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# A Novel Minimum Variance Portfolio Approach to Understanding Dollarization: *Case of Georgia*

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**Abstract**

This paper investigates the Minimum Variance Portfolio (MVP) approach to analyzing dollarization in Georgia, extending the foundational work by Ize and Levy-Yeyati (1998, 2003). Conventional MVP dollarization analyses typically employ a one-period model with restrictive assumptions, particularly regarding serial correlations and the temporal scope of price and exchange rate dynamics. This study relaxes these constraints by incorporating extended time horizons and accounting for more realistic patterns of serial correlation. The enhanced model provides a more nuanced understanding of the interplay between prices and exchange rates, offering valuable insights into the determinants of dollarization. For central banks aiming for long-run de-dollarization while maintaining price stability, these insights are critical for designing effective policy measures. Using historical data from Georgia, the study demonstrates that the modified MVP framework, which incorporates volatilities in price levels and real exchange rates, yields estimates that better align with the observed levels of dollarization in the country, showing an equilibrium value of dollarization at 20-30%. This alignment underscores the model's robustness in capturing the structural dynamics of dollarization and its relevance for policymaking in small, open economies like Georgia.

**JEL Codes:** **E31, E44, E52, E60, G21**

**Keywords:** **MVP, Dollarization, Inflation, CPI, Exchange rate**



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# 1 Introduction

Dollarization remains a persistent challenge for emerging economies, with no straightforward solutions. Consequently, this issue has garnered significant attention from researchers worldwide. Georgia exemplifies one such emerging economy, having adopted the lari (national currency) as its official currency since 1995. Despite grappling with hyperinflation during the 1990s, Georgia has made strides toward stability by managing inflation. After adopting inflation targeting regime in 2009, the inflation has been maintained on average around the target. However, dollarization remains prevalent in Georgia. This takes various forms, including financial dollarization (involving financial assets and liabilities denominated in foreign currency) and real dollarization (involving the indexation of domestic transactions to the exchange rate). Factors such as past macroeconomic instability, geopolitical tensions and recurrent shocks are the origins of dollarization. However, not all of them are in line with actual data.

Dollarization has significant implications for the domestic economy. Levy Yeyati (2006, 2021) highlights its detrimental effect on the interest rate channel of monetary policy transmission. In overly dollarized economies, the financial sector becomes fragile due to rapid exchange rate fluctuations, leading to substantial losses (known as the balance sheet effect) and making them susceptible to banking crises. Moreover, these economies tend to experience slower and more volatile economic growth (Levy Yeyati, 2006). There are also various hedging motives for saving in dollars based on the exchange rate's behavior relative to prices and income processes: the dollar serves as a natural hedge against consumption risk due to dollar currency pricing (Gopinath et al., 2020) and offers insurance against the simultaneous occurrence of financial and currency crises common in these economies (Bocola and Lorenzoni, 2020). When people save in dollars, depreciations in the local currency lead to a domestic wealth transfer from firms to households. While financial dollarization can enhance financial depth by enabling onshore risk hedging, this advantage is evident primarily during periods of high inflation. Although dollarization may stimulate investment by reducing interest rates, it does so at the expense of financial stability. Consequently, policymakers aim to mitigate dollarization to minimize its predominantly negative repercussions. However, in an open economy, a certain level of dollarization is inevitable, as foreign currency deposits offer a means of diversification and can facilitate international trade.

Understanding the intricate relationship between price levels and exchange rate movements holds critical significance in economic analysis, financial market stability, and central bank policy formulation. Typically, inflation and exchange rates are two key drivers of dollarization. One widely accepted model to estimate the drivers of dollarization is the Minimum Variance Portfolio (MVP) model by Ize and Levy Yeyati (1998, 2003). Under certain assumptions, it proposes that dollarization increases with inflation volatility and decreases with the volatility of real exchange rate depreciation. The simple idea of this one-period model is that equilibrium dollarization (which we can interpret as natural dollarization) oscillates around the level of dollarization where the entire portfolio (composed of dollar and domestic currency deposits) achieves minimum variance. The model suggests that if inflation is volatile, then the domestic currency deposits have a lower weight in the portfolio, while if real exchange rate depreciation rate is more volatile, then the dollar deposit gets a lower weight in the portfolio. This is an intuitive result, but, as we show here, only a special (short-term) case of a broader idea.

Our contribution to the literature is twofold. Firstly, we've developed a unified framework that addresses dollarization both in the short and long term. This is achieved by expanding upon the theoretical model proposed by Ize and Levy Yeyati (1998, 2003). Specifically, we introduce a new version of the Minimum Variance Portfolio (MVP) model that incorporates long-term factors and serial correlation influencing the dollarization of interest-bearing deposits.

Secondly, to estimate the optimal level of dollarization in Georgia, we use the Consumer Price Index (CPI) and the nominal exchange rate (adjusted by CPI) into the MVP equation. The rationale for this approach lies in the ability of these variables to reflect the dynamic interplay between price levels and exchange rate fluctuations, providing a more comprehensive and realistic estimation of the MVP dollarization level over the long term. In a small open economy like Georgia, where the population is acutely sensitive to price and exchange rate volatility, nominal exchange rate fluctuations could translate into price variability. This is particularly significant given the rigidity of prices in the short term, where the denomination of prices critically determines how nominal exchange rate shocks are transmitted to domestic prices and broader economic activity.

Our methodology mirrors the original MVP framework by Ize and Levy-Yeyati (1998,2003), leveraging month-on-month changes for consistency and robustness. A comparative analysis of changes over a five-year horizon in price levels and exchange rates reveals a better alignment with actual dollarization levels. This consistency underscores the utility of this timeframe in capturing the underlying financial dollarization trends. Notably, the results suggest that longer time horizons offer a more accurate representation of the economic adjustments associated with dollarization dynamics.

To build on the original framework, we adopt a generalized equation of MVP dollarization, which enhances the traditional model by incorporating the serial correlation in inflation and exchange rate shocks. This addition is a crucial refinement, as it accounts for the negative covariance often observed between consecutive real exchange rate shocks, driven by the mean-reverting nature of real exchange rates. By contrast, inflation dynamics are more heterogeneous: while few countries exhibit mean reversion in price levels, others do not. These differences significantly influence the optimal portfolio composition, directly impacting the degree of dollarization.

The negative covariance between exchange rate shocks and the more unpredictable nature of inflation emphasize the importance of tailoring de-dollarization strategies to reflect country-specific economic conditions. Our findings reinforce the need for policy frameworks that consider both short-term price rigidities and the long-term structural trends in exchange rate and inflation dynamics. This is critical for designing interventions that mitigate the risks associated with high financial dollarization while fostering greater reliance on local currency.

## 2 Overview of Dollarization in Georgia

Dollarization in Georgia has its origins in the economic instability that followed the collapse of the Soviet Union in the early 1990s. During this period, hyperinflation and recurrent currency crises severely undermined confidence in the Georgian lari prompting widespread adoption of the US dollar as a safer store of value and medium of exchange. Moreover, in the early 2000s, the access of the Georgian commercial banks to the international financial markets increased. The commercial banks started attracting foreign currency resources from non-resident banks and international financial institutions, while long-term lari resources were limited. This contributed to the growth of foreign currency lending in the country. This phenomenon, termed financial dollarization, has persisted as a key feature of Georgia's economy. Over time, strong macroeconomic fundamentals, combined with targeted prudential measures, have improved the attractiveness of lari-denominated deposits and loans. Nevertheless dollarization is still excessive and higher than optimal level.

## Key Factors Contributing to Dollarization

- **Macroeconomic Landscape:** : Initially, Georgia has endured numerous episodes of elevated inflation and currency devaluations, most notably in the 1990s, which ingrained a dollarization mindset among the public. Over time, price stability has been achieved, and volatility has decreased, but fully changing this deeply rooted mindset remains a significant challenge. As a small open economy, Georgia remains highly exposed to external shocks, which contribute to economic volatility and reinforce the public's preference for foreign currency holdings. This mindset, rooted in uncertainty regarding the value of the lari, continues to persist despite recent improvements in macroeconomic stability.

- **Financial Markets:** Capital market is still shallow which impediments companies be able to issue long-term bonds and receive capital. Meanwhile, limited access to hedging instruments, restrict savers' options, compelling them to rely on foreign currencies.

- **Structural Factors:** The high degree of openness in the Georgian economy, with international trade constituting nearly 100% of GDP, further drives dollarization.

In 2009, the NBG adopted an inflation-targeting (IT) regime, marking a significant step towards stabilizing inflation. Inflation targeting regime with flexible exchange rate created low and stable inflationary environment that eventually increased attractiveness of domestic currency. Amid to the fact that there were strong macroeconomic fundamentals additional de-dollariztaion reforms were implemented in 2016, which including measures to increase access to long-term lari loans, adequate sharing of FX risks and mandatory pricing of goods and services in lari. Introduction of comprehensive de-dollarization policies has led to a gradual decline. For instance, the prohibition of foreign currency loans for amounts below GEL 100,000 in 2016 (which over the years have been increased to 500,000 laris) contributed significantly to the reduction of loan dollarization. Improvements in macroeconomic stability, particularly through inflation targeting and a floating exchange rate, have also played a role in bolstering public confidence in the lari. Nonetheless, sectors such as real estate and automobiles continue to exhibit real dollarization, where prices are pegged to the US dollar, despite official transactions being conducted in lari.

While notable progress has been made, full de-dollarization remains a long-term challenge. De-dollarization is a gradual process that requires sustained efforts to rebuild trust in the domestic currency. This paper shows that, under the given price level and exchange rate volatility, which determines the optimal level of dollarization, the current dollarization level still remains excessive. The current situation necessitates long-term policy measures to address structural and financial barriers. Future efforts will require deepening financial markets, enhancing access to hedging instruments, and continuing to build the credibility of the lari through consistent monetary policy.

# 3 Related Literature

The primary framework utilized for analyzing dollarization is portfolio allocation theory under the assumption of uncovered interest parity (UIP). Ize and Yeyati (1998, 2003) present a one-period model that establishes the MVP allocation between local and foreign currency deposits (and loans, respectively) based on anticipated future inflation and exchange rate movements. According to this model, financial dollarization is influenced not by the levels of inflation and exchange rate, but rather by the expectations regarding their volatility, as indicated by the MVP ratio. If the anticipated volatility of inflation exceeds that of the real exchange rate depreciation, the proportion of foreign currency deposits in the MVP increases. This MVP ratio closely approximates actual dollarization levels observed in numerous countries (Yeyati, 2021). The authors also indicate that, according to the MVP model, the selection of monetary policy and exchange rate regime plays a crucial role in determining the extent of dollarization within the economy.

Levy Yeyati's (2006) extensive research delves into the phenomenon of financial dollarization (FD), exploring its determinants and assessing whether the observed effects of FD on financial stability and economic performance align with theoretical expectations. The author underscores the significance of balance sheet effects associated with FD, which introduce a currency imbalance across the entire economy. The theoretical framework laid out by the author for studying dollarization, particularly FD, categorizes the literature on asset substitution into distinct groups of explanations: 1) The portfolio perspective views FD as the optimal choice for investors aiming to minimize portfolio return volatility, contingent upon the volatility of inflation and real depreciation rates. This perspective, also discussed by Ize and Levy Yeyati (2003), posits that dollarization arises from investors seeking to optimize their portfolios given varying returns in different currencies. 2) The market failure viewpoint suggests that FD emerges as a response to market imperfections, implying a bias toward dollarization due to market inefficiencies, externalities, and a regulatory framework that fails to adequately address these issues. 3) The institutional perspective posits that increasing FD results from institutional shortcomings. The presence of institutional failures can contribute to a rise in FD.

The author observes that both the portfolio and institutional views effectively explain the cross-country differences in the proportion of dollar-denominated deposits. Consequently, factors such as inflation and exchange rate volatility, coupled with poor institutional quality, can fuel financial dollarization. Additionally, the author explores the relationship between FD and crisis susceptibility, highlighting how exchange rate devaluation and increased FD can heighten the propensity for crises.

De Nicoló, Honohan, and Ize (2005) investigated the factors influencing deposit dollarization across different regions using a broader range of explanatory variables. In addition to the MVP share, these variables encompass inflation rates, institutional quality indicators, and dummy variables reflecting restrictions on dollarization, adoption of inflation targeting, legal protections, etc. Their findings indicate that dollarization is positively influenced by the MVP share and inflation rates, while the credibility of macroeconomic policies and institutional quality have a negative impact on dollarization.

Empirical findings indicate that a significant degree of financial dollarization diminishes the efficacy of monetary policy. Sopromadze et al. (2021) demonstrated that exchange rate pass-through can be twice as pronounced in small dollarized countries compared to those without dollarization. In situations where domestic producers lack hedging, depreciation in the exchange rate escalates their debt-service obligations, resulting in an overall increase in prices. The study concludes that the liability dollarization of domestic producers may necessitate a revision of the optimal monetary policy response to USD exchange rate fluctuations, potentially requiring additional tightening of monetary policy during adverse shocks to effectively manage inflation. This approach, once again, imposes penalties on domestic currency borrowers to address the supplementary inflationary pressures arising from foreign currency borrowers, meaning a presence of negative externalities due to dollarization.

Building upon these foundational studies, our work expands the MVP framework by integrating long-term dynamics and serial correlations.

# 4 Model and Methodology

## 4.1 Domestic Depositors' Asset Choices

Agents in country A make decisions regarding their savings. Residents of country A (depositors) have two alternatives for holding their money:

1. **Home Currency Deposits (HCD)**: These are deposits held in local currency at domestic banks. The annual nominal interest rate for the home-currency deposits at time $t$ is denoted by $i_t^H$. The real return (adjusted for inflation) at time $t$ is denoted by $r_t^H$.

2. **Domestically Held Foreign Currency Deposits (FCD)**: These are deposits held in foreign currency (e.g., USD) at domestic banks. The annual nominal

interest rate for the foreign-currency deposit at time $t$ is denoted by $i_t^F$. The real return at time $t$ is denoted by $r_t^F$, respectively.

3. **Agent's Decision Problem**: The agent observes nominal interest rates and treats them as given without uncertainty. On the one hand, holding home currency deposits carries the risk of losing purchasing power in the event of inflation. Hence, the variance of (one-period) real return is equal to the variance of inflation. On the other hand, holding foreign currency deposits is also risky if the local currency depreciates sharply relative to the foreign counterpart (which is expressed in exchange rate fluctuations), even in the absence of inflation. Hence, the variance of the real return is equal to the variance of real exchange rate change.

We follow the notations used in Ize and Yeyati (1998, 2003).

## 4.2 Depositors' Restrictions

Depositors cannot borrow or lend money in any currency (short-selling deposits is not allowed). Depositors only hold their money in deposits. This paper focuses on choosing between different asset types (HCD, FCD).

## 4.3 Nominal and Real Returns

The nominal rates are compounded once in a period, and gross nominal return at time $n$ is thus $(1+i_t^H)^n$ and $(1+i_t^F)^n$, with $n$ denoting the deposit maturity.

Being at time $t$, we denote the future price index n periods ahead as $P_{t+n}$. The index shows how cheap/expensive is the consumption basket at time $t+n$ compared to the initial period of $t$, and it corresponds to the overall price index. $P_{t+n}$ evolves according to the formula:

$$P_{t+n} = (1+\pi_{t+1})(1+\pi_{t+2})...(1+\pi_{t+n}) \tag{1}$$

where $\pi_t$ denotes the inflation rate at time $t$[1].

[1] By definition, CPI inflation is $\pi_{t+n} = \frac{P_{t+n}}{P_{t+n-1}} - 1$ for all n and $t$. If we normalize initial price index

Thus, the gross real returns for the domestic and foreign currency deposits can be expressed as:

$$(1+r_t^H)^n = \frac{(1+i_t^H)^n}{P_{t+n}} = (1+i_t^H)^n * (1+\pi_{t+1})^{-1} * (1+\pi_{t+2})^{-1} * ... * (1+\pi_{t+n})^{-1} \quad (2)$$

$$(1+r_t^F)^n = (1+i_t^F)^n \frac{S_{t+n}}{P_{t+n}} = (1+i_t^F)^n * \frac{1+\Delta S_{t+1}}{1+\pi_{t+1}} * \frac{1+\Delta S_{t+2}}{1+\pi_{t+2}} * ... * \frac{1+\Delta S_{t+n}}{1+\pi_{t+n}} \quad (3)$$

where $S_{t+n}$ is the nominal exchange rate [2] - for example, amount of GEL that can be bought with 1 unit of USD - that applies to the deposit opened at time $t$ and maturing at $t+n$.

Logarithmic transformation of (2) and (3), and division by n leads to[3]:

$$r_t^H = i_t^H - \frac{\pi_{t+1} + \pi_{t+2} + ... + \pi_{t+n}}{n} \quad (4)$$

$$r_t^F = i_t^F + \frac{(\Delta S_{t+1} - \pi_{t+1}) + (\Delta S_{t+2} - \pi_{t+2}) + ... + (\Delta S_{t+n} - \pi_{t+n})}{n} \quad (5)$$

Denoting the real exchange rate adjusted for the inflation as $s_{t+n} \equiv \Delta S_{t+n} - \pi_{t+n}$ for any $t$ and $n$ leads to the modified version of (5):

$$r_t^F = i_t^F + \frac{s_{t+1} + s_{t+2} + ... + s_{t+n}}{n} \quad (6)$$

Deriving (4) and (6) we demonstrate that the real return on domestic currency deposit is negatively affected by inflation shocks, while the foreign currency deposit real rate positively correlates with the real exchange rate shock. Following Ize and Yeyati (1998, 2003) notation, we introduce these shocks at any $t$ and n as $\mu_{\pi_{t+n}}$ and

$P_t$ to 1, it is easy to demonstrate that $P_{t+n} = \frac{P_{t+1}}{P_t} * \frac{P_{t+2}}{P_{t+1}} * ... * \frac{P_{t+n}}{P_{t+n-1}}$, which, after substituting the inflation formula, gives equation (1).

[2]Similar to the CPI inflation, we can define the change in exchange rate between two consecutive periods as $\Delta S_{t+n} = S_{t+n}/S_{t+n-1} - 1$ for all n and $t$. Normalizing the nominal exchange rate prevailing at deposit opening date to 1 leads to the following expression: $S_{t+n} = \frac{S_{t+1}}{S_t} * \frac{S_{t+2}}{S_{t+1}} * ... * \frac{S_{t+n}}{S_{t+n-1}} = (1+\Delta S_{t+1}) * (1+\Delta S_{t+2}) * ... * (1+\Delta S_{t+n})$, which is used to construct (3).

[3]Here we employed the following property of the log function: $log(1+x) \approx x$ for the sufficiently small values of $x$.

$\mu_{s_{t+n}}$ for inflation and real exchange rate disturbances, respectively. Thus, the real returns can be re-expressed as the function of the shocks in the following manner:

$$r_t^H = E_t[r_t^H] - \mu_{\pi_{t+1}} - \mu_{\pi_{t+2}} - ... - \mu_{\pi_{t+n}} \quad (7)$$

$$r_t^F = E_t[r_t^F] + \mu_{s_{t+1}} + \mu_{s_{t+2}} + ... + \mu_{s_{t+n}} \quad (8)$$

The main idea behind (7) and (8) is that the ex-post real return of the deposit opened at time $t$ with a maturity of $n$ periods, has the expected return at maturity that is affected either with inflation shocks (for domestic currency deposits), or real exchange rate shocks (for the foreign currency deposits).

We assume that the distribution of each disturbance is centered around zero, with variance-covariance matrix $S_{xy}$.

At time $t$ the investor makes a decision about the deposit portfolio. Specifically, he decides what portion of the portfolio should be in the local and foreign currency. Let $w_t^H$ and $w_t^F$ be the share of local- and foreign-currency denominated deposits in the total portfolio obtained at time $t$. By construction, $w_t^H = 1 - w_t^F$. The portfolio real return from the deposit holding would be the weighted average of domestic and foreign currency investments:

$$R_t = w_t^H * r_t^H + w_t^F * r_t^F = (1 - w_t^F)r_t^H + w_t^F * r_t^F = r_t^H + w_t^F * (r_t^F - r_t^H) \quad (9)$$

Thus, the expected real return of the portfolio is expressed as:

$$E_t[R_t] = E_t[r_t^H + w_t^F * (r_t^F - r_t^H)] = E_t[r_t^H] + w_t^F * E_t[r_t^F - r_t^H] \quad (10)$$

The variance of the portfolio return would be:

$$var_t[R_t] = var_t[r_t^H + w_t^F * (r_t^F - r_t^H)] = var_t[r_t^H] + (w_t^F)^2 * var_t[r_t^F - r_t^H] + 2w_t^F * cov_t(r_t^H, r_t^F - r_t^H) \quad (11)$$

Depositor has the utility function that is increasing with respect to expected return.

Since the depositor is risk-averse, he derives disutility from the volatile returns. The depositor's utility function has the standard form:

$$\begin{aligned} U_t &= E_t[R_t] - \frac{c}{2} * var_t[R_t] \\ &= E_t[r_t^H] + w_t^F * E_t[r_t^F - r_t^H] - \frac{c}{2} \\ &\quad * [var_t[r_t^H] + (w_t^F)^2 * var_t[r_t^F - r_t^H] + 2w_t^F * cov_t(r_t^H, r_t^F - r_t^H)] \end{aligned} \tag{12}$$

where $c > 0$ denotes the risk aversion coefficient. To maximize utility, a depositor chooses optimal weight $w_t^F$, considering both the expected return and variance of local and foreign currency deposits. The first order conditions associated with this problem implies:

$$E_t[r_t^F - r_t^H] - \frac{c}{2} * [2w_t^F * var_t[r_t^F - r_t^H] + 2cov_t(r_t^H, r_t^F - r_t^H)] = 0 \tag{13}$$

Solving for the optimal weight leads to:

$$(w_t^F)^* = \frac{E_t[r_t^F - r_t^H]}{c * var_t[r_t^F - r_t^H]} - \frac{cov_t(r_t^H, r_t^F - r_t^H)}{var_t[r_t^F - r_t^H]} \tag{14}$$

When the uncovered interest parity (UIP) holds, which is typically the case in the long run, the first part of (14), which is the expected real interest rate spread, will be zero, and only the second term survives. The variance-covariance terms in the surviving part is simplified as:

$$cov(r_t^H, r_t^F - r_t^H) = cov_t(r_t^H, r_t^F) - var_t(r_t^H) \tag{15}$$

and

$$var_t(r_t^F - r_t^H) = var_t(r_t^F) + var_t(r_t^H) - 2cov_t(r_t^F, r_t^H) \tag{16}$$

To simplify further, we recall (7) and (8), and plug them later in (15) and (16).

$$\begin{aligned} var_t(r_t^H) &= var_t(E_t[r_t^H] - \mu_{\pi_{t+1}} - \mu_{\pi_{t+2}} - ... - \mu_{\pi_{t+n}}) \\ &= \sum_{i=1}^{n} var_t(\mu_{\pi_{t+i}}) + \sum_{i \neq j} \sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{\pi_{t+j}}) \end{aligned} \tag{17}$$

$$\begin{aligned} var_t(r_t^F) &= var_t(E_t[r_t^F] + \mu_{s_{t+1}} + \mu_{s_{t+2}} + ... + \mu_{s_{t+n}}) \\ &= \sum_{i=1}^{n} var_t(\mu_{s_{t+i}}) + \sum_{i \neq j} \sum_{j=1}^{n} cov_t(\mu_{s_{t+i}}, \mu_{s_{t+j}}) \end{aligned} \tag{18}$$

$$cov_t(r_t^H, r_t^F) = cov_t(E_t[r_t^H] - \mu_{\pi_{t+1}} - \mu_{\pi_{t+2}} - ... - \mu_{\pi_{t+n}}, E_t[r_t^F] + \mu_{s_{t+1}} + \mu_{s_{t+2}} + ... + \mu_{s_{t+n}})$$
$$= -\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}}) \tag{19}$$

Thus,

$$\frac{cov_t(r_t^H, r_t^F - r_t^H)}{var_t[r_t^F - r_t^H]} = \frac{cov_t(r_t^H, r_t^F) - var_t(r_t^H)}{var_t(r_t^F) + var_t(r_t^H) - 2cov_t(r_t^F, r_t^H)}$$
$$= -\frac{\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}}) + \sum_{i=1}^{n} var_t(\mu_{\pi_{t+i}}) + \sum_{i\neq j}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{\pi_{t+j}})}{\sum_{i=1}^{n}(var_t(\mu_{s_{t+i}}) + var_t(\mu_{\pi_{t+i}})) + \sum_{i\neq j}\sum_{j=1}^{n}(cov_t(\mu_{s_{t+i}}, \mu_{s_{t+j}}) + cov_t(\mu_{\pi_{t+i}}, \mu_{\pi_{t+j}})) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}})} \tag{20}$$

Assuming that disturbances of inflation and real exchange rate are not serially correlated (in other words, $cov_t(\mu_{s_{t+i}}, \mu_{s_{t+j}}) = cov_t(\mu_{\pi_{t+i}}, \mu_{\pi_{t+j}}) = 0$ for all $t$, and $i \neq j$), (20) reduces to:

$$\frac{cov_t(r_t^H, r_t^F - r_t^H)}{var_t[r_t^F - r_t^H]} = -\frac{\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}}) + \sum_{i=1}^{n} var_t(\mu_{\pi_{t+i}})}{\sum_{i=1}^{n}(var_t(\mu_{s_{t+i}}) + var_t(\mu_{\pi_{t+i}})) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}})} \tag{21}$$

which, in combination with (14), implies:

$$(w_t^F)^* = \frac{E_t[r_t^F - r_t^H]}{c * var_t[r_t^F - r_t^H]} + \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}}) + \sum_{i=1}^{n} var_t(\mu_{\pi_{t+i}})}{\sum_{i=1}^{n}(var_t(\mu_{s_{t+i}}) + var_t(\mu_{\pi_{t+i}})) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}})} \tag{22}$$

Again, recalling that in the long run the UIP holds, the deposit dollarization converges to:

$$\lambda^* = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}}) + \sum_{i=1}^{n} var_t(\mu_{\pi_{t+i}})}{\sum_{i=1}^{n}(var_t(\mu_{s_{t+i}}) + var_t(\mu_{\pi_{t+i}})) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}})} \tag{23}$$

(23) provides a measure of deposit dollarization in the long-run, which is consistent with the literature on the Minimum Variance Portfolio[4] (MVP).

[4] Recall that Ize and Yeyati (1998, 2003) expressed the dollar share of minimum variance portfolio as $\lambda^*_{ILY} = \frac{S_{\pi\pi}+S_{\pi s}}{S_{\pi\pi}+S_{ss}+2S_{\pi s}}$, where $S_{\pi\pi}$ and $S_{ss}$ are the variances of inflation and real exchange rate, respectively, while $S_{\pi s}$ denotes the covariance between inflation and real exchange rate. The dollarization level derived in (23) repeats the structure of Ize-Yeyati's formula, however our model incorporates $n$ periods of analysis, while Ize and Yeyati analyzed 1-period model only. This is why the variances pile-up in our model - the cumulative disturbances are likely to have the more pronounced impact on dollarization decisions.

Next, we consider a case where we deviate from Ize-Yeyati's assumption regarding the absence of serial correlation in inflation and real exchange rate disturbances. In contrast, we assume that when $i \neq j$, $cov(\mu_{s_{t+i}}, \mu_{s_{t+j}}) = \rho_s$ and $cov(\mu_{\pi_{t+i}}, \mu_{\pi_{t+j}}) = \rho_\pi$ for all $t$ and $|i - j| = 1$, and 0 otherwise. The assumption implies that the shocks in consecutive time periods may be correlated. Under these assumptions, the expression for the MVP level of dollarization given the time horizon $n$ is reformulated as:

$$\lambda_n^* = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}}) + \sum_{i=1}^{n} var_t(\mu_{\pi_{t+i}}) + 2(n-1)\rho_\pi}{\sum_{i=1}^{n}(var_t(\mu_{s_{t+i}}) + var_t(\mu_{\pi_{t+i}})) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}}) + 2(n-1)[\rho_\pi + \rho_s]} \tag{24}$$

Since our model focuses on a long time horizon of size $n$, it is challenging to analyze the relationship between $\lambda_n^*$ and $n$. It is interesting aspect since the longer time-horizon might change the riskiness of local or foreign currency deposits and, at the end, may affect the MVP level of dollarization too. To investigate the link between the length of the deposit maturity and the dollarization, we start with (24) and try to further simplify it by introducing additional assumptions.

Suppose, the variance of the disturbances are constant, that is $var_t(\mu_{\pi_{t+i}}) = var_t(\mu_{\pi_{t+j}}) = v_\pi$ and $var_t(\mu_{s_{t+i}}) = var_t(\mu_{s_{t+j}}) = v_s$ for all $t$, $i$ and $j$. In addition, let the $cov_t(\mu_{\pi_{t+i}}, \mu_{s_{t+j}}) = \rho_{\pi s}$ for all $t$ and $i = j$, and zero otherwise. Given these restrictions, $\lambda_n^*$ becomes:

$$\lambda_n^* = \frac{\rho_{\pi s} + v_\pi + 2\rho_\pi - \frac{2\rho_\pi}{n}}{2\rho_{\pi s} + v_\pi + v_s + 2[\rho_\pi + \rho_s] - \frac{2[\rho_\pi + \rho_s]}{n}} \tag{25}$$

One thing that can be noted from (25) is the limit value of $\lambda$ - dollarization level economy converges to when $n \to \infty$:

$$\begin{aligned}\lim_{n \to \infty} \lambda_n^* &= \lim_{n \to \infty} \frac{\rho_{\pi s} + v_\pi + 2\rho_\pi - \frac{2\rho_\pi}{n}}{2\rho_{\pi s} + v_\pi + v_s + 2[\rho_\pi + \rho_s] - \frac{2[\rho_\pi + \rho_s]}{n}} \\ &= \frac{\rho_{\pi s} + v_\pi + 2\rho_\pi}{2\rho_{\pi s} + v_\pi + v_s + 2\rho_\pi + 2\rho_s}\end{aligned} \tag{26}$$

The limiting equation (26) represents a more generalized version of the framework developed by Ize and Levy-Yeyati, with a key distinction: it accounts for the 1-period cross-correlation between inflation and exchange rate shocks, a feature absent in the original model. This distinction is practically significant because the real exchange rate typically exhibits mean-reverting behavior, which implies a negative covariance between consecutive shocks. In contrast, the behavior of prices is more complex; for a few countries, it may exhibit mean reversion, while for most others, this pattern is absent. These differences in the dynamics of inflation and exchange rate shocks can influence the optimal portfolio composition, ultimately affecting the degree of dollarization.

Next, we can analyze the dynamics of $\lambda_n^*$. For that, we differentiate the (25) with respect to $n$ to derive the following condition[5]:

$$\frac{\partial \lambda_n^*}{\partial n} = \frac{2(\rho_\pi v_s + \rho_\pi \rho_{\pi s} - \rho_s v_\pi - \rho_s \rho_{\pi s})}{(n(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s) - 2\rho_\pi - 2\rho_s)^2}$$

The time horizon cannot affect $\lambda_n^*$ iff the derivative is zero, from which follows that necessary condition for the constant dollarization is:

$$\rho_\pi v_s + \rho_\pi \rho_{\pi s} - \rho_s v_\pi - \rho_s \rho_{\pi s} = 0$$

or

$$\rho_\pi v_s + \rho_\pi \rho_{\pi s} = \rho_s v_\pi + \rho_s \rho_{\pi s}$$

That leads to:

$$\rho_\pi = \rho_s \frac{v_\pi + \rho_{\pi s}}{v_s + \rho_{\pi s}} \tag{27}$$

The trivial case when this condition is satisfied would be $\rho_\pi = \rho_s = 0$ that states no serial correlation in inflation and real exchange rate shocks. Putting the trivial case aside, the first order condition implies $\frac{\rho_\pi}{\rho_s} = \frac{v_\pi + \rho_{\pi s}}{v_s + \rho_{\pi s}}$. Considering the non-negativity of variances, if we assume that $\rho_{\pi s} > 0$ (which is typically the case for small open economies), the right hand-side term would be strictly positive. Meanwhile, the mean reverting behavior of the real exchange rate leaves two options: either the country follows the price level targeting and $\rho_\pi$ is negative too, or inflation in a country has some persistency that implies positive $\rho_\pi$. In the latter case, the left hand-side is negative and always less than the right counterpart, which implies decreasing dollarization. Also, we can state that $\lambda_n^*$ is increasing function of $n$ when $\frac{\rho_\pi}{\rho_s} > \frac{v_\pi + \rho_{\pi s}}{v_s + \rho_{\pi s}}$.

[5]Please, refer to the appendix for the technical details.

# 5 MVP Dollarization: Empirical Findings for Georgia

For the empirical analysis, we utilize monthly data on CPI inflation and the nominal exchange rate (adjusted by CPI) to conduct our assessment. The rationale for using price levels over a long-term horizon is grounded in the idea that they provide a more realistic measure of how economic agents respond to changes in both prices and exchange rates, as demonstrated by our multi-period model above. Over time, individuals and businesses adjust their behavior in response to sustained inflationary pressures and currency fluctuations, which are often not fully captured in short-term data. By focusing on a longer horizon, we can better observe the cumulative effects of inflation and exchange rate movements, thereby offering a more comprehensive understanding of how these factors influence economic decisions, particularly in the context of dollarization and inflationary expectations. This approach is crucial for capturing the long-term dynamics that drive adjustments in consumer behavior and monetary policy responses. We present our adjusted version of the initial equation as follows:

$$\lambda = \frac{V_{\Delta 1/CPI} + V_{\Delta 1/CPI,\Delta E/CPI} + 2 * V_{\Delta 1/CPI,\Delta 1/CPI(-1)}}{V_{\Delta 1/CPI} + V_{\Delta E/CPI} + 2 * V_{\Delta 1/CPI,\Delta 1/CPI} + 2 * (V_{\Delta 1/CPI,\Delta 1/CPI(-1)} + V_{\Delta E/CPI,\Delta E/CPI(-1)})} \tag{28}$$

where CPI[6] is a consumer price index in Georgia; $E$ is a nominal exchange rate; USD price in terms of Georgian lari; $E/CPI$ is a bilateral exchange rate adjusted by CPI, similar to $s$ in the original paper; $\Delta$ is a change in respective variable; $V_{ab}$ is a variance-covariance operator (with $a = b$ it will be variance and covariance otherwise); $V_{\Delta 1/CPI,\Delta 1/CPI(-1)}$ is a $\rho_\pi$ in (26) and the term $V_{\Delta E/CPI,\Delta E/CPI(-1)}$ is a similar to the $\rho_s$ in the (26) - representing the serial correlations for price level and exchange rate shocks. We calibrate these parameters ($\rho_\pi$ and $\rho_s$) for Georgia, and took a values of 0.03 and -0.02 respectively for 5-year price changes; However, for 10-year price changes these parameters are 0.0003 and -0.0002 respectively.

In this study, we employ the volatility of inflation and the bilateral USD/GEL

[6] we use $1/CPI$ which reflects the purchasing power of one lari.

exchange rate adjusted by CPI to assess the level of MVP dollarization. While the original paper utilizes the volatility in inflation and the real exchange rate for a single period, our analysis extends over multiple periods, providing a more comprehensive perspective on price levels and dollarization. additionally, we allow for serial correlation between consecutive shocks.

We examine changes in the CPI over 5- and 10-year periods, as well as changes in the exchange rate over corresponding durations. This means that we evaluate changes over these 5- and 10-year periods in CPI and the adjusted exchange rate, which we believe is a more realistic approach calculating the MVP dollarization level in countries similar to Georgia. We compare the results with monthly changes, following Ize and Yeyati's (1998, 2003) MVP model. Our extended version of the original model for longer-term periods provides theoretical support for our empirical estimations.

We used historical data on the volatilities of the exchange rate (adjusted by CPI) and CPI as proxies for expected volatilities, which are not directly observable. To calculate the variance-covariance matrix for MVP, we experimented with various time series lengths. Our findings indicate that the results are more stable with longer time frames.

As a baseline, we assess the original version of this model by examining changes in the real exchange rate and inflation over a single period (i.e., month-on-month).

**Figure 1:** MVP dollarization for Monthly Changes

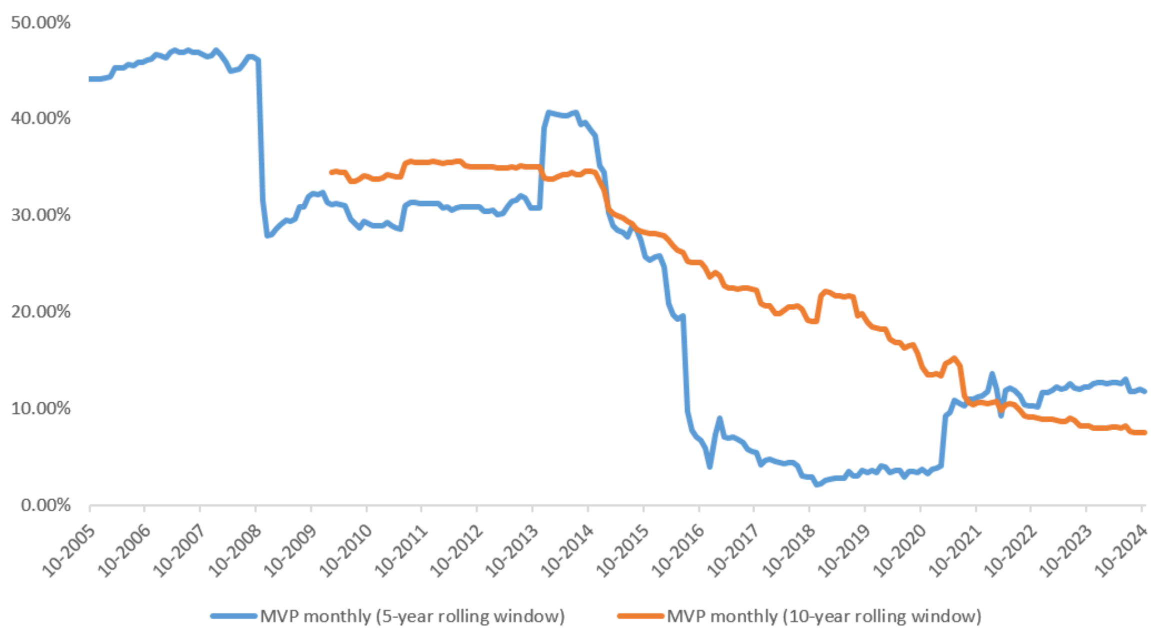
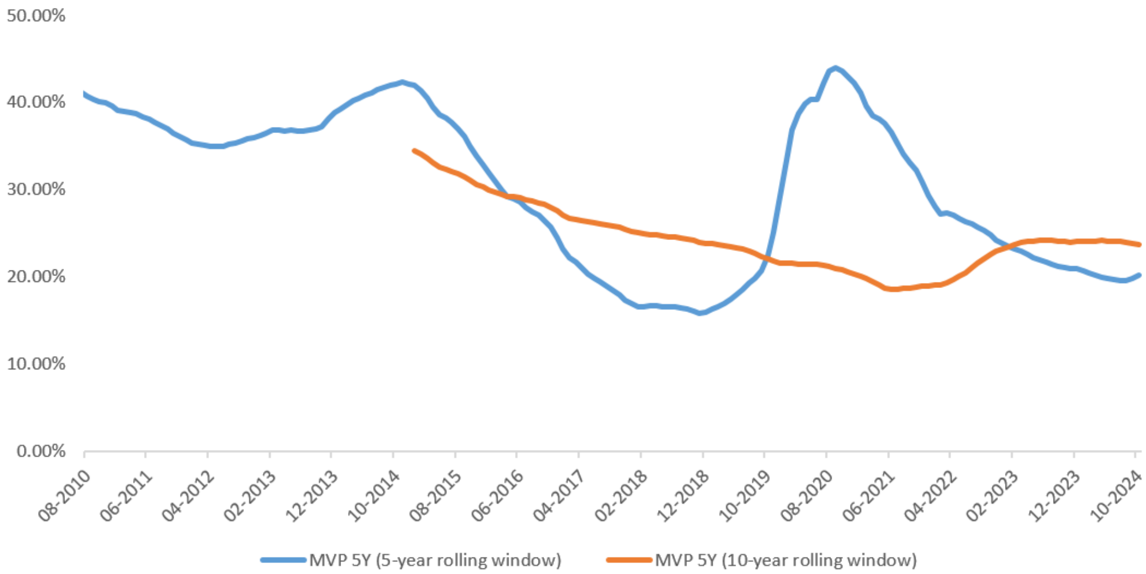


Figure 1 presents the monthly fluctuations of the variables in (28), where variance and covariances are computed using 5-year and 10-year rolling windows. The analysis reveals that, under a 5-year rolling window, the optimal level of dollarization based on the MVP is approximately 12%. In contrast, when employing a 10-year rolling window, this optimal level decreases to around 7.5%. Both estimates appear significantly lower compared to the actual levels of dollarization, highlighting the need to estimate MVP dollarization over a longer time horizon to capture actual dynamics more realistically.

**Figure 2:** MVP dollarization for 5-Year Changes

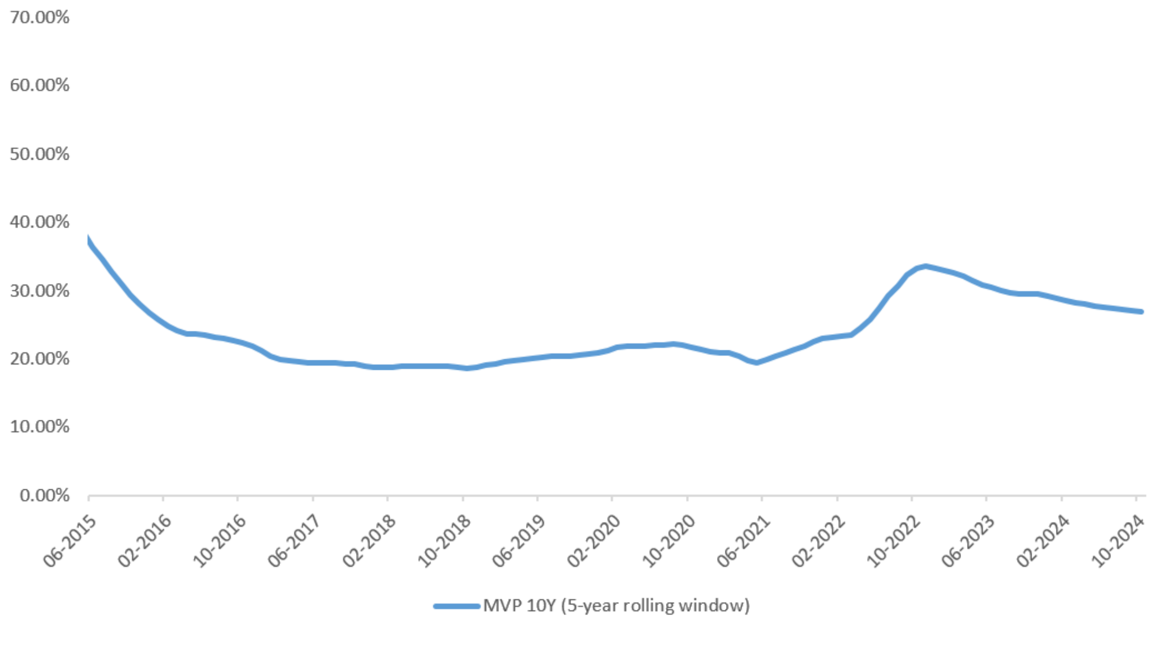



Therefore, to calculate the long-term MVP dollarization, we utilize a 5-year horizon between price levels and the exchange rate (Figure 2). In this approach, both 5-year and 10-year rolling windows are employed to estimate the variances and covariances. The results derived from the longer time horizon suggest that the extended timeframe captures more stable relationships between the variables. Specifically, the 5-year rolling window estimates the optimal dollarization level at approximately 21%, while the 10-year window indicates a higher optimal level, around 24% percent. This difference between monthly changes and 5-year changes underscores the importance of considering longer-term horizons and serial correlations, which may better capture the structural and persistent dynamics that influence dollarization levels over time. Such findings emphasize the utility of extended periods for capturing volatility and covariance patterns, offering a more robust basis for policy recommendations.

In addition to the analysis above, we further explore the 10-year changes in the price level and the exchange rate to evaluate how the extended timeframe affects the optimal level of dollarization in Georgia (Figure 4). The results indicate that employing a longer time horizon captures deeper, more persistent economic trends, leading to a higher MVP dollarization level. Specifically, when using a 5-year rolling window within the 10-year changes, the optimal dollarization level rises to approximately 27%. This increase suggests that the extended period accounts for more structural and cyclical dynamics in the economy, which may not be fully reflected in shorter horizons.

The findings underscore the critical role of time in determining the relationship between inflation, exchange rate volatility, and dollarization. Longer horizons allow for the incorporation of gradual shifts in economic fundamentals, such as changes in policy regimes, external shocks, or shifts in market expectations. These factors contribute to a more accurate estimation of the MVP, aligning it more closely with the actual levels of dollarization observed in practice. Moreover, this analysis suggests that short-term volatilities may not fully capture the underlying risks and behaviors influencing dollarization, thereby reinforcing the importance of long-term perspectives in both empirical research and policy design.

**Figure 3:** MVP dollarization for 10-Year Changes




To sum up, by expanding the original framework, we propose a generalized MVP dollarization equation that integrates serial correlation in inflation and exchange rate shocks. This refinement enhances the model by addressing the commonly observed negative covariance between consecutive real exchange rate shocks, usually unlike inflation shocks, attributable to the mean-reverting behavior of real exchange rates. These dynamic differences play a critical role in shaping the optimal portfolio composition and, consequently, the degree of dollarization.

The interplay between negatively correlated exchange rate shocks and the unpredictable nature of inflation highlights the necessity of customizing de-dollarization strategies to reflect the unique economic conditions of each country. Our results underscore the importance of policy frameworks that account for both the short-term rigidities in price dynamics and the long-term structural trends in exchange rate and inflation behavior. Such an approach is essential for developing strategies to better fight financial dollarization and promote a greater reliance on local currency.

# 6 Conclusion and Policy Recommendations

This study introduces a modified version of the original Minimum Variance Portfolio (MVP) model to evaluate dollarization dynamics in long-term horizon. The findings indicate that this modified model, which incorporates price level and exchange rate volatilities together with the serial correlations, better approximates actual levels of dollarization in the country. This underscores the importance of integrating CPI and exchange rate (adjusted by CPI) dynamics into the design of policies aimed at regulating dollarization.

The analysis reveals that the optimal dollarization level, as estimated by the original MVP using month on month data, varies depending on the time horizon in rolling windows. Using a five-year rolling window, the optimal level is around 12%, whereas it decreases to 7.5% over a 10-year horizon. Extended horizons in price level and exchange rate (adjusted by CPI) yield estimates of 21–27%, closer to actual dollarization dynamics, suggesting that longer-term perspectives provide more stable and reliable estimates. These findings highlight the importance of structural trends in developing effective policy recommendations to manage dollarization risks.

In cases where actual financial dollarization exceeds MVP levels, the study emphasizes the difficulty of escaping from a high-dollarization equilibrium. This situation often arises due to the positive correlation between default risk and currency devaluation, reinforcing the comparative advantage of the dollar. Policy measures such as tighter regulations on dollar-denominated loans, particularly for sectors without dollar-based revenues, and penalties for unhedged dollar exposure can improve financial resilience and enable greater exchange rate flexibility. These prudential measures can transition the economy from a "fear of floating" equilibrium to one characterized by reduced dollarization and stable floating exchange rates.

On the other hand, when financial dollarization is below MVP levels, the emphasis shifts to strengthening the nominal stability and credibility of monetary policy. This includes gradual enhancements to the central bank's autonomy, anti-inflationary measures, and prudential policies such as imposing limits on dollar-denominated debt, differentiated reserve requirements, and liquidity management strategies to mitigate exchange rate risks. While this has always been emphasized in the literature, by using one-period models, the role of monetary policy credibility was not fully appreciated.

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# A Technical Derivations

We start with the differentiation of Equation (26) with respect to $n$ to derive the following condition:

$$\frac{\partial \lambda_n^*}{\partial n} = \frac{B(v_\pi + \rho_{\pi s} + 2\rho_\pi) - A(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s)}{B^2},$$

where

$$A \equiv n\rho_{\pi s} + nv_\pi + 2(n-1)\rho_\pi, \quad B \equiv 2n\rho_{\pi s} + nv_\pi + nv_s + 2(n-1)(\rho_\pi + \rho_s).$$

Substituting $A$ and $B$ into the derivative expression implies:

$$\frac{\partial \lambda_n^*}{\partial n} = \frac{[n(v_\pi+v_s+2\rho_{\pi s}+2\rho_\pi+2\rho_s)-2\rho_\pi-2\rho_s](v_\pi+\rho_{\pi s}+2\rho_\pi)-[n(v_\pi+\rho_{\pi s}+2\rho_\pi)-2\rho_\pi](v_\pi+v_s+2\rho_{\pi s}+2\rho_\pi+2\rho_s)}{[n(v_\pi+v_s+2\rho_{\pi s}+2\rho_\pi+2\rho_s)-2\rho_\pi-2\rho_s]^2}.$$

Expanding the numerator:

$$\begin{gathered}[n(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s) - 2\rho_\pi - 2\rho_s]\,(v_\pi + \rho_{\pi s} + 2\rho_\pi) - \\ [n(v_\pi + \rho_{\pi s} + 2\rho_\pi) - 2\rho_\pi]\,(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s) = \\ n(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s)(v_\pi + \rho_{\pi s} + 2\rho_\pi) - (2\rho_\pi + 2\rho_s)(v_\pi + \rho_{\pi s} + 2\rho_\pi) - n(v_\pi + \\ \rho_{\pi s} + 2\rho_\pi)(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s) + 2\rho_\pi(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s)\end{gathered}$$

Simplify each term step by step and combining terms involving $n$ gives:

$$\begin{gathered}[(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s)(v_\pi + \rho_{\pi s} + 2\rho_\pi) - (v_\pi + \rho_{\pi s} + 2\rho_\pi)(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s)] + \\ 2\rho_\pi(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s) - 2\rho_\pi(v_\pi + \rho_{\pi s} + 2\rho_\pi) - 2\rho_s(v_\pi + \rho_{\pi s} + 2\rho_\pi)\end{gathered}$$

Notice that the term in the square brackets simplifies to zero. Thus, we are left with:

$$2\rho_\pi(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s) - 2\rho_\pi(v_\pi + \rho_{\pi s} + 2\rho_\pi) - 2\rho_s(v_\pi + \rho_{\pi s} + 2\rho_\pi)$$

which further reduces to:

$$\begin{gathered}2\rho_\pi v_\pi + 2\rho_\pi v_s + 4\rho_\pi\rho_{\pi s} + 4(\rho_\pi)^2 + 4\rho_\pi\rho_s - 2\rho_\pi v_\pi - 2\rho_\pi\rho_{\pi s} - 4(\rho_\pi)^2 - 2\rho_s v_\pi - 2\rho_s\rho_{\pi s} - 4\rho_\pi\rho_s = \\ 2(\rho_\pi v_s + \rho_\pi\rho_{\pi s} - \rho_s v_\pi - \rho_s\rho_{\pi s})\end{gathered}$$

Finally, substitute back into the original expression for the derivative:

$$\frac{\partial \lambda_n^*}{\partial n} = \frac{2(\rho_\pi v_s + \rho_\pi \rho_{\pi s} - \rho_s v_\pi - \rho_s \rho_{\pi s})}{(n(v_\pi + v_s + 2\rho_{\pi s} + 2\rho_\pi + 2\rho_s) - 2\rho_\pi - 2\rho_s)^2}$$

The time horizon cannot affect $\lambda_n^*$ iff the derivative is zero, from which follows that necessary condition for the constant dollarization is:

$$\rho_\pi v_s + \rho_\pi \rho_{\pi s} - \rho_s v_\pi - \rho_s \rho_{\pi s} = 0$$

or

$$\rho_\pi v_s + \rho_\pi \rho_{\pi s} = \rho_s v_\pi + \rho_s \rho_{\pi s}$$

That leads to:

$$\rho_\pi = \rho_s \frac{v_\pi + \rho_{\pi s}}{v_s + \rho_{\pi s}} \tag{29}$$

The trivial case when this condition is satisfied would be $\rho_\pi = \rho_s = 0$ that states no serial correlation in inflation and real exchange rate shocks. Putting the trivial case aside, the condition (29) implies $\frac{\rho_\pi}{\rho_s} = \frac{v_\pi + \rho_{\pi s}}{v_s + \rho_{\pi s}}$. Also, we can state that $\lambda_n^*$ is increasing function of $n$ when $\frac{\rho_\pi}{\rho_s} > \frac{v_\pi + \rho_{\pi s}}{v_s + \rho_{\pi s}}$, and vice-versa.

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