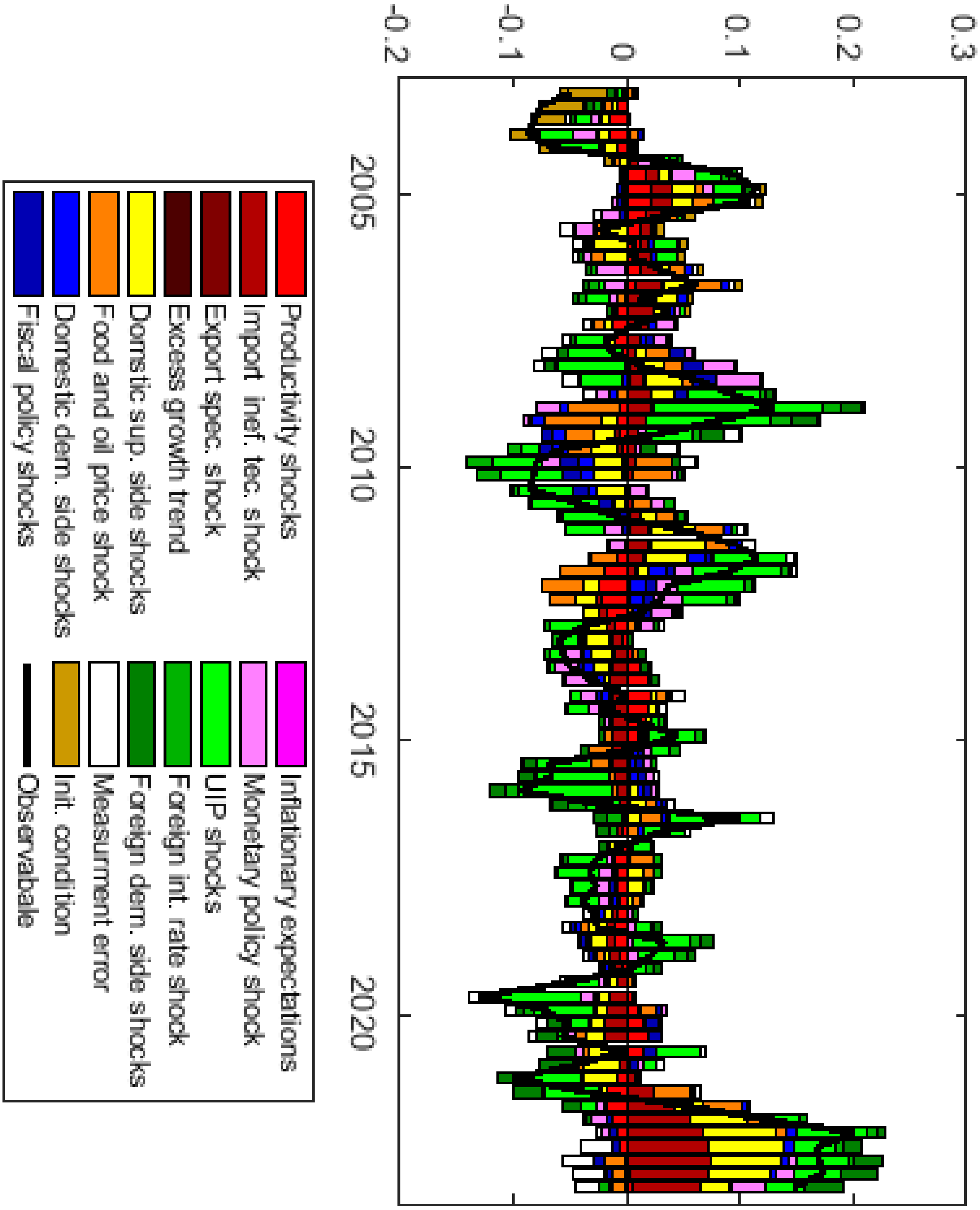
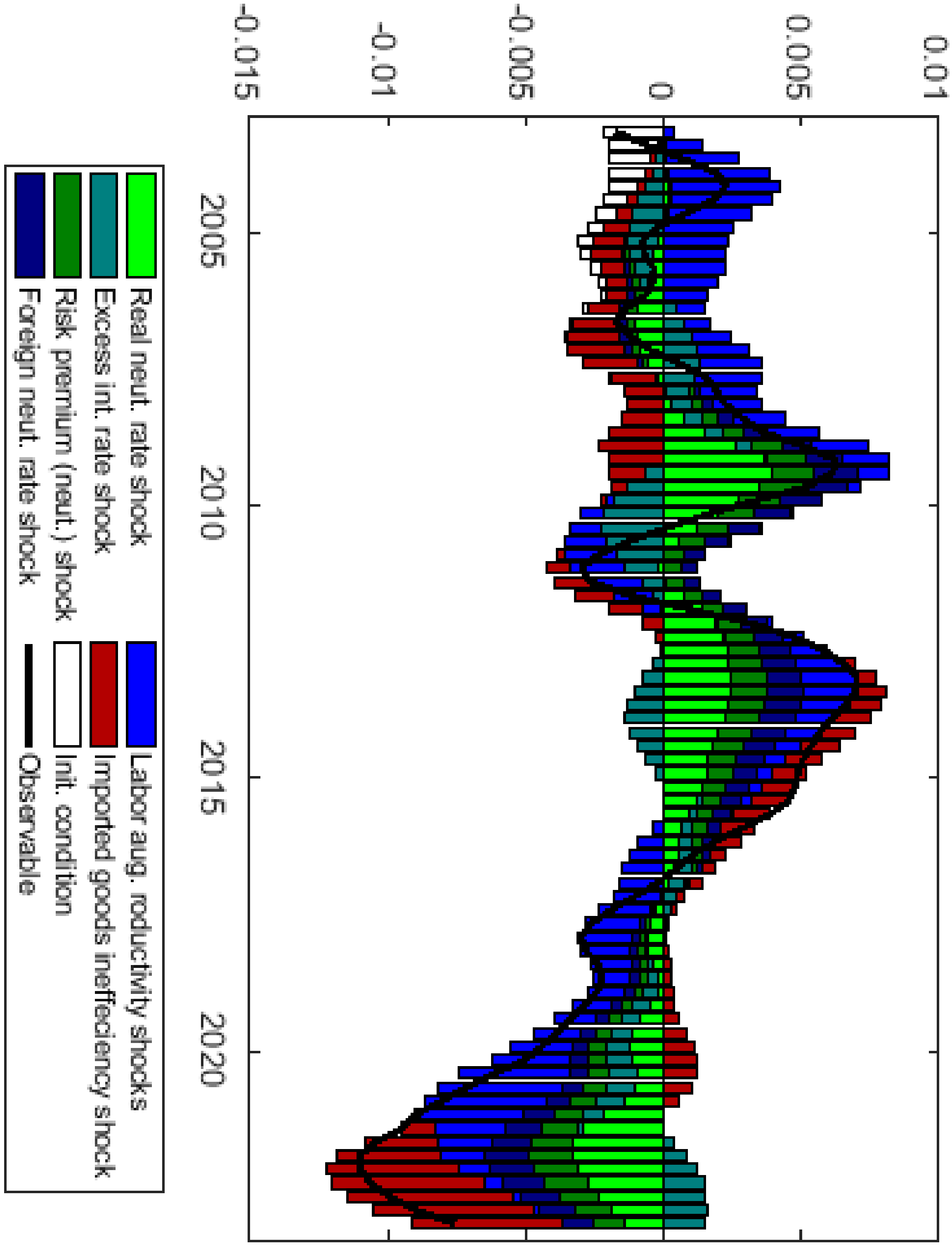


Figure 27: Real neutral interest rate



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